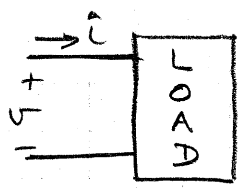
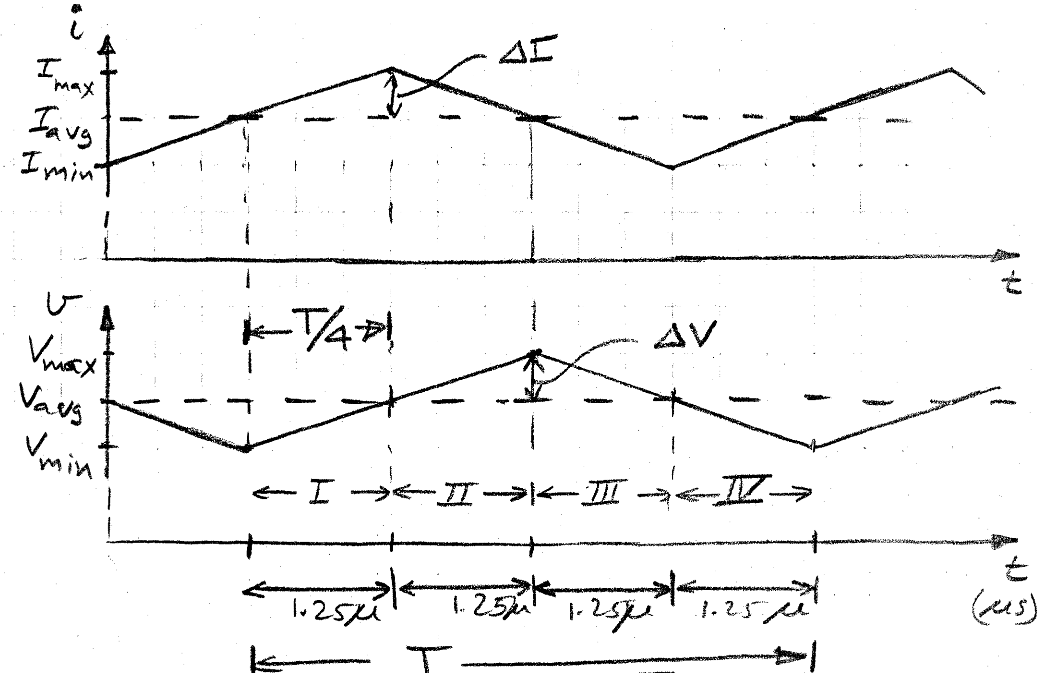
Problem 3-9





$p(t) = v(t) \cdot i(t)$ and $P_{avg} = \frac{1}{T}\int_0^T v \cdot i \cdot dt$.

The slopes (rate of change) of v and i are as follows:

$$(\text{slope})_v = \frac{V_{max} - V_{min}}{T/2} \quad V/\mu s,$$

$$(\text{slope})_i = \frac{I_{max} - I_{min}}{T/2} \quad A/\mu s$$

Considering the 4 segments shown above,

$$\begin{aligned} P_{avg} = \frac{1}{T}\Bigg[& \left(\int_0^{T/4} [V_{min} + \text{slope}_v \cdot t] \cdot [I_{avg} + \text{slope}_i \cdot t] \cdot dt\right) \\ & + \left(\int_0^{T/4} [V_{avg} + \text{slope}_v \cdot t] \cdot [I_{max} - \text{slope}_i \cdot t] \cdot dt\right) \\ & + \left(\int_0^{T/4} [V_{max} - \text{slope}_v \cdot t] \cdot [I_{avg} - \text{slope}_i \cdot t] \cdot dt\right) \\ & + \left(\int_0^{T/4} [V_{avg} - \text{slope}_v \cdot t] \cdot [I_{min} + \text{slope}_i \cdot t] \cdot dt\right)\Bigg] \end{aligned}$$

$$= \frac{1}{T}\Big[\left(V_{min}\cdot I_{avg} + V_{avg}\cdot I_{max} + V_{max}\cdot I_{avg} + V_{avg}\cdot I_{min}\right)\times\frac{T}{4}$$

$$+\left(\cancel{I_{avg}\cdot slope_v} + I_{max}\cdot slope_v - \cancel{I_{avg}\cdot slope_v} - I_{min}\cdot slope_v\right)\times\frac{(T/4)^2}{2}$$

$$+\left(V_{min}\cdot slope_i - \cancel{V_{avg}\cdot slope_i} - V_{max}\cdot slope_i + \cancel{V_{avg}\cdot slope_i}\right)\times\frac{(T/4)^2}{2}$$

$$+\left(\cancel{slope_v\cdot slope_i} - \cancel{slope_v\cdot slope_i} + \cancel{slope_v\cdot slope_i} - \cancel{slope_v\cdot slope_i}\right)\frac{(T/4)^3}{3}\Big]$$

$$= \frac{1}{4}\left[\left(V_{min}+V_{max}\right)\cdot I_{avg} + \left(I_{min}+I_{max}\right)\cdot V_{avg}\right]$$

$$+\frac{T}{32}\left[slope_v\left(I_{max}-I_{min}\right) - slope_i\left(V_{max}-V_{min}\right)\right]$$

(Noting that $V_{min}+V_{max} = 2\,V_{avg}$ and

$I_{min}+I_{max} = 2\,I_{avg}$)

$$= V_{avg}\cdot I_{avg}$$

$$+\frac{1}{16}\Big[\left(V_{max}-V_{min}\right)\left(I_{max}-I_{min}\right)$$

$$-\left(I_{max}-I_{min}\right)\cdot\left(V_{max}-V_{min}\right)\Big]$$

$$= V_{avg}\cdot I_{avg}$$

<u>Another Approach:</u> In terms of fourier components

$$i(t) = I_{avg} + \frac{8}{\pi^2}\Delta I \sum_{h=1,3,5} \frac{(-1)^{\frac{h-1}{2}}}{h^2}\sin h\omega t$$

and $$v(t) = V_{avg} + \frac{8}{\pi^2}\Delta V \sum_{h=1,3,5} \frac{(-1)^{(h-1)/2}}{h^2}\sin h\left(\omega t - \frac{\pi}{2}\right)$$

Since $v(t)$ lags $i(t)$ by $T/4$, or by $90°$ at the fundamental frequency.

$p(t) = v(t) \cdot i(t)$ which contains products of sine terms.

$$\sin h\theta \cdot \sin(h\theta - h90^\circ) = \frac{1}{2}\left[\cos(\cancel{h\theta - h\theta} + h90^\circ) - \cos(2h\theta - h90^\circ)\right]$$

$$= -\frac{1}{2}\cos(2h\theta - h90^\circ) \quad \text{for } h = 1, 3, 5, \ldots$$

The integral of the above term over one time period is zero.

Therefore,

$$P_{avg} = \frac{1}{T}\int_0^T v(t) \cdot i(t) \cdot dt = V_{avg} \cdot I_{avg}.$$