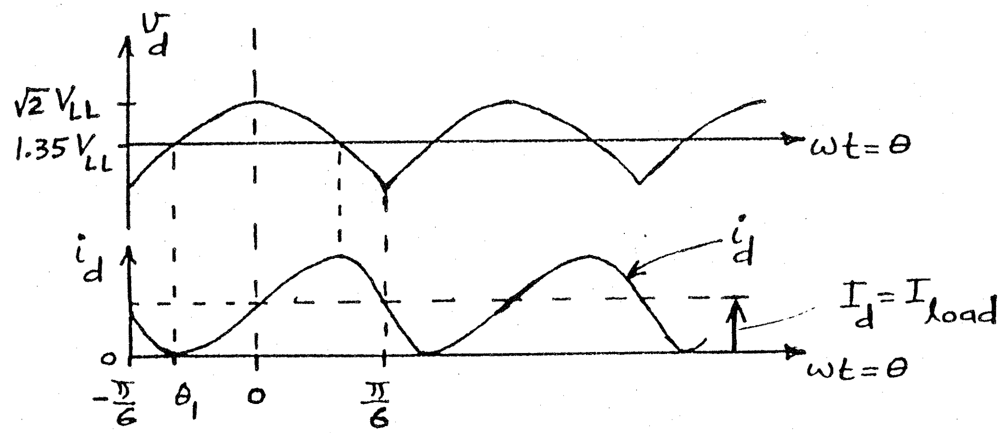
Problem 5-26



Derive equation for $i_d(\theta)$:

$V_d = 1.35\ V_{LL}$ (continuous conduction)

$$i_d(\theta) = \frac{1}{\omega L_d} \int (\sqrt{2}\ V_{LL} \cos\theta - 1.35 V_{LL})\, d\theta \qquad -\frac{\pi}{6} < \theta < \frac{\pi}{6}$$

$$i_d(\theta) = \frac{V_{LL}}{\omega L_d} (\sqrt{2}\ \sin\theta - 1.35\theta + K)$$, K is a constant of integration.

Calculate K:

At $\theta = \theta_1$, $i_d = 0$. To calculate (θ_1) defined in the figure above

$\sqrt{2}\ V_{LL} \cos(\theta_1) = 1.35\ V_{LL}$ or $\theta_1 = -0.302$ rad

Using $\theta = \theta_1$ in the expression for $i_d(\theta)$

$$i_d(\theta_1) = \frac{V_{LL}}{\omega L_d}(\sqrt{2}\ \sin(-0.302) - 1.35\ (-0.302) + K) = 0 \qquad \therefore\ K = 0.0129$$

and,

$$i_d(\theta) = \frac{V_{LL}}{\omega L_d}(\sqrt{2}\ \sin\theta - 1.35\ \theta + 0.0129)$$

$$-\frac{\pi}{6} < \theta < \frac{\pi}{6}$$

Calculate I_d:

$$I_d = \int_{-\pi/6}^{\pi/6} i_d(\theta)d\theta \bigg/ \frac{\pi}{3}$$

$$= \frac{3}{\pi}\frac{V_{LL}}{\omega L_d}\int_{-\pi/6}^{\pi/6}(\sqrt{2}\ \sin\theta - 1.35\theta + 0.0129)d\theta$$

$$= \frac{3V_{LL}}{\pi\omega L_d}\left[-\sqrt{2}\ \cos\theta - \frac{1.35}{2}\ \theta^2 + 0.0129\theta\right]_{-\pi/6}^{\pi/6}$$

$$= \frac{3V_{LL}}{\pi\omega L_d}\ (0.0129(\pi/6 - (-\pi/6))) = \frac{3V_{LL}}{\pi\omega L_d}\ (0.0129\ \frac{\pi}{3})$$

$$I_d = 0.0129\ \frac{V_{LL}}{\omega L_d}$$

Therefore,

$$L_{d,min} \simeq \frac{0.013}{\omega I_d}\ V_{LL}$$