Solutions Manual

Power Electronics: Converters, Applications and Design

SECOND EDITION

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Problem 1-1	Change: (per-phase current of 10 A) to (line current of 10 A). Note: 230 V is the rms value.
Problem 3-8	In Fig. P3-8, v_i waveform pulsates between 0 V and 15 V.
Problem 3-13	(b) 0.8 (lagging)
Problem 6-15	Change: ($V_d = V_{do}$) to ($V_d = 0.5\ V_{do}$).
Problem 7-18	Note: Calculate in terms of given V_d and I_o. Assume that $i_o(t) = I_o$.
Problem 7-22	Assume that R_a is negligible.
Problem 8-3	Assume a bipolar voltage switching.

PSPICE SIMULATIONS FOR TEACHING AND DESIGN

As a companion to this book, a large number of computer simulations are available directly from Minnesota Power Electronics Research and Education, P.O. Box 14503, Minneapolis, MN 55414 (Phone/Fax: 612-646-1447) to aid in teaching and in the design of power electronic systems. The simulation package comes complete with a diskette with 76 simulations of power electronic converters and systems using the classroom (evaluation) version of PSpice for IBM-PC-compatible computers, a 261-page detailed manual that describes each simulation and a number of associated exercises for home assignments and self-learning, a 5-page instruction set to illustrate PSpice usage using these simulations as examples, and two high-density diskettes containing a copy of the classroom (evaluation) version of PSpice. This package (for a cost of $395 plus a postage of $4 within North America and $25 outside) comes with a site license, which allows it to be copied for use at a single site within a company or at an educational institution in regular courses given to students for academic credits.

Acknowledgment

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Chapter 1

Problem 1-1

Output: $V_{LL} = 200\,V$ (3-ϕ, rms) at 52 Hz,

$I_o = 10\,A$ at 0.8 lagging power factor

energy efficiency $\eta = 95\%$

Input: $V_{in} = 230\,V$ (1-ϕ, rms) at 60 Hz,

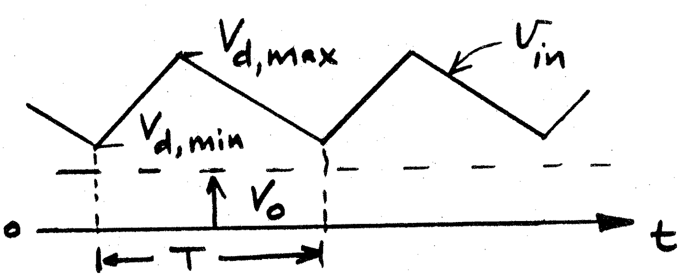
input power factor = 1.0

$$P_{out} = \sqrt{3}\, V_{LL} I_o (\text{power factor}) = 2.77\ kW$$

$$P_{in} = \frac{P_{out}}{\eta} = 2.916\ kW$$

$$I_{in} = \frac{P_{in}}{V_{in}\,(\text{power factor})_{input}} = 12.68\ A\ (\text{rms})$$

Problem 1-2



$V_o = 15\,V$, I_o = Constant

$V_{d,min} = 20\,V$, $V_{d,max} = 30\,V$

$$P_o = V_o I_o = 15\, I_o$$

$$p_{in}(t) = V_{in} I_o \ ; \quad P_{in}(avg) = \frac{I_o \int_0^T V_{in}(t) \cdot dt}{T}$$

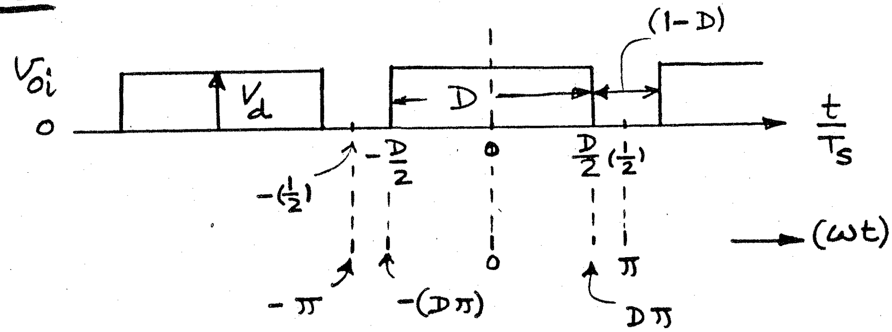
Graphically: $$P_{in} = \frac{I_o}{T}\left[V_{d,min}\, T + \frac{1}{2} T (V_{d,max} - V_{d,min}) \right]$$

$$= \frac{1}{2} I_o \left[V_{d,min} + V_{d,max} \right]$$

$$= 25\, I_o$$

$\therefore$ efficiency $\eta = \frac{P_o}{P_{in}} = \frac{15}{25} = 0.6$.

Problem 1-3



With the choice of origin as above, V_{oi} waveform has even symmetry.

$$\therefore \; b_h = 0 \quad \text{and} \quad a_h = \frac{2}{\pi}\int_0^{\pi} v_{oi}(t)\cos(h\omega t)\, d(\omega t)$$

$$h = 0 \text{ to } \infty$$

$$\therefore \; a_h = \frac{2V_d}{\pi}\int_0^{D\pi} \cos(h\omega t)\cdot d(\omega t) = \frac{2V_d}{h\pi}\sin(Dh\pi) \quad h = 1 \to \infty$$

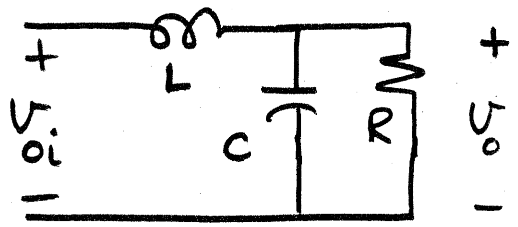
$$\left(V_{oi}\right)_{average} = \frac{1}{2}a_0 = \frac{1}{\pi}\int_0^{D\pi} V_d \cdot d(\omega t) = DV_d \quad \text{(average value)}$$

$$= 15\,V \quad \text{at} \quad D = 0.75$$

h	$(\hat{V}_{oi})_h$ ← peak value
1	9.0 V
2	6.37 V
3	3.0 V
4	0.0
5	1.8 V
⋮	

Problem 1-4

$V_d = 20V$, $D = 0.75$, $f_s = 300$ kHz, $P_o = 240W$.



$L = 1.3\mu H$,
$C = 50\mu F$.
$V_{oi} = 15V = V_o(\text{avg})$.

Assuming the ripple in the output voltage to be negligible,

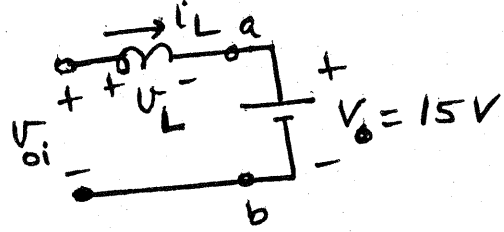
$$R = \frac{V_o^2}{P_o} = 0.9375\Omega .$$

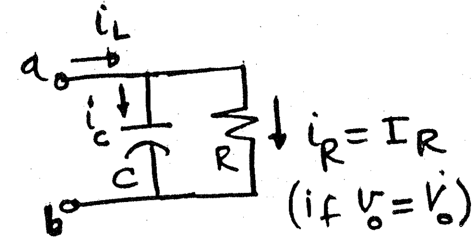
$$\therefore \frac{V_o(s)}{V_{oi}(s)} = \frac{(R) \| \left(\frac{1}{Cs}\right)}{sL + \left[R \| \left(\frac{1}{Cs}\right)\right]} = \frac{R}{(RLC)s^2 + Ls + R}$$

h	$20\log_{10}\left\lvert\frac{V_o(s)}{V_{oi}(s)}\right\rvert$
1	−47.4 db
2	−59.4
3	−66.4
4	−71.4
5	−75.3
⋮	

$s = j\omega_h$
$= j(2\pi \times h \times f_s)$

Problem 1-5





$T_s = \frac{1}{f_s} = 3.33\,\mu s$

$DT_s = 0.75 \times T_s = 2.5\,\mu s$

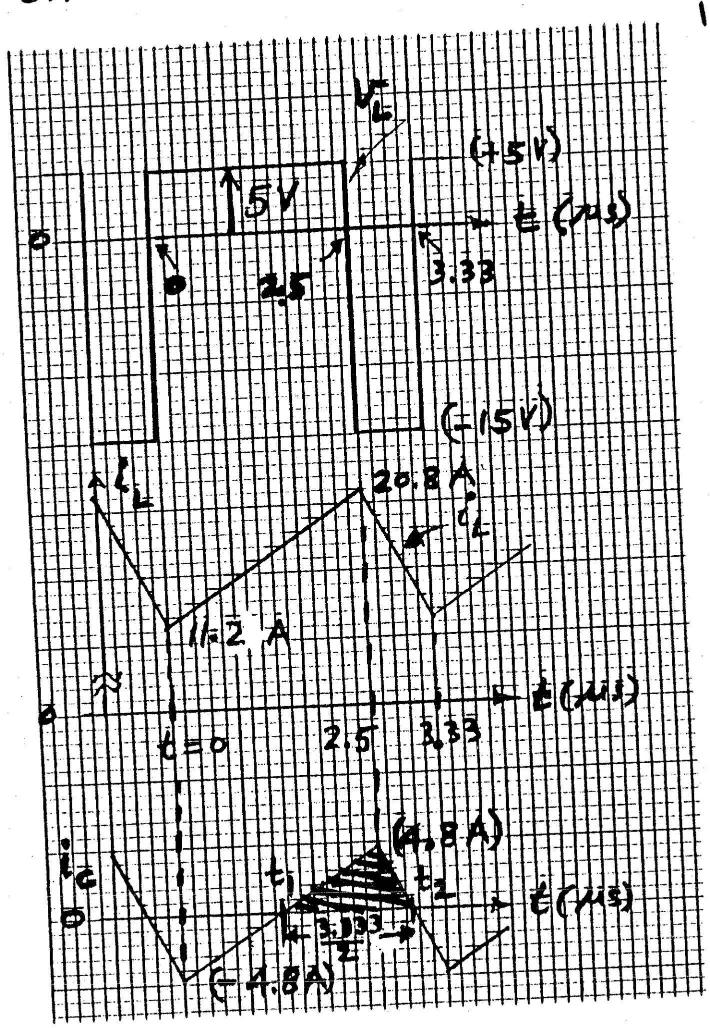
$I_R = \frac{240\,W}{15\,V} = 16\,A$

$i_L = i_c + i_R$

$\therefore\ I_L = I_R$ (Since $I_c = 0$) ← average

$= 16\,A$

During $0 < t < 2.5\,\mu s$ when the switch is ON, $v_L = V_d - V_o = 20 - 15 = 5V$. The waveforms are shown below. The peak-to-peak ripple $(\Delta i_L)_{pp}$ in the inductor current i_L can be calculated during this interval.



$$(\Delta i_L)_{pp} = \frac{1}{L}\int_0^{2.5\mu s} v_L \cdot dt$$

$$= \frac{5 \times 2.5}{1.3}$$

$$\simeq 9.6\,A$$

$\&\ \frac{(\Delta i_L)_{pp}}{2} = 4.8A$

$\therefore\ I_{L,min} = I_L - 4.8$

$= 11.2\,A$

and

$I_{L,max} = I_L + 4.8$

$= 20.8\,A$

$$i_c(t) = i_L(t) - I_R \quad [\text{assuming } i_R \simeq I_R]$$

The i_c waveform is shown. Due to i_c, the capacitor voltage v_c will be at its minimum at t_1 and its maximum at t_2, where $(t_2 - t_1) = \frac{T_s}{2} = \frac{3.33}{2}\,\mu s$.

$$\therefore (\Delta V_c)_{pp} = \frac{1}{C}\int_{t_1}^{t_2} i_c \cdot dt = \frac{1}{C}\left[\begin{array}{l}\text{Area shown}\\ \text{under } i_c\\ \text{waveform}\end{array}\right]$$

$$= \frac{1}{50} \cdot \left[\frac{4.8 \times \frac{3.33}{2}}{2}\right]$$

$$\simeq 80 \text{ mV}$$

Problem 1-6

From Problem 1-3, $(\hat{V}_{oi})_1 = 9\text{V}$

From Problem 1-4,
$$20 \log_{10}\left|\frac{V_o(s)}{V_{oi}(s)}\right| = -47.4$$

$$\therefore (\hat{V}_o)_1 = 0.00427\, (\hat{V}_{oi})_1$$
$$= 38.4 \text{ mV}$$

$$\therefore (\Delta V_c)_{pp} = 2 \times (\hat{V}_o)_1 = 76.8 \text{ mV}$$

The above value is fairly close to the more accurate estimation of 80mV. [An error of 4%.]

Problem 1-7

100 billion kWhr saved in one year:

(a) $\therefore \quad P = \dfrac{100 \times 10^9 \times 10^{-3} \text{ MW-hr}}{(24 \times 365) \text{ hr}} = 11,415 \text{ MW}$

Approximately equal to the output of

$11\frac{1}{2}$ generating plants (1000 MW) each.

(b) Savings $= 0.1 \times 100 \times 10^9$

$= 10 \times 10^9$ \$

Chapter 2

Problem 2-1

$$P_s = \frac{1}{2} V_d I_o f_s[t_c(on) + t_c(off)] ; \qquad (2\text{-}6)$$

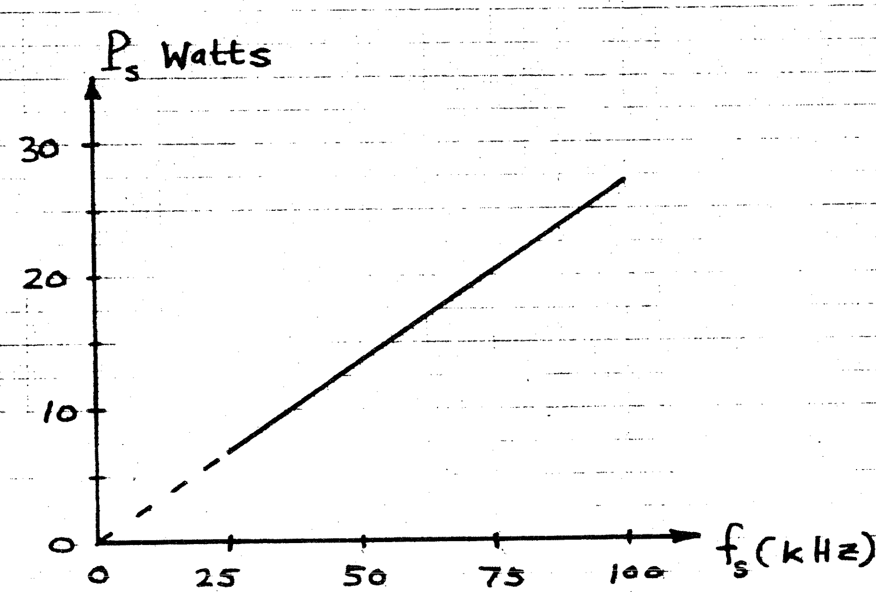
$$t_{c(on)} = t_{ri} + t_{fv} ; \qquad (2\text{-}1)$$

$$t_{c(off)} = t_{rv} + t_{fi} ; \qquad (2\text{-}4)$$

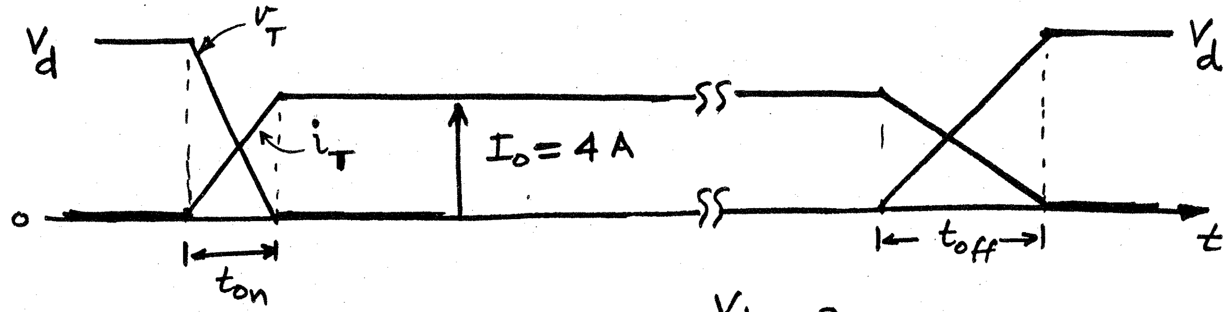
$$P_s = \frac{1}{2}(300V)(4A)\, f_s\, [100\text{ ns} + 50\text{ ns} + 100\text{ ns} + 200\text{ ns}]$$

$P_s = 0.27\text{x}10^{-3} f_s$ Watts

P_s varies linearly from 6.75 W at 25 kHz to 27 W at 100 kHz.



Problem 2-2



$$I_o = \frac{V_d}{R} = \frac{300}{75} = 4.0\ A$$

$$t_{on} = t_{ri} + t_{fv} = 150\ ns$$

$$t_{off} = t_{rr} + t_{fi} = 300\ ns$$

$0 < t < t_{on}$

$$i_T = I_o \frac{t}{t_{on}}$$

$$v_T = V_d\left(1 - \frac{t}{t_{on}}\right)$$

$$\therefore (p_T)_{on} = V_d \frac{I_o}{t_{on}}\left(1 - \frac{t}{t_{on}}\right)t \qquad \therefore (W_T)_{on} = \int_0^{t_{on}} (p_T)_{on}\, dt$$

$$\therefore (W_T)_{on} = \frac{V_d I_o}{t_{on}} \int_0^{t_{on}} \left(1 - \frac{t}{t_{on}}\right) t \cdot dt$$

$$= \frac{V_d I_o t_{on}}{6}$$

Similarly,

$$(W_T)_{off} = \frac{V_d I_o t_{off}}{6}$$

$$\therefore (W_T)_{switching} = (W_T)_{on} + (W_T)_{off} = \frac{V_d I_o}{6}[t_{on} + t_{off}]$$

$$P_s = f_s (W_T)_{switching} = \frac{V_d I_o}{6} f_s [t_{on} + t_{off}]$$

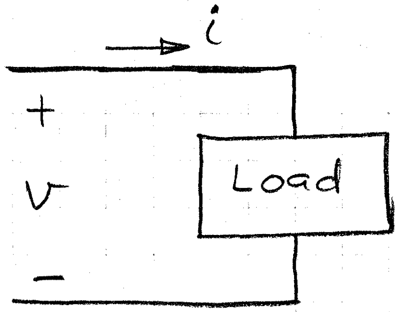
avg. switching power loss

$$\therefore P_s = \frac{300 \times 4}{6} \times 100 \times 10^3 [150 + 300] \times 10^{-9}\ W = 9\ \text{Watts}$$

[at 100 kHz]

In this circuit, P_s is 1/3 of that in Problem 2-1.

Problem 3-1



$\bar{V} = V\angle 0^\circ$

$\bar{I} = I\angle -\phi^\circ$

$v = \sqrt{2}\, V \cos \omega t \quad , \quad i = \sqrt{2}\, I \cos(\omega t - \phi)$

$$p(t) = v(t) \cdot i(t) = 2\, V I \cos \omega t \cdot \cos(\omega t - \phi)$$

$$= VI\left[\cos\phi + \cos(2\omega t - \phi)\right]$$

$$= VI \cos\phi + VI \cos(2\omega t - \phi)$$

$$= VI \cos\phi + VI \cos\phi \cdot \cos 2\omega t + VI \sin\phi \cdot \sin 2\omega t$$

$$= P + P \cos 2\omega t + Q \sin 2\omega t$$

where, $P = VI \cos\phi$ and,

$Q = VI \sin\phi$

Problem 3-2

(a) $v = \sqrt{2}\, 120 \cos \omega t \quad , \quad i = \sqrt{2} \cos(\omega t - 30^\circ)$

$\phi = 30^\circ$

$p(t) = v(t) \cdot i(t)$

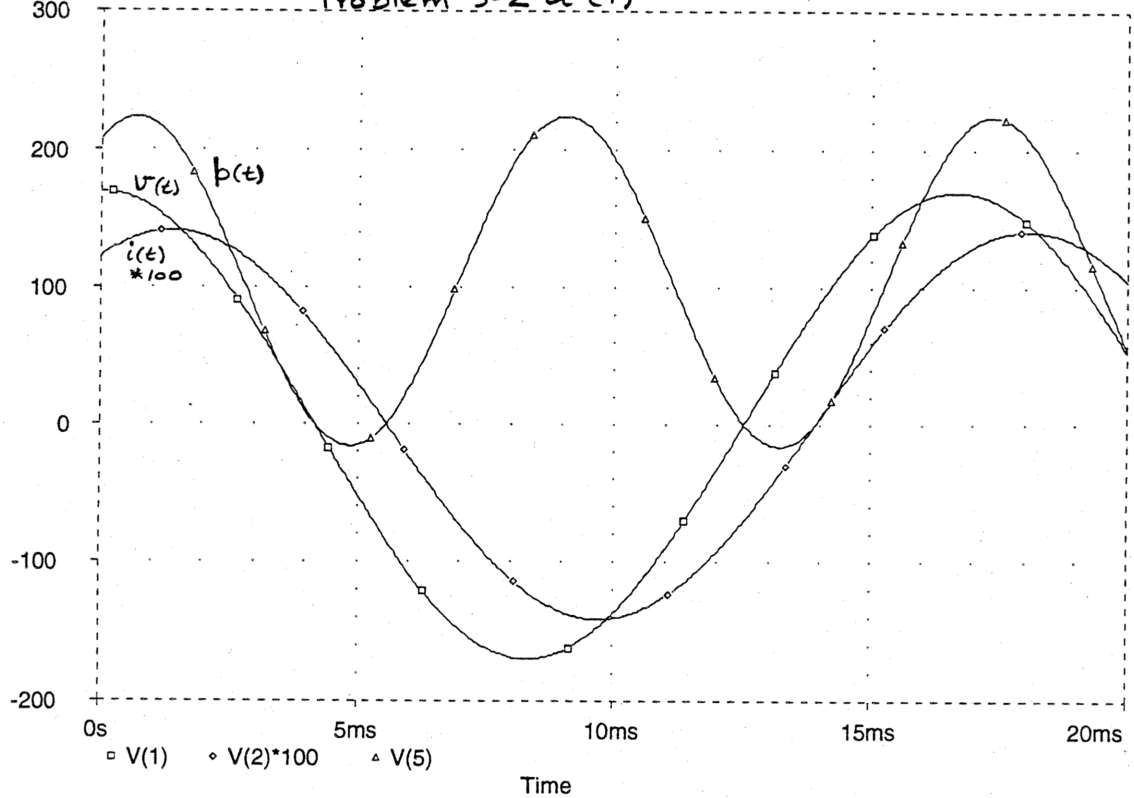
$i_p = (\sqrt{2}\, I \cos\phi) \cos \omega t$ Eq 3-12

$i_q = (\sqrt{2}\, I \sin\phi) \sin \omega t$ Eq 3-13

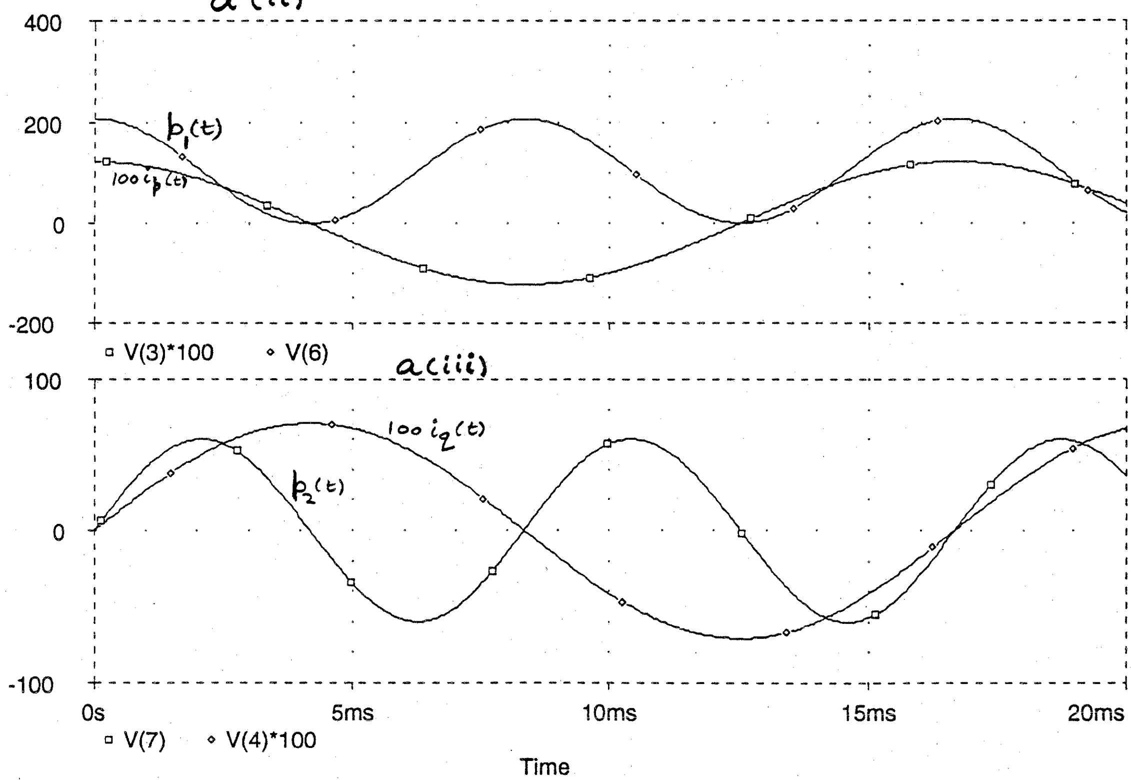
$p_1 = v \cdot i_p$

$p_2 = v \cdot i_q$

Problem 3-2 a (i)



a (ii)



(b) $P = VI\cos\phi = 120 \times 1 \times \cos 30^\circ = 103.92\ W$

(c) $Q = VI \sin\phi$ Eq 3-14

$= 120 \times 1 \times \sin 30^\circ$

$= 60\ VA$

Also from peak of $p_2(t)$ in part a(iii).

(d) $PF = \cos\phi = 0.866$ (lagging)

The load is inductive and it draws positive vars from the source.

Problem 3-3

F_o = Avg value, F_h = rms value

(a) $F_o = 0$, $F_h = \frac{1}{\sqrt{2}} \frac{4}{\pi h} A$ $h = 1, 3, 5, \ldots$ (odd)

(b) $F_o = 0$ $F_h = \frac{4A}{\sqrt{2}\pi h} \cos\left(h\frac{u}{2}\right)$ $h = 1, 3, 5, \ldots$ (odd)

(c) $F_o = 0$ $F_h = \frac{2A(2\pi)}{\sqrt{2}\pi^2 \frac{u}{2} h^2} \sin\left(h\frac{u}{2}\right)$ $h = 1, 3, 5, \ldots$ (odd)

(d) $F_o = 0$ $F_h = \frac{2A(2\pi)}{\sqrt{2}u_2\pi^2 h^2}\left[\sin h(u_1+u_2) - \sin\omega h u_1\right]$ $h = 1, 3, 5, \ldots$ odd

(e) $F_o = 0$ $F_h = \frac{8}{\sqrt{2}\pi^2} \frac{A}{h^2}$ $h = 1, 3, 5, \ldots$ odd

(f) $F_o = \frac{2A}{\pi}$, $F_h = \frac{4A}{\sqrt{2}\pi(h^2-1)}$ $h = 2, 4, 6, \ldots$ even

(g) $F_o = DA$, $F_h = \frac{2A}{\sqrt{2}h\pi}\sin(Dh\pi)$ $h = 1, 2, 3, \ldots$

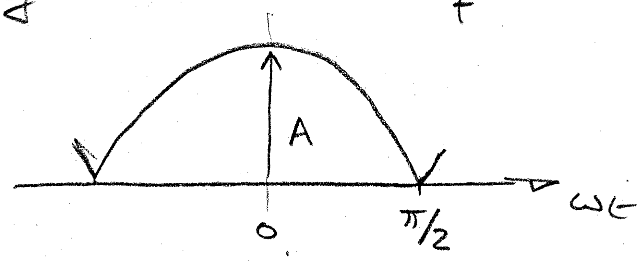
Problem 3-4

Using $A = 10$, this problem is only illustrated for the waveform in Prob 3-3(f).

(a) Using the result of Prob 3-3(f) and Eq 3-28,

$$F_{rms} = 7.071$$

(b) Using the definition of rms given in Eq. 3-5:



$$f = A\cos\omega t \qquad 0 < \omega t < \pi/2$$

$$f^2 = A^2\cos^2\omega t$$ Using the waveform symmetry

$$\therefore \quad F_{rms} = \sqrt{\frac{1}{\frac{\pi}{2}}\int_0^{\pi/2} A^2\cos^2\omega t \cdot d(\omega t)}$$

$$= \sqrt{\frac{2}{\pi}A^2\int_0^{\pi/2}\cos^2\omega t \cdot d(\omega t)}$$

$$= \sqrt{\frac{2}{\pi}A^2\left[\frac{\omega t}{2} + \frac{\sin 2\omega t}{4}\right]\Bigg|_0^{\pi/2}}$$

$$= \sqrt{\frac{2}{\pi}A^2\left[\frac{\pi}{4}\right]} = \sqrt{\frac{A^2}{2}} = 0.7071\,A$$

$$= 7.071$$

Both methods yield the same answer. The same procedure can be used for the rest of the waveforms.

Problem 3-5

(a) This is illustrated for only the waveform in Prob 3-3(a).

(i) $F_1 = \frac{1}{\sqrt{2}} \frac{4}{\pi} A = 0.9\, A$

$F_{rms} = A$

$\therefore \frac{F_1}{F_{rms}} = \frac{1}{\sqrt{2}} \frac{4}{\pi} = 0.9$

(ii) $F_{dis} = \sqrt{F_{rms}^2 - F_1^2} = 0.436\, A$ Using Eq 3-35

$\therefore \frac{F_{dis}}{F_{rms}} = 0.436$

(b) This is illustrated only for the waveform in Prob 3-3(f).

$F_o = \frac{20}{\pi}$ [From Prob 3-3 f solution]

$F_{rms} = 7.071$ [from Prob 3-4 Solution]

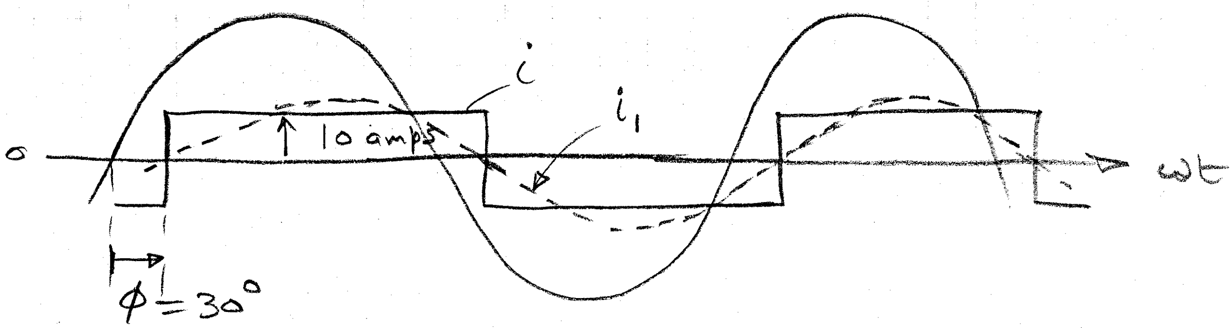
$\therefore \frac{F_o}{F_{rms}} = 0.9$

The same procedure can be used for the rest of the waveforms.

Problem 3-6

$v = \sqrt{2}\, V \sin \omega t$, $V = 120\, V$

This problem is illustrated only for the waveform shown in Prob 3-3 a.



$I_1 = 0.9 \times 10 = 9\, A$ form Prob 3-5

$\phi_1 = \phi$, $\therefore$ $DPF = \cos \phi_1 = \cos \phi = \cos 30^\circ = 0.866$

$P = V I_1 \cos \phi_1 = 120 \times 9 \times 0.866 = 935.28\, W$

$I_{rms} = 10\, A$

$\therefore$ $THD = \dfrac{\sqrt{I^2 - I_1^2}}{I_1} \times 100$

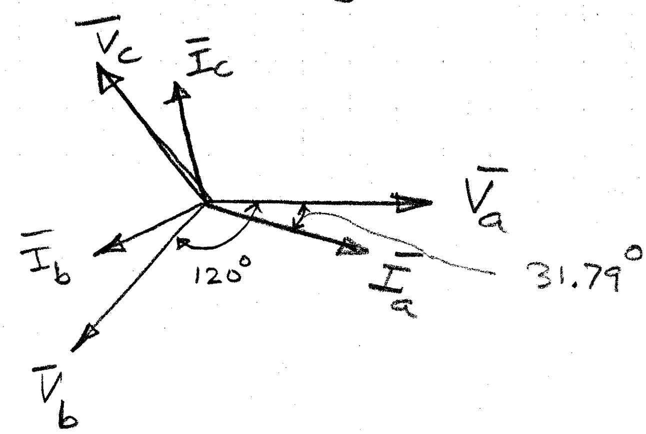
$= 48.43\%$

Problem 3-7

3-ϕ system, $V_{ph} = 120\,V$ (rms).

$\underline{P} = 10\,kW$ at 0.85 PF (lagging).

power factor angle $\phi = \cos^{-1}(0.85) = 31.79^\circ$



$$\underline{P} = 3\,V_{ph}\,I_{ph}\,\cos\phi = 10{,}000$$

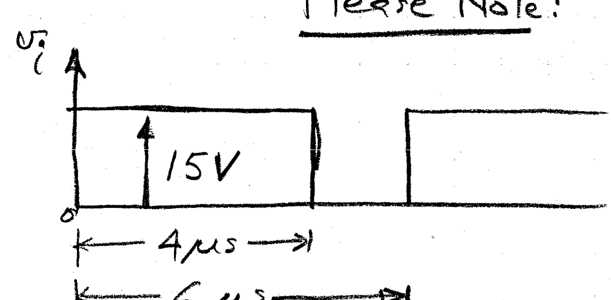
$$\therefore\ I_{ph} = 32.68\,A$$

$$|\bar{Z}| = \frac{V_{ph}}{I_{ph}} = \frac{120}{32.68} = 3.67\,\Omega$$

assuming it to be Y- connected.

Problem 3-8

Please Note: Amplitude of v_i is 15V.



(a)

avg output $V_o = V_i = \frac{15 \times 4}{6} = 10V$

(b) Assuming that $v_o(t) \simeq V_o$

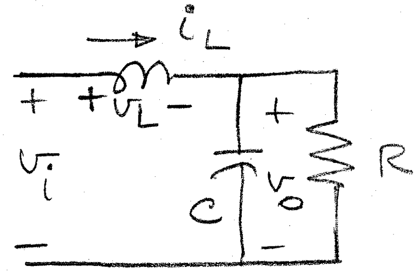
$$i_{Load} \simeq I_{Load} = \frac{V_o}{R}$$

$$P_{Load} = \frac{V_o^2}{R} = 250W \text{ (given)}$$

$$\therefore R = \frac{V_o^2}{250} = 0.4\Omega$$

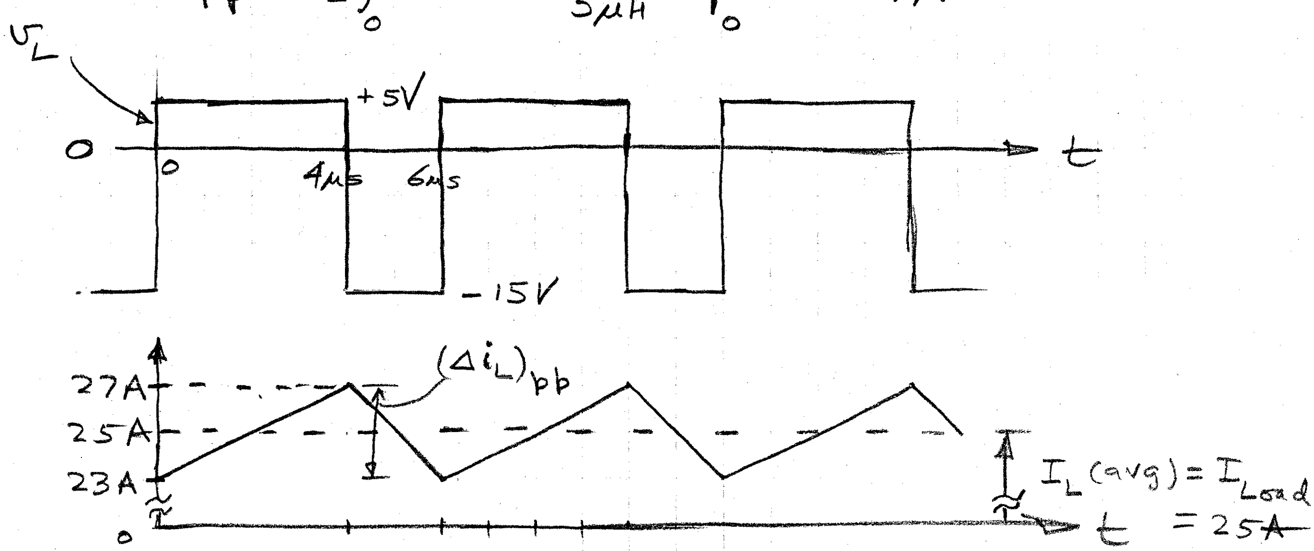
$$\therefore I_{Load} = \frac{V_o}{R} = 25A$$

(C) $v_L = v_i - V_o$

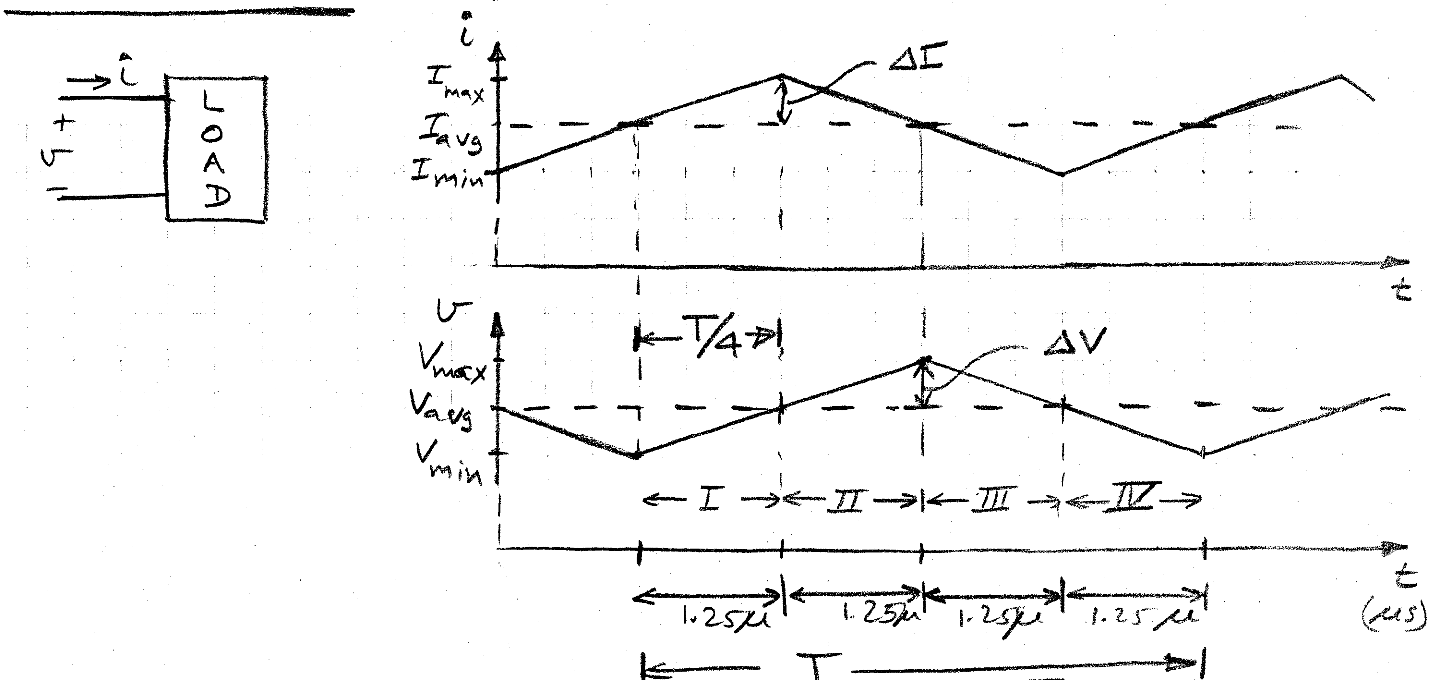


and $i_L = \frac{1}{L}\int v_L \cdot dt$

$$(\Delta i_L)_{pp} = \frac{1}{L}\int_0^{4\mu s} v_L \cdot dt = \frac{5V}{5\mu H} \cdot t\Big|_0^{4\mu s} = 4A$$



Problem 3-9



$$p(t) = v(t) \cdot i(t) \quad \text{and} \quad P_{avg} = \frac{1}{T}\int_0^T v \cdot i \cdot dt \,.$$

The slopes (rate of change) of v and i are as follows:

$$(\text{slope})_v = \frac{V_{max} - V_{min}}{T/2} \quad V/\mu s\,,$$

$$(\text{slope})_i = \frac{I_{max} - I_{min}}{T/2} \quad A/\mu s$$

Considering the 4 segments shown above,

$$\begin{aligned} P_{avg} = \frac{1}{T}\Big[& \Big(\int_0^{T/4} [V_{min} + \text{slope}_v \cdot t] \cdot [I_{avg} + \text{slope}_i \cdot t] \cdot dt\Big) \\ & + \Big(\int_0^{T/4} [V_{avg} + \text{slope}_v \cdot t] \cdot [I_{max} - \text{slope}_i \cdot t] \cdot dt\Big) \\ & + \Big(\int_0^{T/4} [V_{max} - \text{slope}_v \cdot t] \cdot [I_{avg} - \text{slope}_i \cdot t] \cdot dt\Big) \\ & + \Big(\int_0^{T/4} [V_{avg} - \text{slope}_v \cdot t] \cdot [I_{min} + \text{slope}_i \cdot t] \cdot dt\Big)\Big] \end{aligned}$$

$$= \frac{1}{T}\Big[\left(V_{min}\cdot I_{avg} + V_{avg}\cdot I_{max} + V_{max}\cdot I_{avg} + V_{avg}\cdot I_{min}\right)\times\frac{T}{4}$$

$$+\left(\cancel{I_{avg}\cdot slope_v} + I_{max}\cdot slope_v - \cancel{I_{avg}\cdot slope_v} - I_{min}\cdot slope_v\right)\times\frac{(T/4)^2}{2}$$

$$+\left(V_{min}\cdot slope_i - \cancel{V_{avg}\cdot slope_i} - V_{max}\cdot slope_i + \cancel{V_{avg}\cdot slope_i}\right)\times\frac{(T/4)^2}{2}$$

$$+\left(\cancel{slope_v\cdot slope_i} - \cancel{slope_v\cdot slope_i} + \cancel{slope_v\cdot slope_i} - \cancel{slope_v\cdot slope_i}\right)\frac{(T/4)^3}{3}\Big]$$

$$= \frac{1}{4}\left[\left(V_{min}+V_{max}\right)\cdot I_{avg} + \left(I_{min}+I_{max}\right)\cdot V_{avg}\right]$$

$$+\frac{T}{32}\left[slope_v\left(I_{max}-I_{min}\right) - slope_i\left(V_{max}-V_{min}\right)\right]$$

(Noting that $V_{min}+V_{max} = 2\,V_{avg}$ and

$I_{min}+I_{max} = 2\,I_{avg}$)

$$= V_{avg}\cdot I_{avg}$$

$$+\frac{1}{16}\Big[\left(V_{max}-V_{min}\right)\left(I_{max}-I_{min}\right)$$

$$-\left(I_{max}-I_{min}\right)\cdot\left(V_{max}-V_{min}\right)\Big]$$

$$= V_{avg}\cdot I_{avg}$$

<u>Another Approach:</u> In terms of fourier components

$$i(t) = I_{avg} + \frac{8}{\pi^2}\Delta I \sum_{h=1,3,5} \frac{(-1)^{\frac{h-1}{2}}}{h^2}\sin h\omega t$$

and $$v(t) = V_{avg} + \frac{8}{\pi^2}\Delta V \sum_{h=1,3,5} \frac{(-1)^{(h-1)/2}}{h^2}\sin h\left(\omega t - \frac{\pi}{2}\right)$$

Since $v(t)$ lags $i(t)$ by $T/4$,
or by $90°$ at the fundamental frequency.

$p(t) = v(t) \cdot i(t)$ which contains products of sine terms.

$$\sin h\theta \cdot \sin(h\theta - h90^\circ) = \frac{1}{2}\left[\cos(\cancel{h\theta} - \cancel{h\theta} + h90^\circ) - \cos(2h\theta - h90^\circ)\right]$$

$$= -\frac{1}{2}\cos(2h\theta - h90^\circ) \quad \text{for } h = 1, 3, 5, \ldots$$

The integral of the above term over one time period is zero.

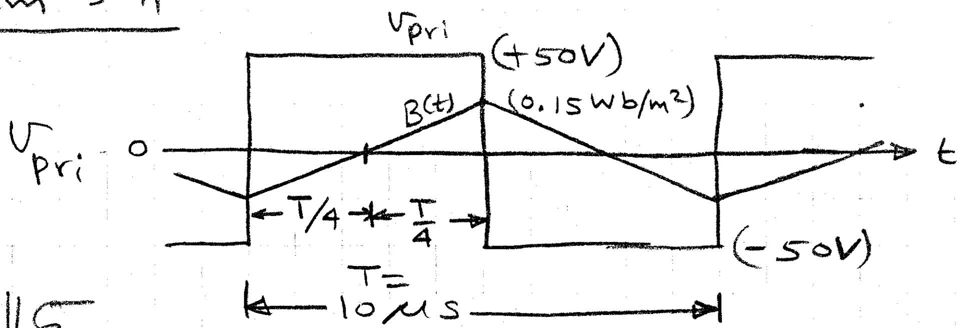
Therefore,

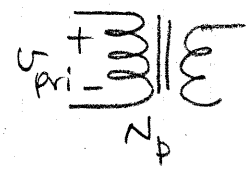
$$P_{avg} = \frac{1}{T}\int_0^T v(t) \cdot i(t) \cdot dt = V_{avg} \cdot I_{avg}.$$

Problem 3-10

$P_{avg} = V_{avg} \cdot I_{avg}$ is exact if the two waveforms are displaced, as shown in Fig P3-9, by T/4. Otherwise, the error will depend on the displacement of the two waveforms.

Problem 3-11





$T = 10\,\mu s$

$$v_{pri} = N_{pri} \cdot \frac{d\phi}{dt} = N_{pri}\, A_c \frac{dB}{dt}$$

In the plot above, the flux density reaches its peak from zero in $T/4$ seconds. Therefore, from the above equations,

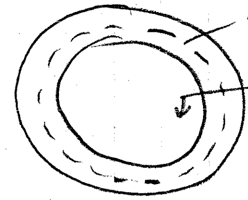
$$B_{peak} = \frac{1}{N_{pri} A_c} \int_0^{T/4} v_{pri} \cdot dt$$

$$0.15 = \frac{1}{N_{pri}\ 0.635 \times 10^{-4}} \ 50 \times \left(\frac{10}{4} \times 10^{-6}\right)$$

$$\therefore \ N_{pri} = 13.12$$

Therefore, N_{pri} = 14 turns minimum to keep flux density below the saturation level.

Problem 3-12



$\ell_c = 3.12$ cm

$A_c = 0.113\ cm^2$

$L = 25\,\mu H$

$\mu_r = 125$

$$L = N^2 P = N^2 \frac{A_c(\mu_0 \mu_r)}{l_c}$$

$$\therefore N = \left[\frac{L\, l_c}{A_c\, \mu_0 \mu_r}\right]^{1/2} \qquad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$= \left[\frac{25 \times 10^{-6} \times 3.12 \times 10^{-2}}{0.113 \times 10^{-4} \times 4\pi \times 10^{-7} \times 125}\right]^{1/2}$$

$$\simeq 21 \text{ turns}$$

Problem 3-13

$V_1 = 110\text{V}$ $\qquad \frac{N_2}{N_1} = 2$

$f = 60\text{ Hz}$

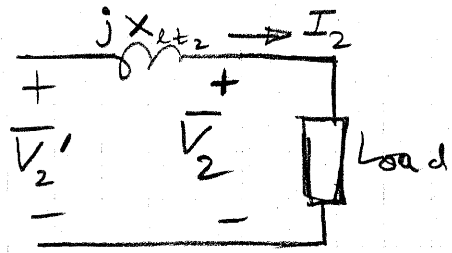
$L_{lt_2} = 5.15 \text{ mH}$

$$x_{lt_2} = \frac{5.15}{1000} \times (2\pi \times 60) = 1.94\,\Omega$$

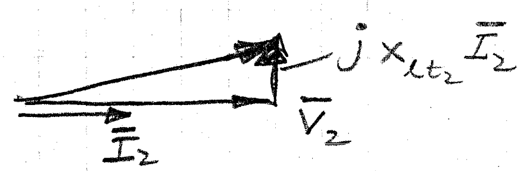
$$V_2' = 110 \times \frac{N_2}{N_1} = 220\text{V}$$

(V_2 at no-load)

Full-Load kVA = 1



(a) at unity power factor



$$V_2 I_2 = 1000$$

$$V_2' = \left[V_2^2 + (x_{lt_2} I_2)^2\right]^{1/2}$$

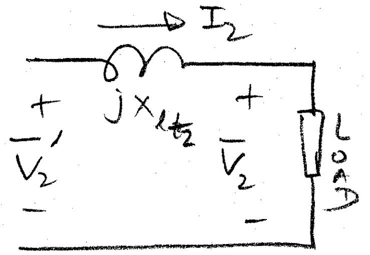
$$(V_2')^2 = V_2^2 + \left(x_{lt_2} \frac{V_2}{1000}\right)^2$$

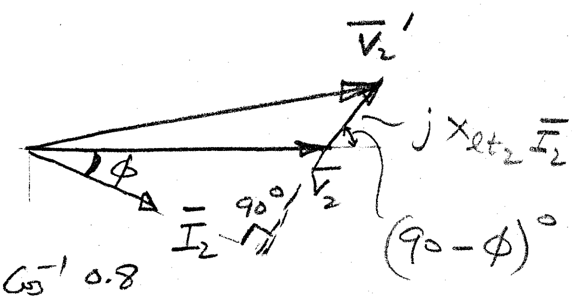
or $$V_2^2\left[1+\left(\frac{X_{lt_2}}{1000}\right)^2\right] = (V_2')^2$$

$\therefore$ $$V_2 \simeq V_2'$$

& $$\%\ \text{regulation} = 100 \times \frac{V_2' - V_2}{V_2'} \simeq 0$$

(b) at 0.8 power factor (assumed to be lagging)





$\phi = \cos^{-1} 0.8$
$= 36.87°$

$$V_2 I_2 = 1000$$

and $$V_2' = V_2 + X_{lt_2} I_2 \cos(90-\phi)°$$

or, $$V_2\left(1 + X_{lt_2}\frac{1}{1000}\,0.6\right) = 220$$

$\therefore$ $$V_2 = 219.744\ V$$

$\therefore$ $$\%\ \text{Regulation} = \frac{220 - 219.744}{220} \times 100$$

$$= 0.116\ \%$$

Problem 3-14

In Problem 3-11, it was calculated that

$N_{pri} = 14$, $A_c = 0.634\ cm^2$

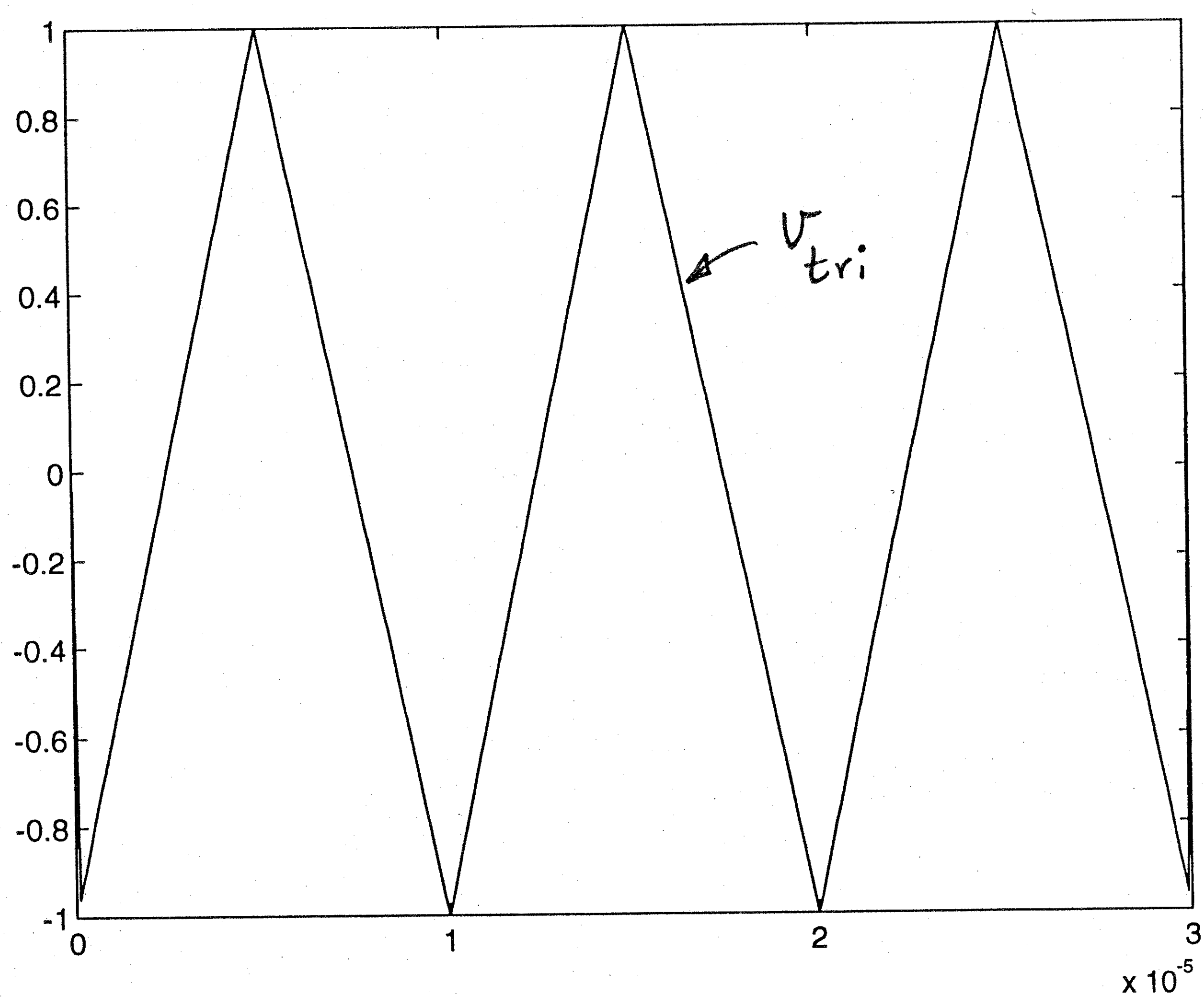
$$\therefore \quad L_m = N^2 P = N^2 \frac{A_c (\mu_o \mu_r)}{l_c}$$

$$= 14^2 \times \frac{0.634 \times 10^{-4} \times 4\pi \times 10^{-7} \times 2500}{3.15 \times 10^{-2}}$$

$$= 1.24\ mH$$

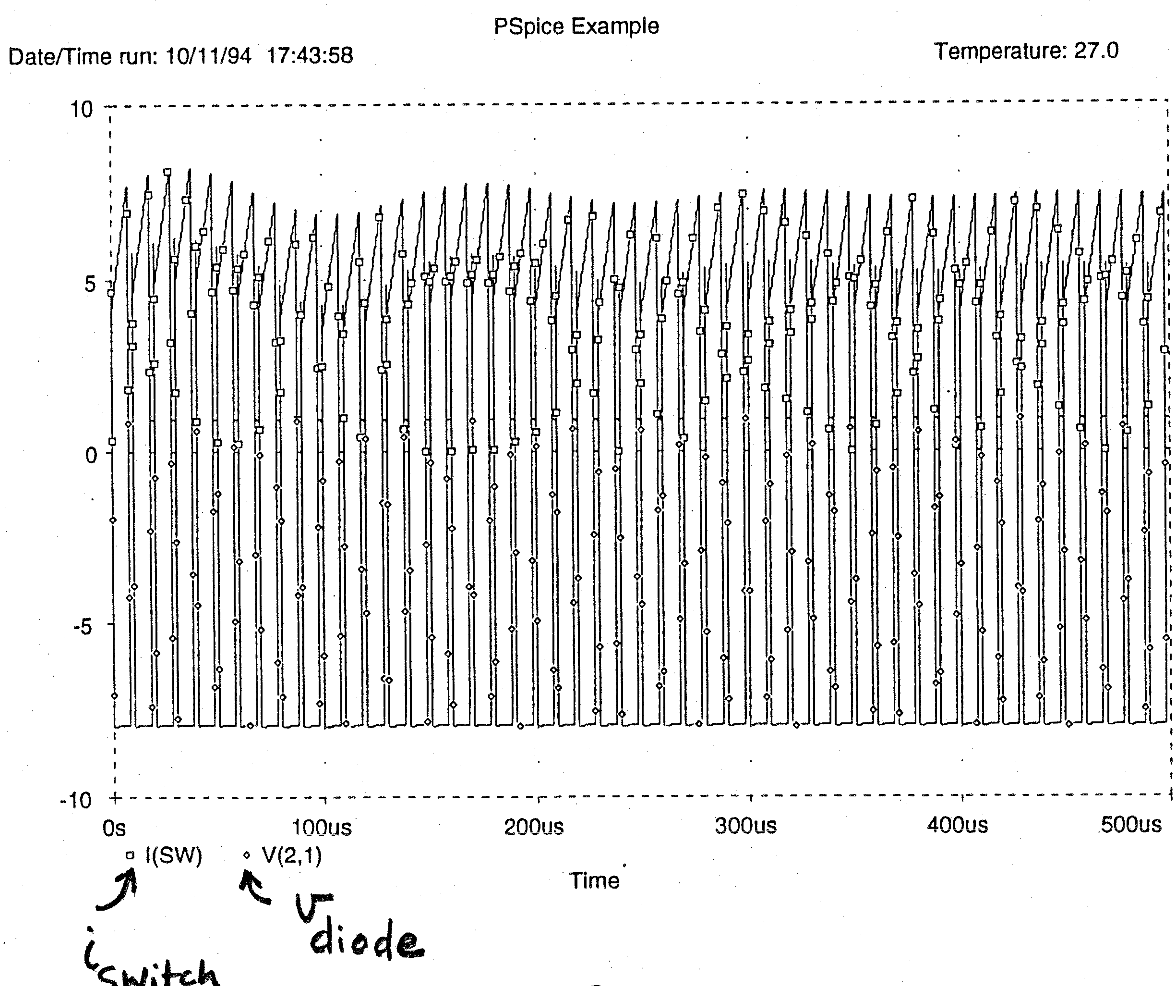
Problem 4-1

```
% Generate Triangular Waveform at 100 kHz
clc,clg,clear
f=100e3;w=2*pi*f;deltat=1/(f*100);Tmax=3/f;
%
time=0:deltat:Tmax;
wt=w*time;
tri=sign(sin(wt)).*(2/pi*rem(wt,pi) - 1);
plot(time,tri)
```



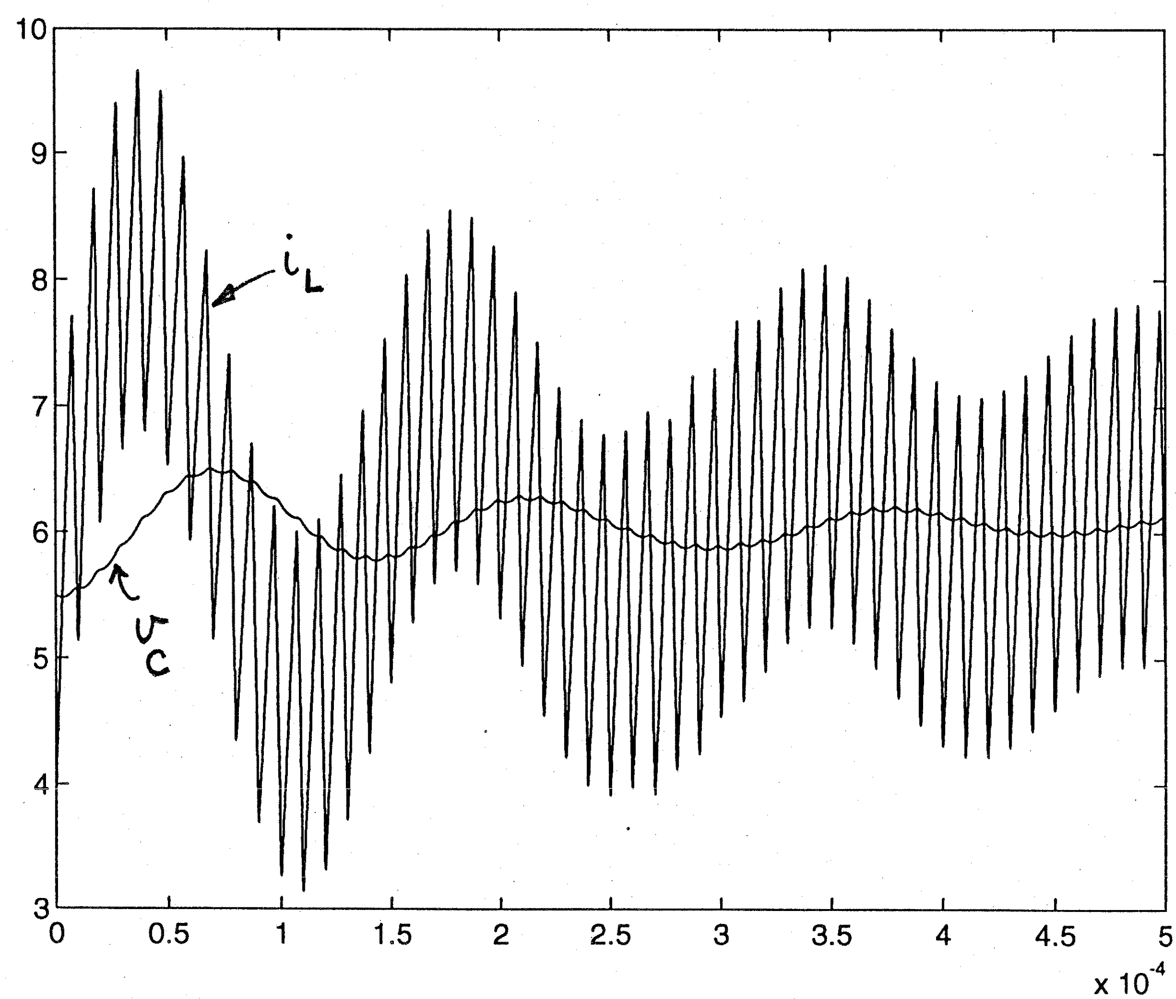
Problem 4-2

```
PSpice Example
*
DIODE   2  1   POWER_DIODE
Rsnub   1  5   100.0
Csnub   5  2   0.1uF
*
SW      2  0   6  0  SWITCH
VCNTL   6  0   PULSE(0V, 1V, 0s, 1ns, 1ns, 7.5us, 10us)
*
L       1  3   5uH    IC=4A
rL      3  4   1m
C       4  2   100uF  IC=5.5V
RLOAD   4  2   1.0
*
VD      1  0   8.0V
*
.MODEL  POWER_DIODE D(RS=0.01, CJO=10pF)
.MODEL  SWITCH VSWITCH(RON=0.01)
.TRAN  10us   500.0us   0s   0.2us   uic
.PROBE
.END
```



Problem 4-3


```
% Listing in Fig 4-11b
clc,clg,clear
% Input Data
Vd=8; L=5e-6; C=100e-6; rL=1e-3; R=1.0; fs=100e3; Vcontrol=0.75;
Ts=1/fs; tmax=50*Ts; deltat=Ts/50;
%
time=0:deltat:tmax;
vst=time/Ts - fix(time/Ts);
voi=Vd*(Vcontrol > vst);
%
A=[-rL/L -1/L; 1/C -1/(C*R)];
b=[1/L 0]';
MN=inv(eye(2) - deltat/2 * A);
M=MN*(eye(2)+deltat/2 * A);
N=MN*deltat/2*b;
%
iL(1)=4.0; vC(1)=5.5;
timelength=length(time);
%
for k = 2:timelength
x=M*[iL(k-1) vC(k-1)]' + N*(voi(k) + voi(k-1));
iL(k)=x(1); vC(k)=x(2);
end
%
plot(time,iL,time,vC)
```



Problem 4-4

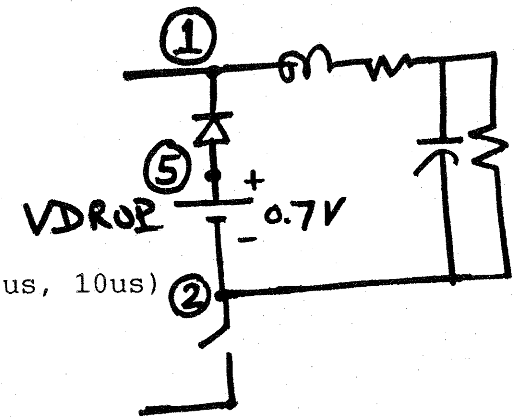
(a) Removal of snubber does not change the simulation time. However, in many simulations, it can lead to very large increase in the execution times. For example, at light loads (make RLOAD = 10.0 Ω) where the inductor current becomes discontinuous, there is a very large increase in the execution time.

(b) Convergence problems at 10 μs.

(c) With the suggested changes, even the case with a discontinuous inductor current (and the R-C Snubber removed) executes normally.

(d) Convergence problems at 10 μs.

Problem 4-5

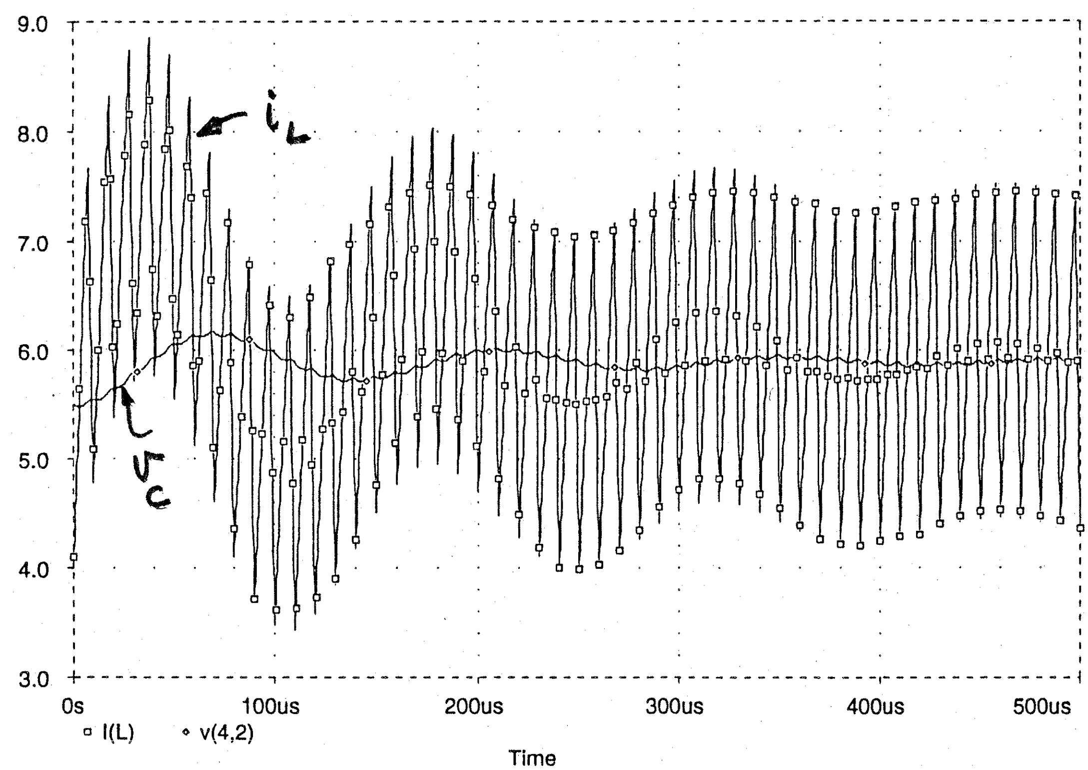



```
PSpice Example
*
DIODE   5  1   POWER_DIODE
VDROP   5  2   0.7V
Rsnub   1  5   100.0
Csnub   5  2   0.1uF
*
SW      2  0   6  0  SWITCH
VCNTL   6  0   PULSE(0V, 1V, 0s, 1ns, 1ns, 7.5us, 10us)
*
L       1  3   5uH     IC=4A
rL      3  4   1m
C       4  2   100uF   IC=5.5V
RLOAD   4  2   1.0
*
VD      1  0   8.0V
*
.MODEL  POWER_DIODE D(RS=0.01, CJO=10pF)
.MODEL  SWITCH VSWITCH(RON=0.01)
.TRAN   10us   500.0us   0s   0.2us   uic
.PROBE
.END
```

PSpice Example

Date/Time run: 10/11/94 18:12:56

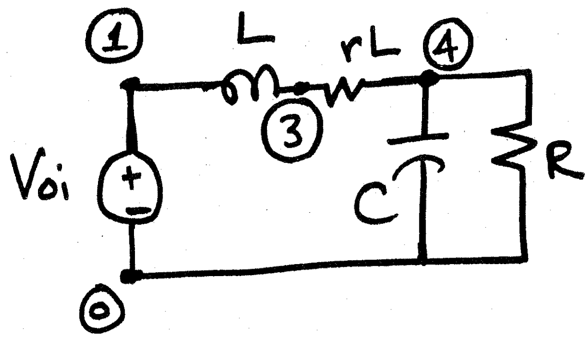
Temperature: 27.0

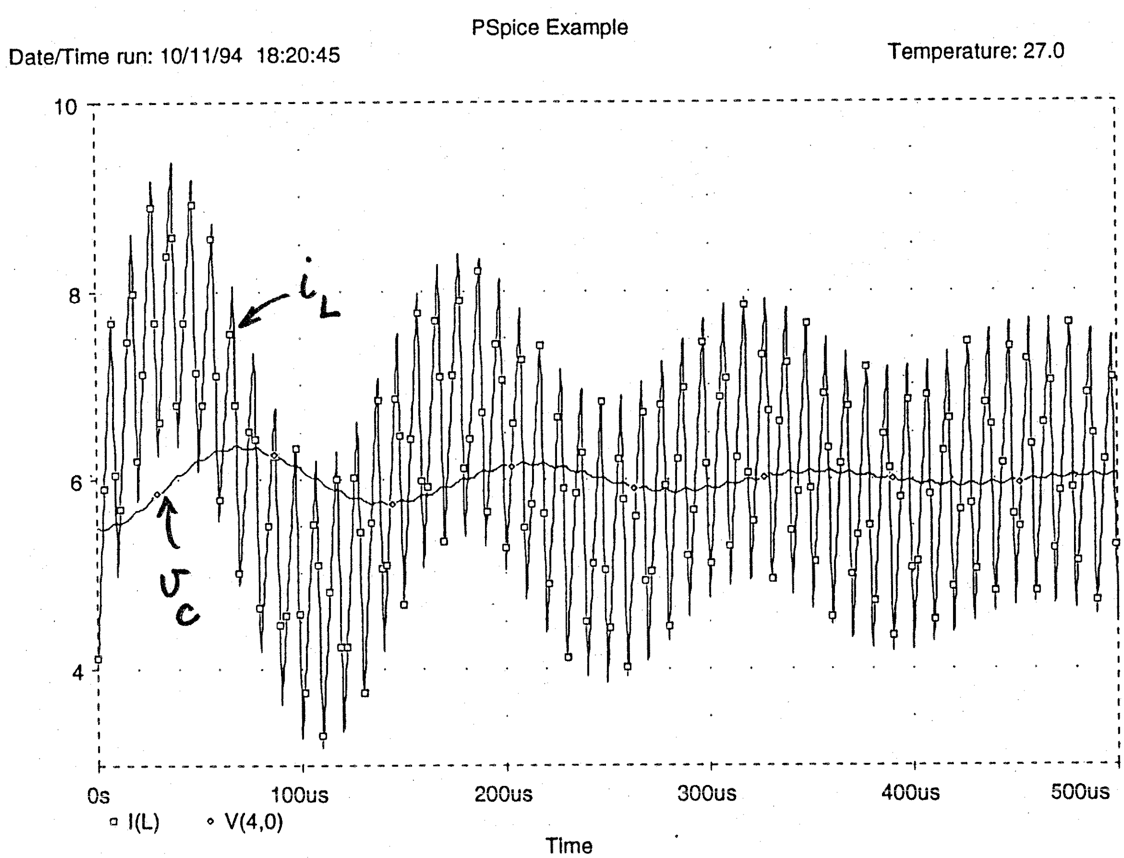


These results are closer to the MATLAB simulation results.

Problem 4-6

```
PSpice Example
Voi     1  0  PULSE(0V, 8V, 0s, 1ns, 1ns, 7.5us, 10us)
*
L       1  3  5uH     IC=4A
rL      3  4  1m
C       4  0  100uF   IC=5.5V
RLOAD   4  0  1.0
*
.TRAN  10us  500.0us  0s  0.2us  uic
.PROBE
.END
```





These results are much closer to the MATLAB simulation results.

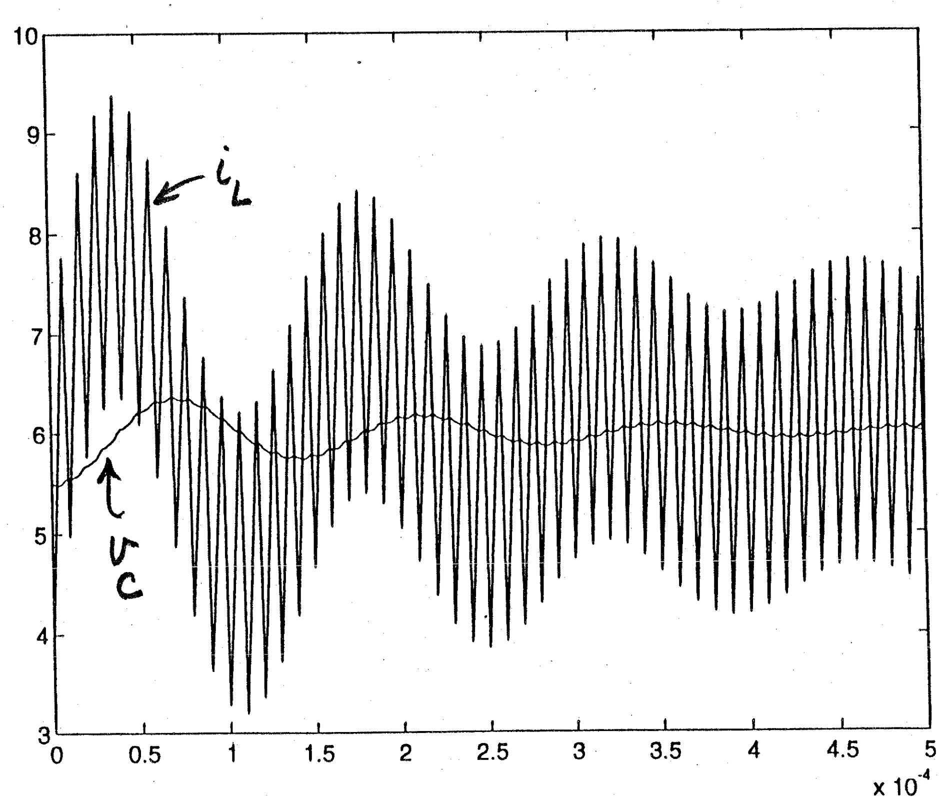
Problem 4-7

PROB4_7A.M

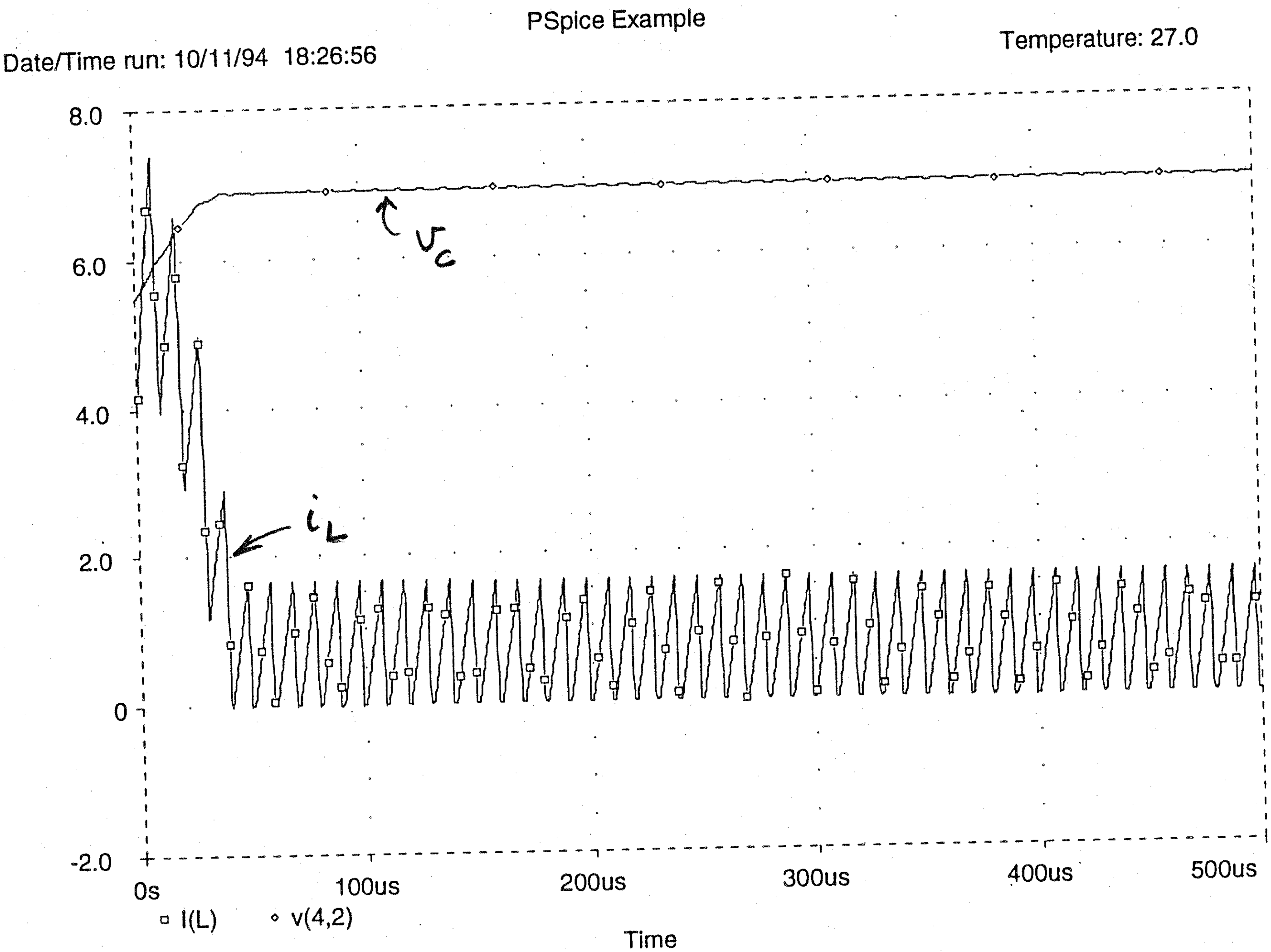
```
% use of ODE45
clc,clg,clear
% Input Data
Vd=8; L=5e-6; C=100e-6; rL=1e-3; R=1.0; fs=100e3; Vcontrol=0.75;
Ts=1/fs; tmax=50*Ts; deltat=Ts/50;
%
x0=[4.0 5.5]';
t0=0;
tf=500e-6;
[t, x] = ode45('xdot',t0,tf,x0);
plot(t,x)
```

XDOT.M

```
function xdot=xdot(t, x)
% used with Prob4_7.m
% Input Data
Vd=8; L=5e-6; C=100e-6; rL=1e-3; R=1.0; fs=100e3; Vcontrol=0.75;
Ts=1/fs; tmax=50*Ts; deltat=Ts/50;
%
vst=t/Ts - fix(t/Ts);
voi=Vd* (Vcontrol > vst);
%
xdot(1) = -rL/L*x(1) -(1/L)*x(2) + (1/L)*voi;
xdot(2) = 1/C*x(1) -1/(C*R)*x(2);
```



Problem 4-8



The inductor current i_L becomes discontinuous at light load (RLOAD = 10.0 Ω).

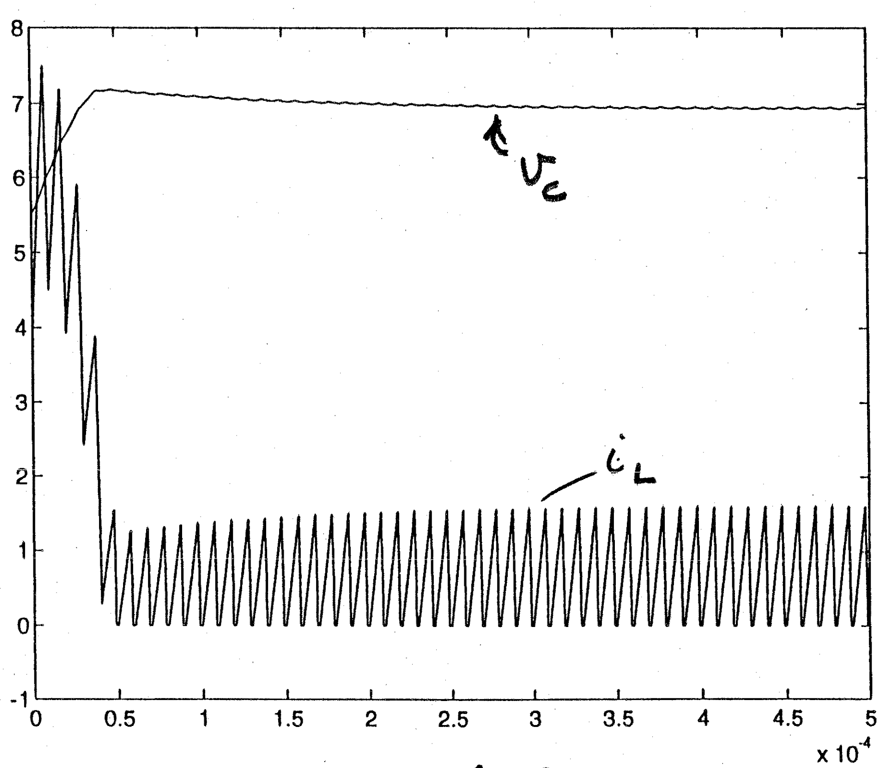
Problem 4-9

PROB4_9.M

```
% use of ODE45
clc,clg,clear
% Input Data
Vd=8; L=5e-6; C=100e-6; rL=1e-3; R=1.0; fs=100e3; Vcontrol=0.75;
Ts=1/fs; tmax=50*Ts; deltat=Ts/50;
%
x0=[4.0 5.5]';
t0=0;
tf=500e-6;
[t, x] = ode45('xdot_dis',t0,tf,x0);
plot(t,x)
```

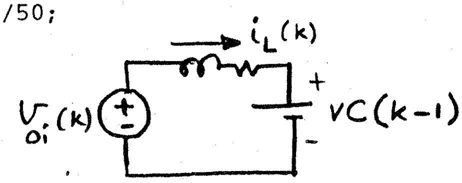
XDOT_DIS.M

```
function xdot=xdot_dis(t, x)
% used with Prob4_9.m
% Input Data
Vd=8; L=5e-6; C=100e-6; rL=1e-3; R=10; fs=100e3; Vcontrol=0.75;
Ts=1/fs; tmax=50*Ts; deltat=Ts/50;
%
vst=t/Ts - fix(t/Ts);
voi=Vd* (Vcontrol > vst);
%
xdot(1) = -rL/L*x(1) -(1/L)*x(2) + (1/L)*voi;
%
  if x(1) <= 0
     if xdot(1) <= 0
       x(1)=0; xdot(1)=0; voi=x(2);
     end
  end
%
xdot(2) = 1/C*x(1) -1/(C*R)*x(2);
```

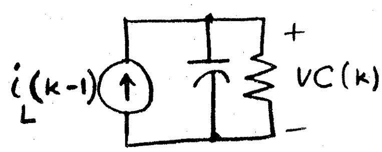


Problem 4-10

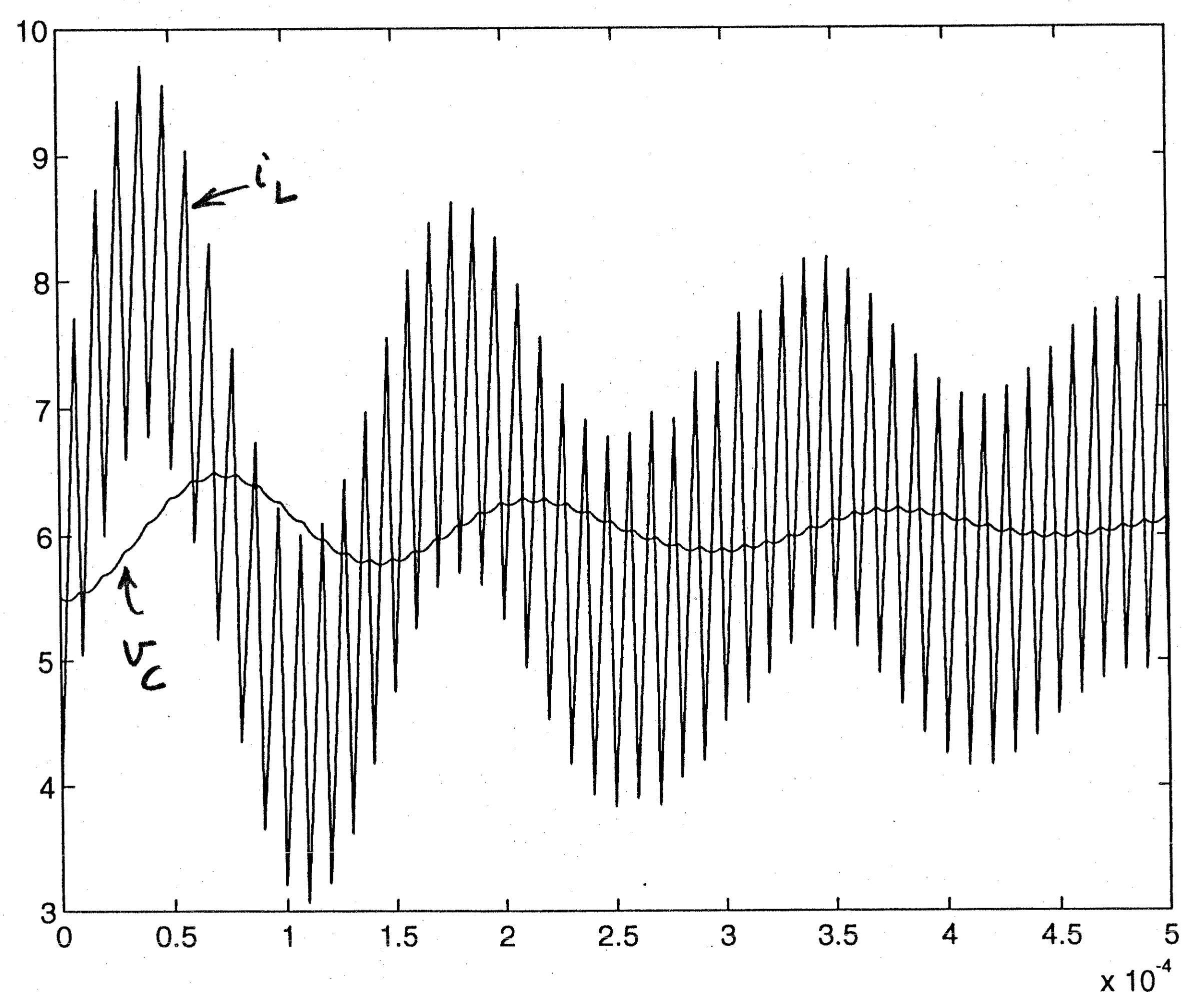
```
%
clc,clg,clear
% Input Data
Vd=8; L=5e-6; C=100e-6; rL=1e-3; R=1.0; fs=100e3; Vcontrol=0.75;
Ts=1/fs; tmax=50*Ts; deltat=Ts/50;
TrL= L/rL;
TRC= R*C;
%
time=0:deltat:tmax;
vst=time/Ts - fix(time/Ts);
voi=Vd*(Vcontrol > vst);
%
iL(1)=4.0; vC(1)=5.5;
timelength=length(time);
%
for k = 2:timelength
 vin=voi(k) - vC(k-1);
 iL(k)=iL(k-1)*exp(-deltat/TrL) + vin/rL*(1-exp(-deltat/TrL));
 vC(k)=R*iL(k-1)*(1-exp(-deltat/TRC)) + vC(k-1)*exp(-deltat/TRC);
end
%
plot(time,iL,time,vC)
```



Calculate $i_L(k)$ in the above circuit.

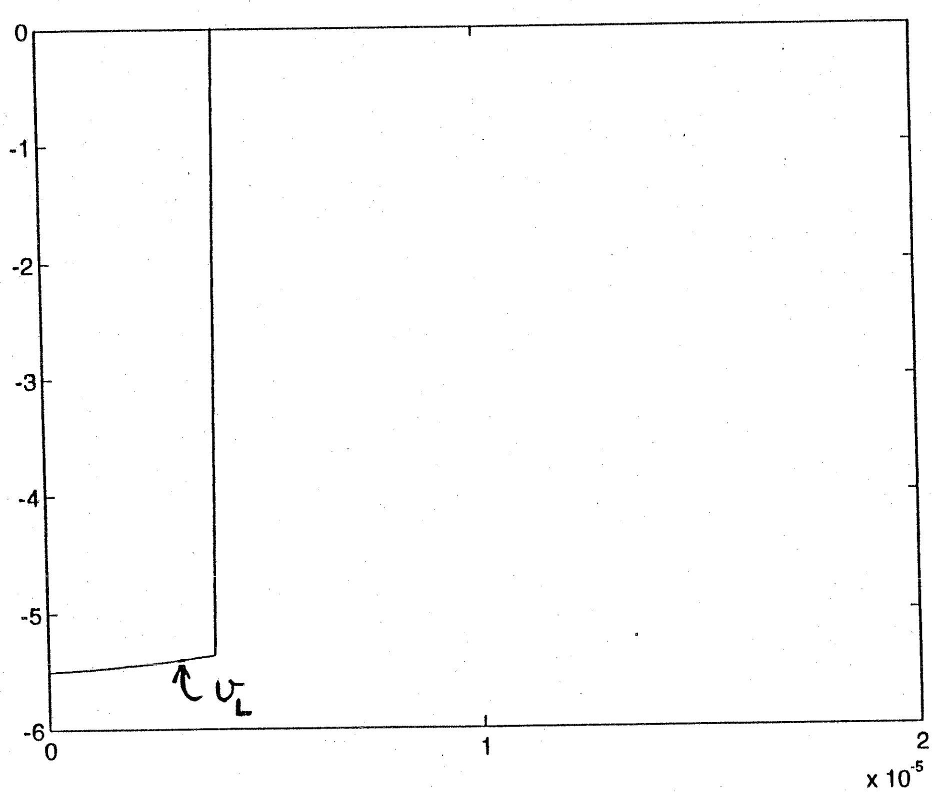


Calculate $vC(k)$.

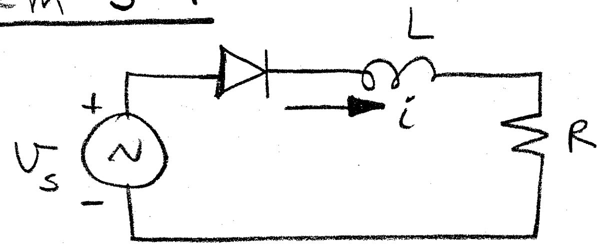


Problem 4-11


```
% Listing in Fig 4-11b
clc,clg,clear
% Input Data
L=5e-6; C=100e-6; rL=1e-3; R=1.0; fs=100e3; Vcontrol=0.75;
Ts=1/fs; tmax=2*Ts; deltat=Ts/50;
%
time =0:deltat:tmax;
A=[-rL/L -1/L; 1/C -1/(C*R)];
MN=inv(eye(2) - deltat/2 * A);
M=MN*(eye(2)+deltat/2 * A);
%
iL(1)=4.0; vC(1)=5.5;
vL(1)= -rL*iL(1) - vC(1);
timelength=length(time);
%
for k = 2:timelength
  if iL(k-1) > 0
      x=M*[iL(k-1) vC(k-1)]';
      iL(k)=x(1); vC(k)=x(2);
      vL(k)= -rL*iL(k) - vC(k);
  else
      iL(k)=0;
      vC(k)=(vC(k-1)-1/(C*R)*deltat/2*vC(k-1))/(1+deltat/(2*C*R));
      vL(k)=0;
  end
end
%
plot(time,vL)
```



Problem 5-1

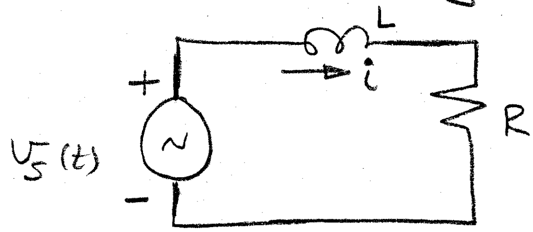


$V_s = 120V$ at $60Hz$

$L = 10mH$, $R = 5\Omega$

$\omega = 2\pi f = 377$ rad/s

To obtain $i(t)$, we can consider the following equivalent circuit, which applies only when the current is flowing



$$v_s(t) = \left[\hat{V}_s \sin \omega t\right] u(t)$$

In the above circuit, for $t > 0$ while the current flows

$$Ri + L\frac{di}{dt} = \hat{V}_s \sin \omega t$$

or $$\frac{di}{dt} + \frac{R}{L} i = \frac{\hat{V}_s}{L} \sin \omega t$$

Adding the forced response and the natural response,

$$i = Ae^{-\frac{R}{L}t} + \frac{\hat{V}_s}{Z} \sin(\omega t - \phi)$$

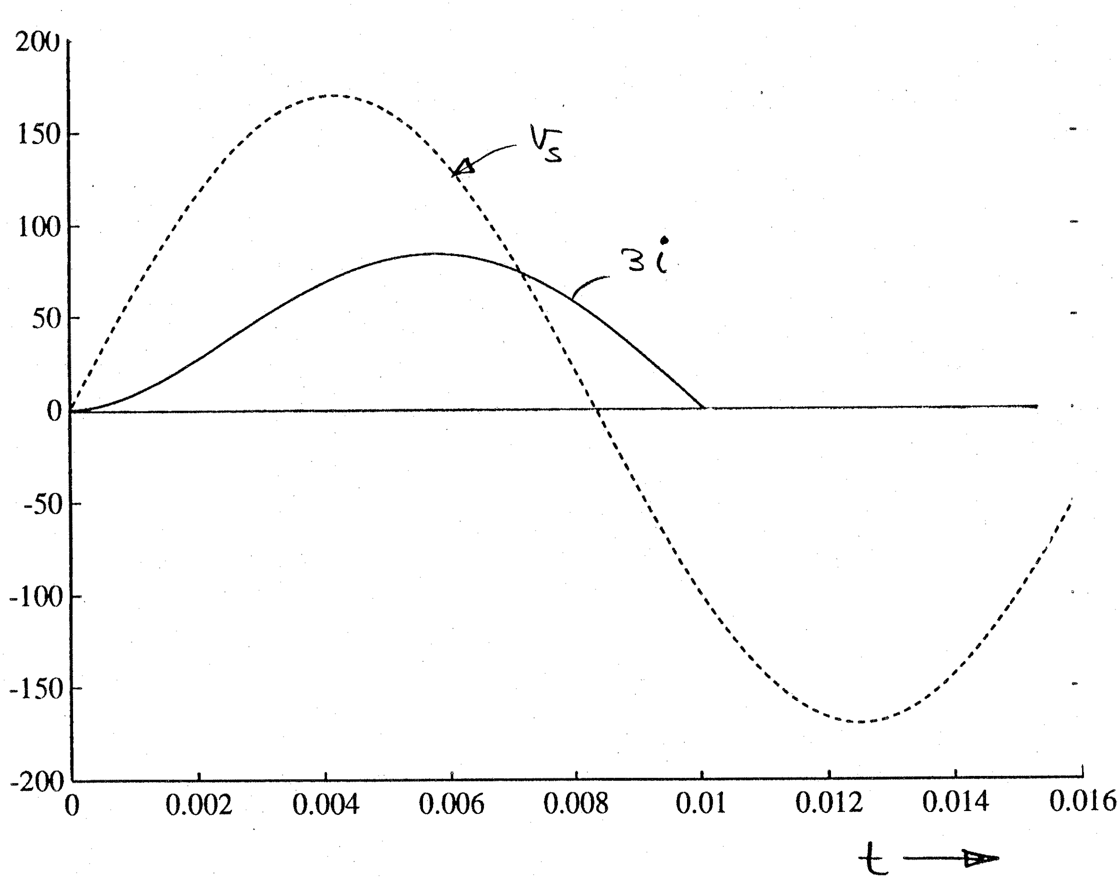
where $Z = \sqrt{R^2 + (\omega L)^2}$, $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ and A is the coefficient to be determined.

Initially at $t=0$, $i(0^-) = i(0^+) = 0$. We will use this initial condition to calculate A in the previous equation:

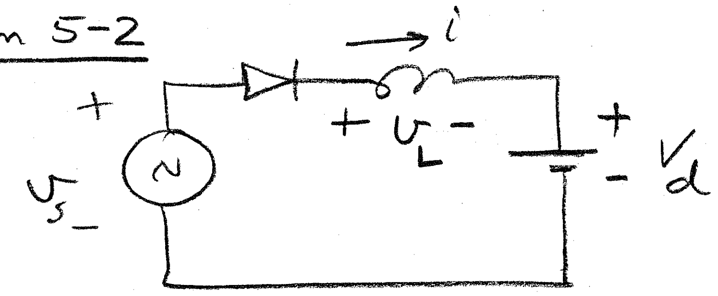
$$0 = A + \frac{\hat{V}_s}{Z}\sin(-\phi)$$

$$\therefore \quad A = \frac{\hat{V}_s}{Z}\sin\phi = \hat{V}_s\left(\frac{\omega L}{Z^2}\right)$$

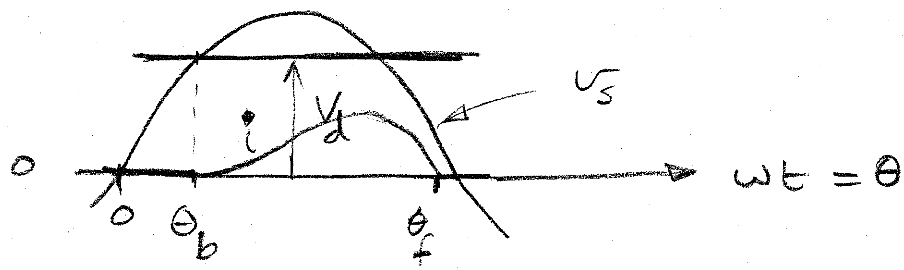
$$\therefore \quad i(t) = \frac{\hat{V}_s\,\omega L}{Z^2}e^{-\frac{R}{L}t} + \frac{\hat{V}_s}{Z}\sin(\omega t - \phi)$$



Problem 5-2



$$v_L = v_s - V_d \qquad i > 0$$



$$\sqrt{2}\,V_s \sin\theta_b = V_d$$

$$\therefore \quad \theta_b = \sin^{-1}\left(\frac{150}{\sqrt{2}\times 120}\right) = 1.084 \text{ rad } (62.11°)$$

$$i(\theta) = \frac{1}{\omega L}\int_{\theta_b}^{\theta} v_L \cdot d\theta = \frac{1}{\omega L}\int_{\theta_b}^{\theta}\left(\sqrt{2}\,V_s \sin\theta - V_d\right) d\theta$$

$$i(\theta) = \frac{1}{\omega L}\left[\sqrt{2}\,V_s\left(-\cos\theta + \cos\theta_b\right) - V_d\left(\theta - \theta_b\right)\right]$$

$$= -45.01\cos\theta - 39.79\,\theta + 64.19 \qquad \theta_b < \theta < \theta_f$$

final value

Calculate θ_f

$$i(\theta_f) = 0 = -45.01\cos\theta - 39.79\,\theta + 64.19$$

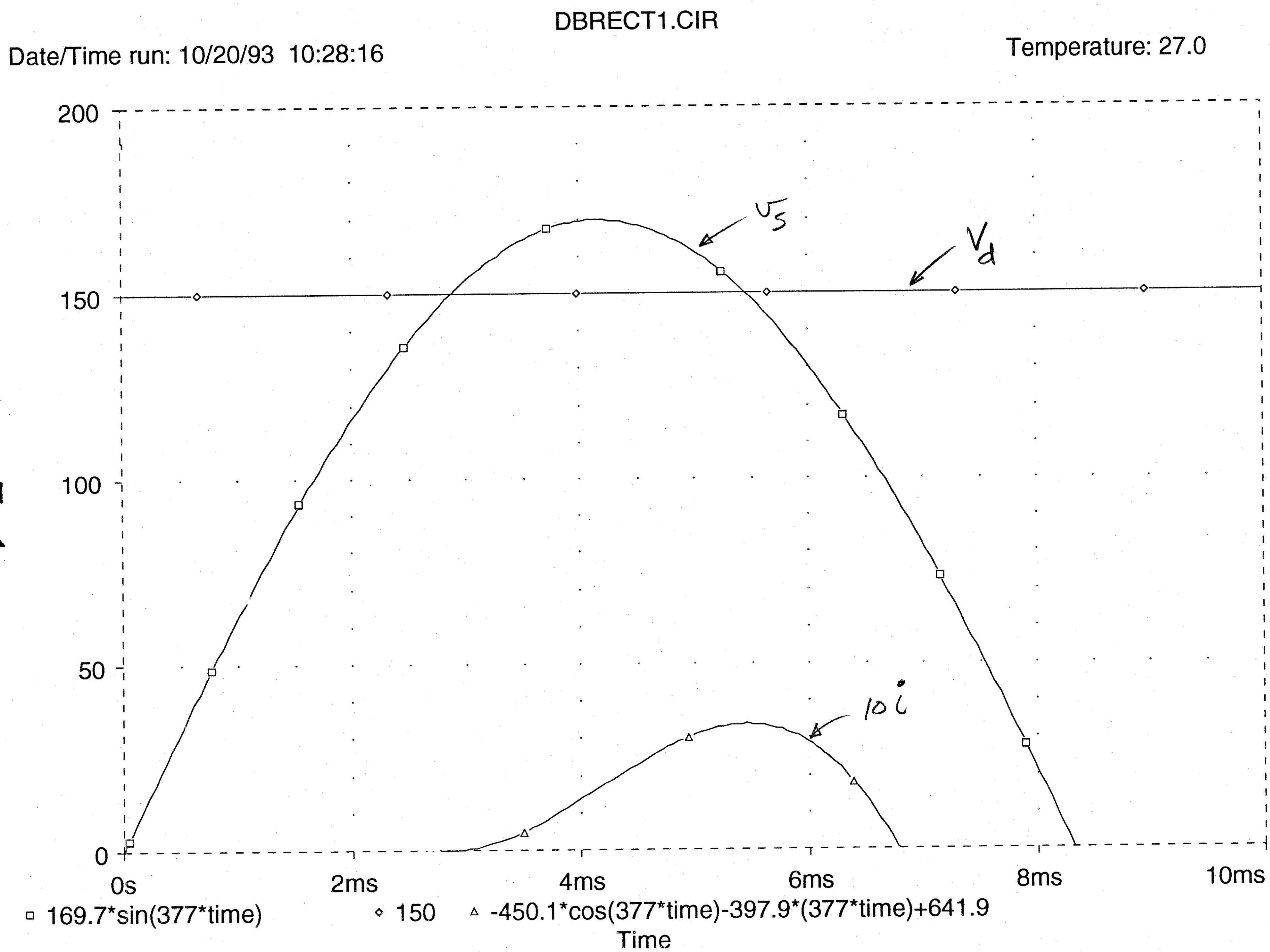
$$\cos\theta_f + 0.884\,\theta_f = 1.426$$

(by trial-and-error, or some other procedure)

$$\therefore \quad \theta_f = 2.56 \text{ rad}$$

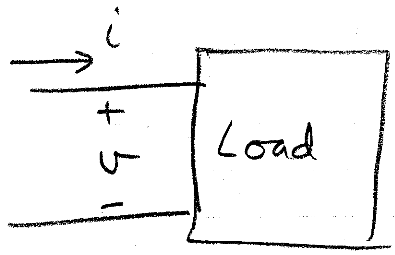
$i(\theta)$ can be calculated and plotted between $\theta_b = 1.084$ rad and $\theta_f = 2.56$ rad from the above equation $i(\theta)$.

5-4



DBRECT1.CIR
Date/Time run: 10/20/93 10:28:16
Temperature: 27.0
200
150
100
50
0
0s
2ms
4ms
6ms
8ms
10ms
v_s
V_d
$10i$
□ 169.7*sin(377*time)
◇ 150
△ -450.1*cos(377*time)-397.9*(377*time)+641.9
Time

Problem 5-3



$$v = V_d + \sqrt{2}\, V_1 \cos \omega_1 t + \sqrt{2}\, V_1 \sin(\omega_1 t) + \sqrt{2}\, V_3 \cos \omega_3 t$$

$$i = I_d + \sqrt{2}\, I_1 \cos \omega_1 t + \sqrt{2}\, I_3 \cos(\omega_3 t - \phi_3)$$

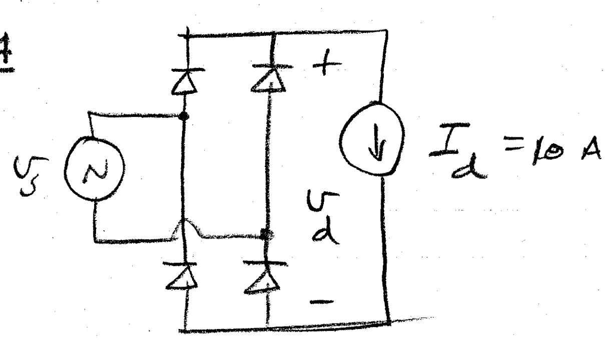
(a) $P = V_d I_d + V_1 I_1 + V_3 I_3 \cos \phi_3$

(b) $V = \sqrt{V_d^2 + (\sqrt{2} V_1)^2 + V_3^2}$

$I = \sqrt{I_d^2 + I_1^2 + I_3^2}$

(c) $PF = \dfrac{P}{V\, I}$; substitute the expressions from parts (a) and (b).

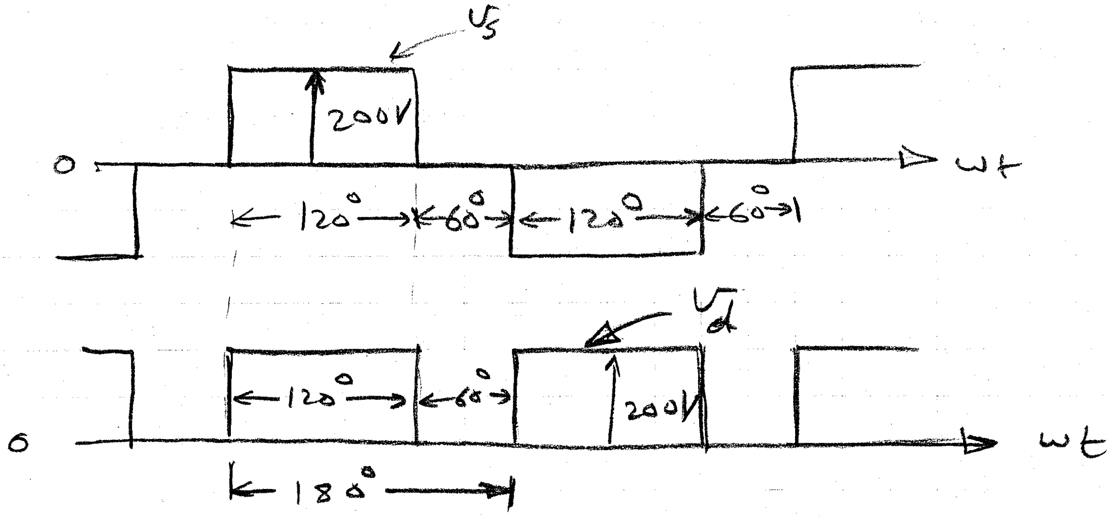
Problem 5-4



(a) v_s: Sinusoidal, $V_s = 120V$ $\therefore V_d = 0.9 V_s = 108\ V$

$P_d = V_d I_d = 1080\ W.$

(b)

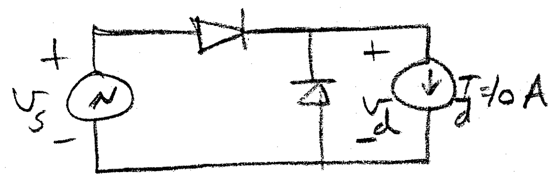


$$V_d = \frac{(200\times 120^\circ) + (0\times 60^\circ)}{180^\circ} = 200 \times \frac{2}{3} = 133.33\ V$$

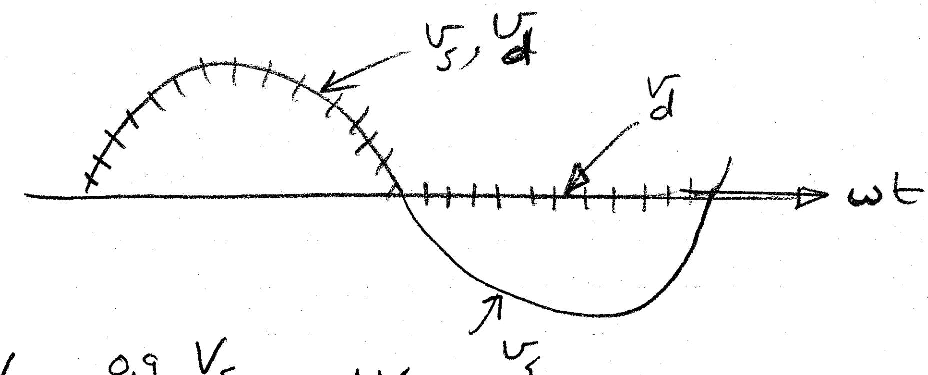
$$P_d = V_d I_d = 1333.3\ W$$

Problem 5-5

(a)



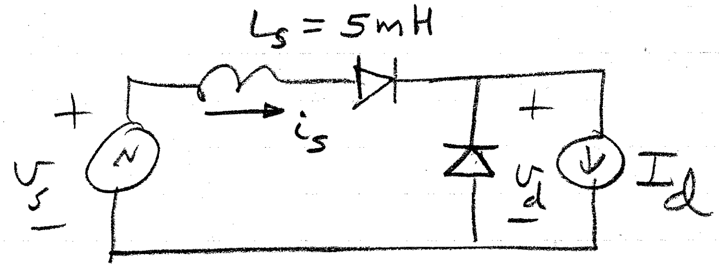
$V_s = 120V$



$$V_d = \frac{0.9}{2} V_s = 54V$$

$$P_d = V_d I_d = 540\ W$$

(b)



$u, V_d, P_d = ?$

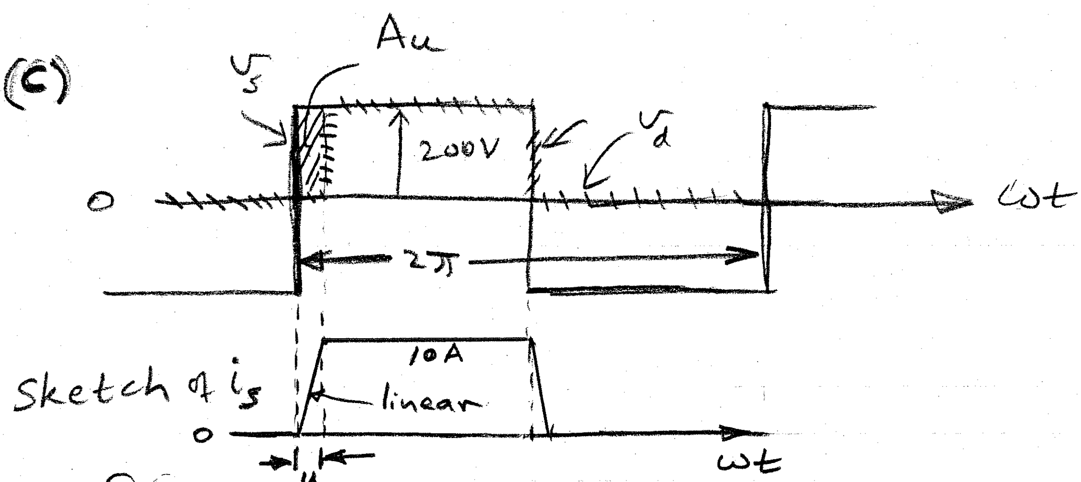
$$\cos u = 1 - \frac{\omega L_s I_d}{\sqrt{2} V_s} \qquad (5\text{-}22)$$

$$V_d = 0.45 V_s - \frac{\omega L_s}{2\pi} I_d \qquad (5\text{-}26)$$

$$P_d = V_d I_d$$

$\therefore \; u = 27.26^\circ, \quad V_d = 51\ V, \quad P_d = 510\ W$

(c)



$\omega = 2\pi \times 60$
$= 377\ \frac{rad}{s}$
$L_s = 5mH$

Similar development leading up to Eq. (5-21) - - -

$$\int_0^u 200 \cdot d(\omega t) = \omega L_s I_d$$

$$200\, u = 377 \times 5 \times 10^{-3} \times 10$$

$$\therefore \; u = 0.094 \text{ rad}$$
$$= 5.4^\circ$$

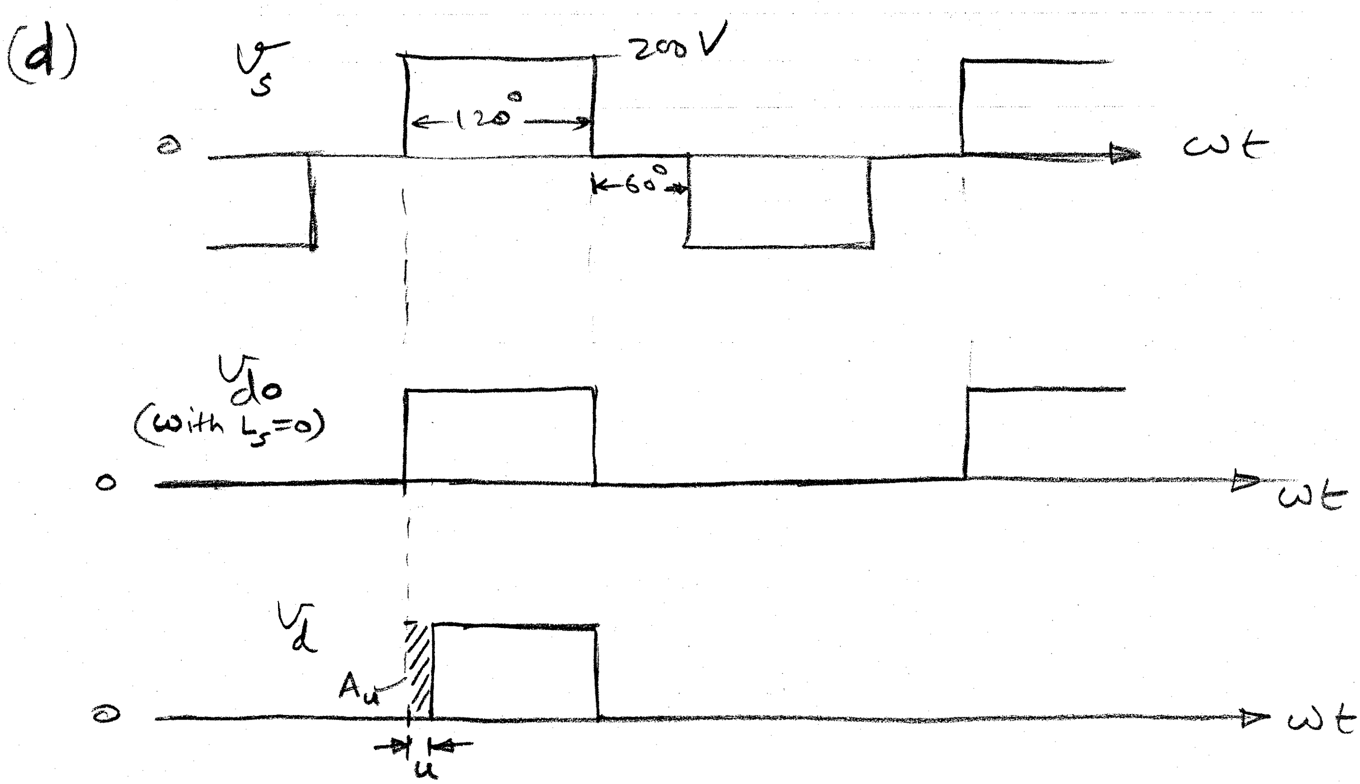
From Eq. 5-21

$$A_u = \omega L_s I_d = 18.85 \text{ V}\cdot\text{rad}$$

$$\therefore \; V_d = 100 - \frac{18.85}{2\pi} = 97\ V$$

$$P_d = V_d I_d = 970\ W$$

(d)



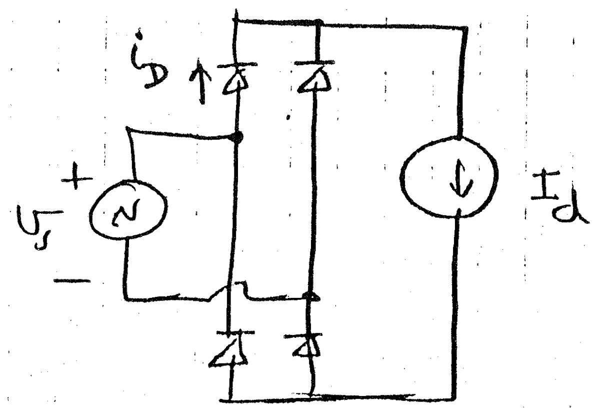
$$V_{do} = \frac{200 \times 120}{360} = 66.67\ V$$

$$A_u = \omega L_s I_d = 18.85\ V.rad \text{ (as in part c)}$$

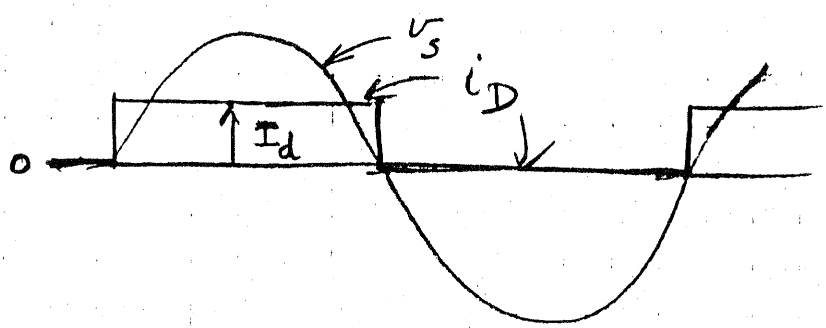
$$\therefore \; V_d = V_{do} - \frac{A_u}{2\pi} = 63.67\ V$$

$$P_d = V_d I_d = 636.7\ W$$

Problem 5-6



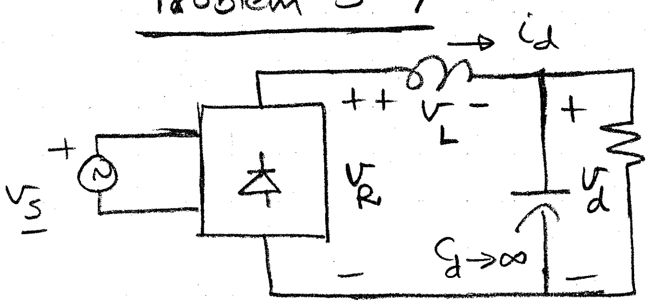
i_D = diode current



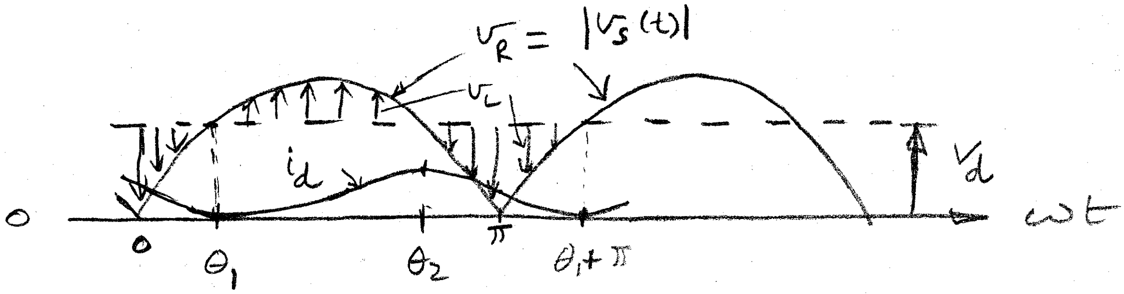
$$I_D\,(avg) = \frac{I_d}{2}$$

$$I_D\,(rms) = \sqrt{\frac{I_d^2}{2}} = \frac{I_d}{\sqrt{2}}$$

Problem 5-7



$$v_d(t) \simeq V_d = 0.9 V_s$$



For $0 < \theta < \pi$

$$i_d(\theta) = i_d(0) + \frac{1}{\omega L_d} \int_0^\theta v_L \cdot d(\omega t)$$

$$= i_d(0) + \frac{1}{\omega L_d} \left[\int_0^\theta (\sqrt{2} V_s \sin \omega t - V_d) \cdot d(\omega t) \right]$$

$$= i_d(0) + \frac{1}{\omega L_d} \left[\sqrt{2} V_s \cos(\omega t) \Big|_\theta^0 - V_d \theta \right]$$

$$= i_d(0) + \frac{1}{\omega L_d} \left[\sqrt{2} V_s (1 - \cos \theta) - V_d \theta \right]$$

To obtain $i_d(0)$; we know that $i_d(\theta_1) = 0$ in the limiting case. Therefore, at $\theta = \theta_1$

$$i(\theta_1) = 0 = i_d(0) + \frac{\sqrt{2} V_s}{\omega L_d} - \frac{\sqrt{2} V_s}{\omega L_d} \cos \theta_1 - \frac{V_d \theta_1}{\omega L_d}$$

[Note: $\sqrt{2} V_s \cos \theta_1 = V_d$]

$$= i_d(0) + \frac{\sqrt{2} V_s}{\omega L_d} - \frac{V_d}{\omega L_d} (1 + \theta_1)$$

$$\therefore \quad i_d(0) = \frac{V_d}{\omega L_d}(1+\theta_1) - \frac{\sqrt{2}V_s}{\omega L_d}$$

and,

$$i_d(\theta) = \frac{V_d}{\omega L_d}(1+\theta_1) - \cancel{\frac{\sqrt{2}V_s}{\omega L_d}} + \cancel{\frac{\sqrt{2}V_s}{\omega L_d}} - \frac{\sqrt{2}V_s}{\omega L_d}\cos\theta - \frac{V_d}{\omega L_d}\theta$$

$$= \frac{V_d}{\omega L_d}(1+\theta_1-\theta) - \frac{\sqrt{2}V_s}{\omega L_d}\cos\theta$$

Average current I_d:

$$I_d = \frac{1}{\pi}\int_0^{\pi} i_d(\theta)\cdot d\theta$$

$$= \frac{1}{\pi}\left[\frac{V_d}{\omega L_d}\left\{(1+\theta_1)\pi - \frac{\pi^2}{2}\right\} - \cancel{\frac{\sqrt{2}V_s}{\omega L_d}\sin\theta\Big|_0^{\pi}} \rightarrow 0\right]$$

$$= \frac{V_d}{\omega L_d}\left(1+\theta_1-\frac{\pi}{2}\right)$$

$$V_d = 0.9\,V_s \quad \text{and} \quad \theta_1 = \cos^{-1}\left(\frac{V_d}{\sqrt{2}V_s}\right) = 0.881 \text{ rad}$$

$$\therefore \quad I_d = \frac{0.9V_s}{\omega L_d}\left(1+0.881-\frac{\pi}{2}\right) = 0.279\frac{V_s}{\omega L_d}$$

$$\therefore \quad L_{d,min} = 0.279\frac{V_s}{\omega I_d}$$

Problem 5-8

u form Eq. 5-32:

$$\cos u = 1 - \frac{2\omega L_s}{\sqrt{2}V_s} I_d$$

$\therefore u = 0.3$ rad $= 17.14$ deg

V_d form Eq. 5-33

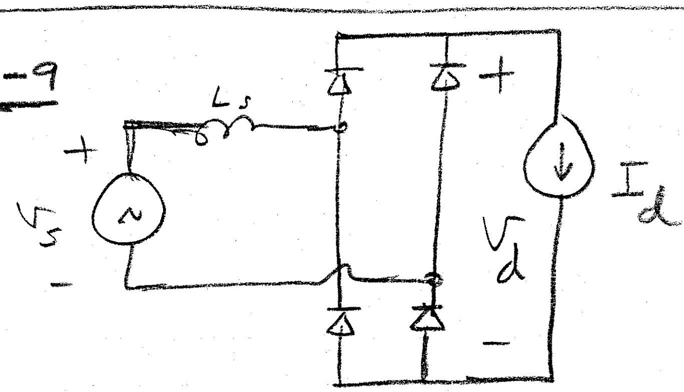
$$V_d = 0.9V_s - \frac{2\omega L_s}{\pi} I_d = 105.6\ V$$

$$P_d = V_d I_d = 1056\ W$$

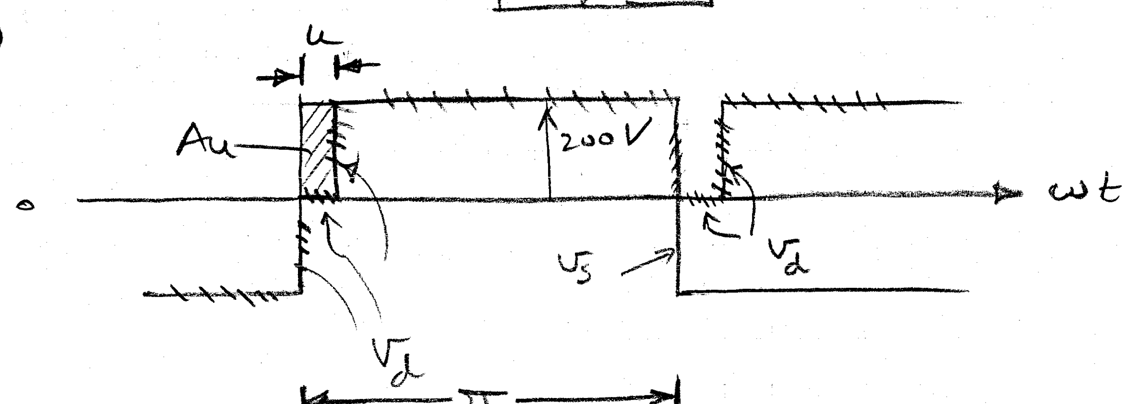
$$\text{drop } \Delta V_d\% = \frac{V_{do} - V_d}{V_{do}} \times 100 = 2.22\%$$

where $V_{do} = 0.9V_s$

Problem 5-9



(a)



$$A_u = 200\,u = 2\omega L_s I_d$$

$$\therefore\ u = \frac{2\omega L_s I_d}{200}\ \text{rad} = 0.0377\ \text{rad} = 2.16\ \text{deg}$$

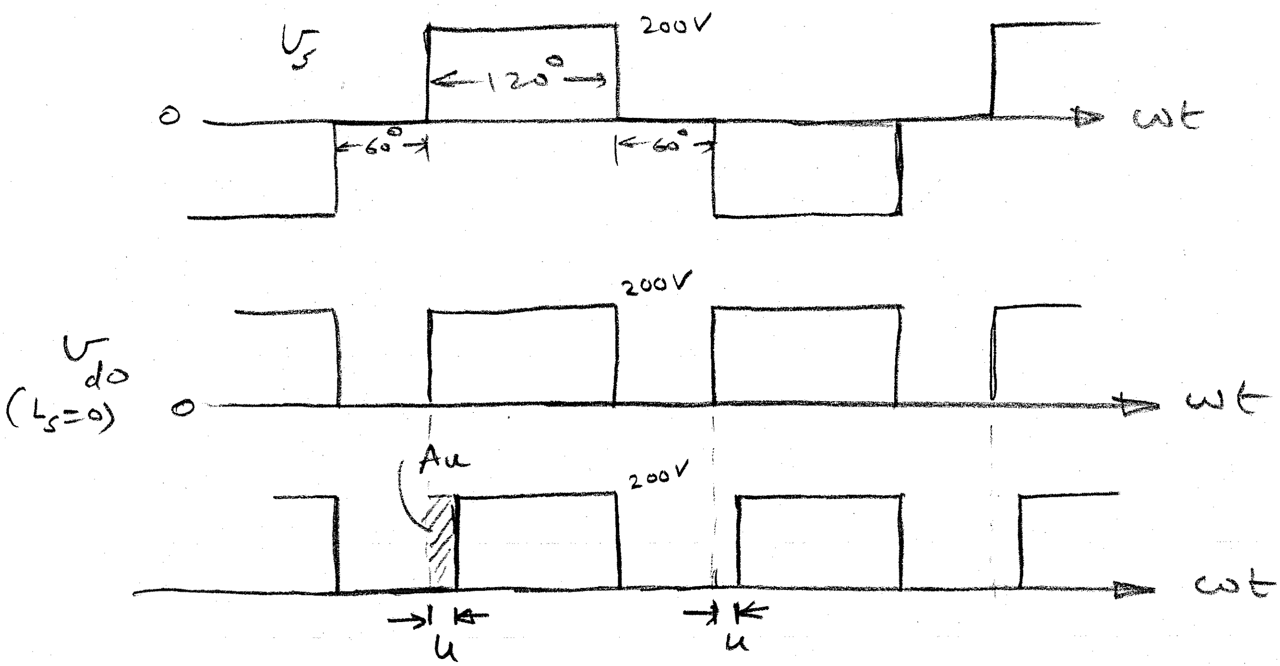
$$V_{do} = 200\ V$$

$$V_d = V_{do} - \frac{A_u}{\pi} = 200 - \frac{2\omega L_s I_d}{\pi} = 197.6$$

$$P_d = V_d I_d = 1976\ W$$

$$\text{drop } \Delta V_d\% = \frac{V_{do} - V_d}{V_{do}} \times 100 = 1.2\%$$

(b)



$$V_{do} = 200 \times \frac{120}{180} = 133.33\ V$$

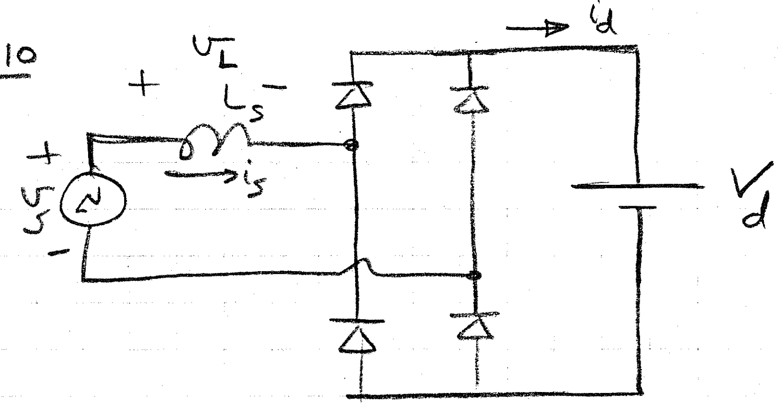
$$A_u = 200\,u = 2\omega L_s I_d$$

$$u = \frac{2\omega L_s I_d}{200}\ \text{rad} = 0.0377\ \text{rad} = 2.16\ \text{deg}$$

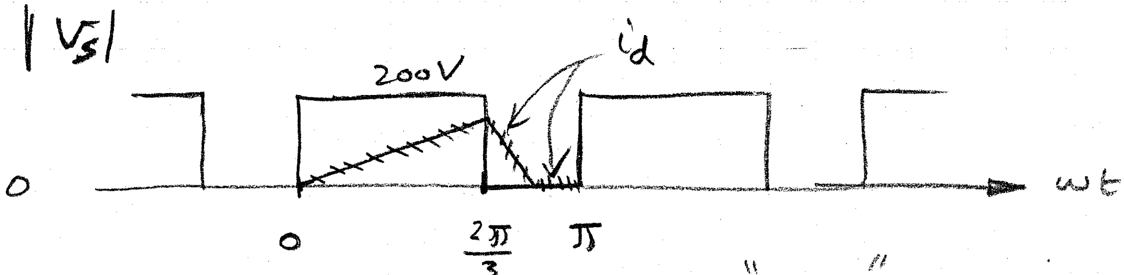
$$V_d = V_{do} - \frac{A_u}{\pi} = 133.33 - \frac{2\omega L_s I_d}{\pi} = 130.93\ V$$

$$P_d = V_d I_d = 1309.3\ W \qquad \text{drop } \Delta V_d\% = 1.8\%$$

Problem 5-10



$\omega = 2\pi f = 377 \frac{rad}{s}$



$i_d(0) = 0$ ⟵ see "Hint."

$0 < \omega t < \frac{2\pi}{3}$ $L_s \frac{di_d}{dt} = 200 - 160 = 40$

$$\therefore \frac{di_d}{d(\omega t)} = \frac{40}{\omega L_s} = \frac{40}{377 \times 10^{-3}}$$

$$i_d(\omega t = \tfrac{2\pi}{3}) = \frac{40 \times (2\pi/3)}{377 \times 10^{-3}} = 222.2\,A$$

$\omega t > \frac{2\pi}{3}$

$$L_s \frac{di_d}{dt} = -160$$

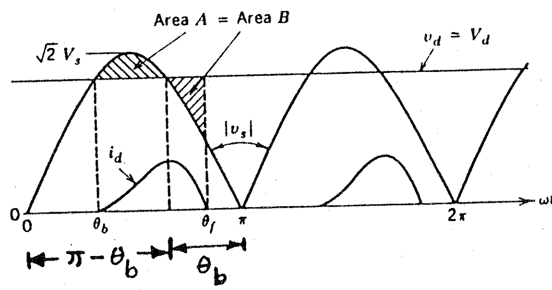
$$\therefore \frac{di_d}{d(\omega t)} = -\frac{160}{377 \times 10^{-3}} = -424.4 \frac{A}{rad}$$

$\therefore$ i_d comes down to zero in

$$\frac{222.2}{424.4} = 0.523 \text{ rad} \simeq 30^\circ$$

and it stays at zero until $\omega t = \pi$.

Problem 5-11



Calculate θ_b

$$\sqrt{2}\; V_s \sin\theta_b = V_d$$

$$\therefore\; \theta_b = \sin^{-1}\left(\frac{150V}{\sqrt{2}\; 120V}\right) = 1.084 \text{ rad.}$$

Derive $i_d(\theta)$

$$i_d(\theta) = \frac{1}{\omega L_s} \int_{\theta_b}^{\theta} v_L \; d\theta$$

$$= \frac{1}{\omega L_s} \int_{\theta_b}^{\theta} (\sqrt{2}\; V_s \sin\theta - V_d)\; d\theta$$

$$i_d(\theta) = \frac{1}{\omega L_s} [\sqrt{2}\; V_s(-\cos\theta + \cos\theta_b) - V_d(\theta - \theta_b)]$$

$$\therefore\; i_d(\theta) = -450.1 \cos\theta - 397.9\theta + 641.8$$

Calculate θ_f

$$i_d(\theta_f) = 0 = -450.1 \cos\theta_f - 397.9\; \theta_f + 641.8$$

$$\cos\theta_f + 0.884 \cdot \theta_f = 1.426$$

$$\therefore\; \theta_f = 2.56 \text{ rad}$$

Calculate $I_{d,peak}$:

The peak current occurs at $(\pi - \theta_b)$ since V_L is positive between θ_b and $\pi - \theta_b$.

$$I_{d,peak} = i_d(\pi - \theta_b) \quad ; \quad \pi - \theta_b = 2.058 \text{ rad}$$

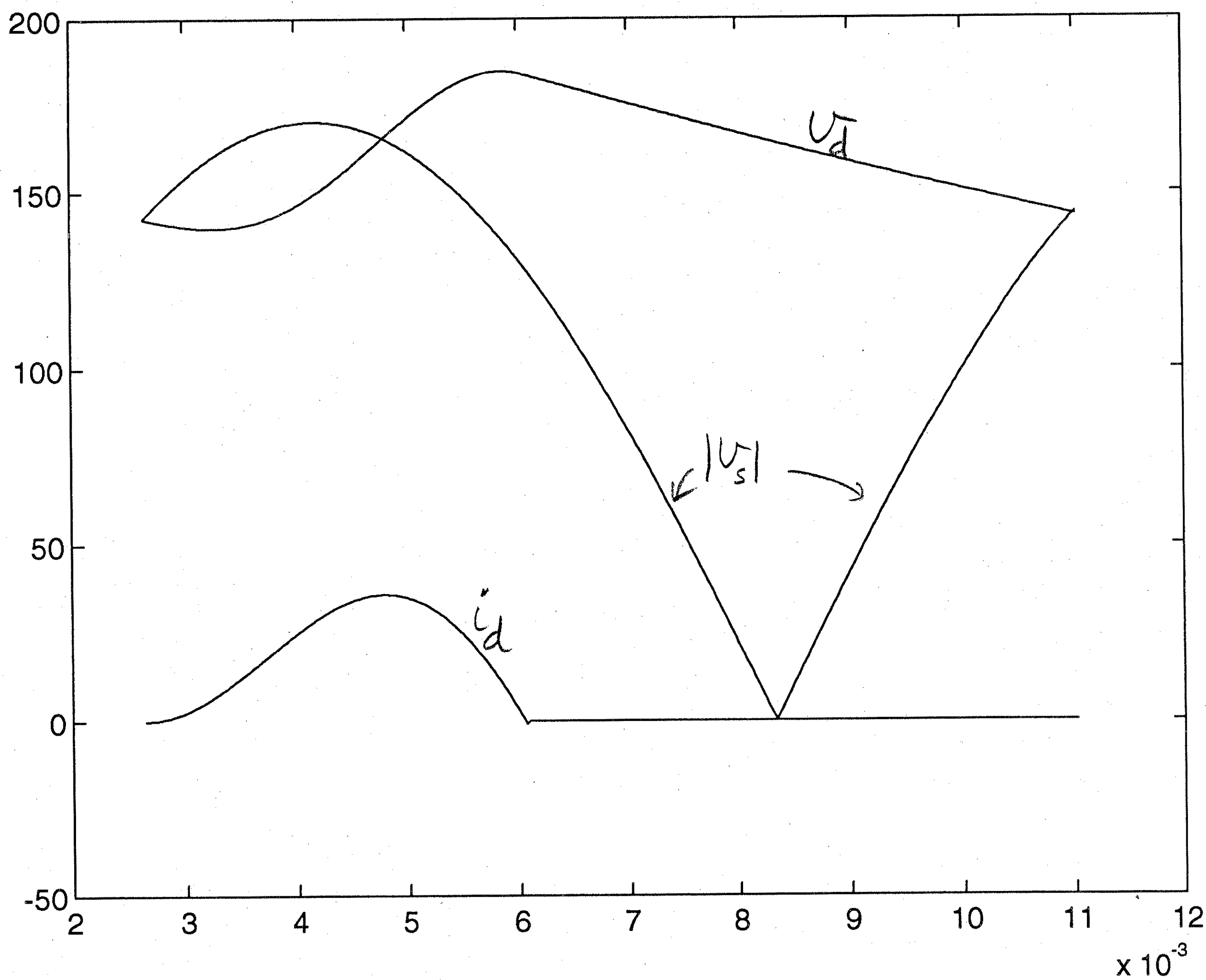
$$\therefore\; I_{d,peak} = -450.1\cos(2.058) - (397.9)(2.058) + 641.8 = 33.6A$$

Problem 5-12

Note that v_d in the MATLAB program listing is referred as the capacitor voltage v_c.

$V_d \simeq 160.87\ V$

and $P_d \simeq 1304\ W$



Note that the waveforms are identical to those in Fig. 5-22.

Problem 5-13

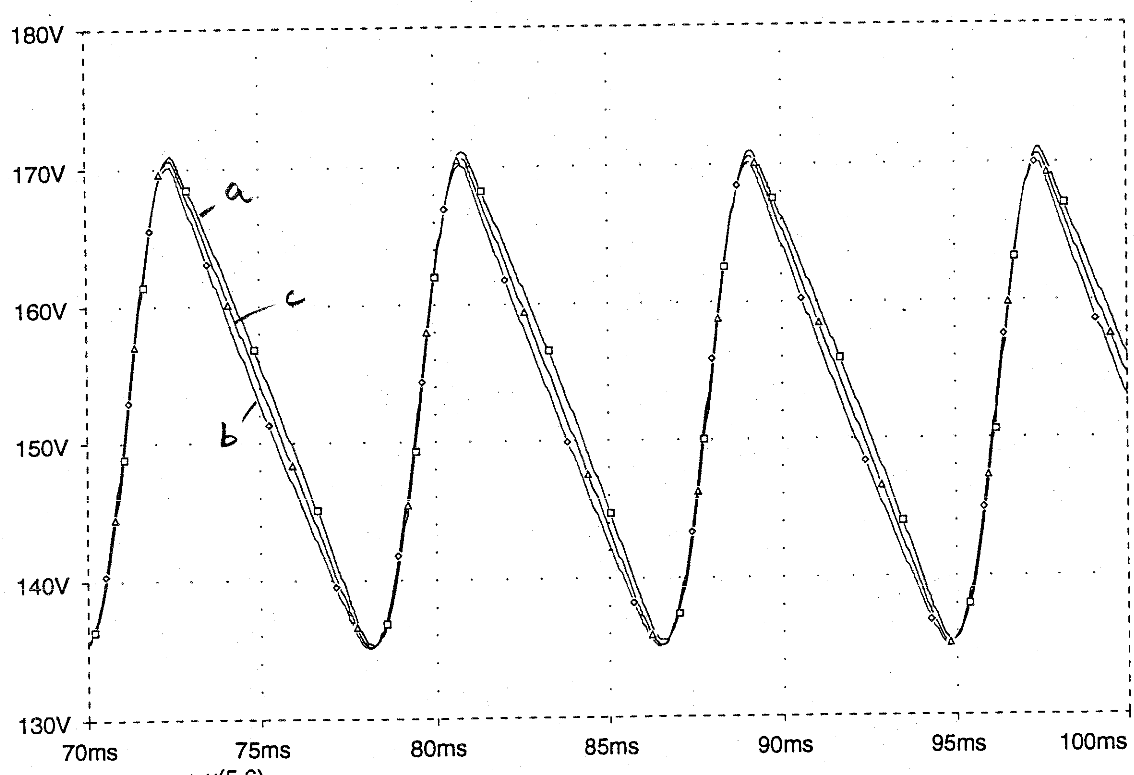
The simulation in part a results in $V_d = 153.04V$. Therefore, for a power of 1 kW, the following values are calculated:

$$R_{Load} = \frac{V_d^2}{1000W} = 23.42\Omega \text{ in part b}$$

and

$$I_{Load} = \frac{1000\ W}{V_d} = 6.534A$$

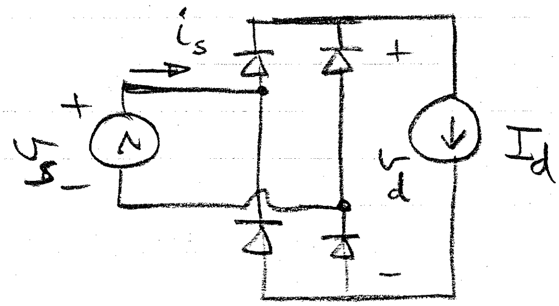
To compare results form these three parts, all cases are run in the same file, as the listing indicates. The waveforms are very similar.

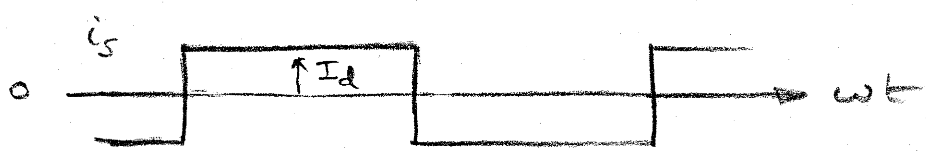


```
Prob5_13a
* Single-Phase, Diode-Bridge Rectifier
.LIB PWR_ELEC.LIB
.PARAM   FREQ = 60.0Hz
*
LS       1  2  1mH
RS       2  3  0.4
*
rdc      4  5  1u
GDC      5  6  VALUE={1000/V(5,6)}
CD       5  6  1000uF IC=160V
*
XD1      3  4  DIODE_WITH_SNUB
XD3      0  4  DIODE_WITH_SNUB
XD2      6  0  DIODE_WITH_SNUB
XD4      6  3  DIODE_WITH_SNUB
*
VS       1  0  SIN(0 170V {FREQ} 0 0 0)
*
.TRAN    50us   100ms   0s    50us   UIC
.PROBE
.FOUR    60.0  v(1) i(LS)   i(rdc)   v(5,6)
.END
Prob5_13b
* Single-Phase, Diode-Bridge Rectifier
.LIB PWR_ELEC.LIB
.PARAM   FREQ = 60.0Hz
*
LS       1  2  1mH
RS       2  3  0.4
*
rdc      4  5  1u
RLOAD    5  6  23.42
CD       5  6  1000uF IC=160V
*
XD1      3  4  DIODE_WITH_SNUB
XD3      0  4  DIODE_WITH_SNUB
XD2      6  0  DIODE_WITH_SNUB
XD4      6  3  DIODE_WITH_SNUB
*
VS       1  0  SIN(0 170V {FREQ} 0 0 0)
*
.TRAN    50us   100ms   0s    50us   UIC
.PROBE
.FOUR    60.0  v(1) i(LS)   i(rdc)   v(5,6)
.END
Prob5_13c
* Single-Phase, Diode-Bridge Rectifier
.LIB PWR_ELEC.LIB
.PARAM   FREQ = 60.0Hz
*
LS       1  2  1mH
RS       2  3  0.4
*
rdc      4  5  1u
ILOAD    5  6  6.534A
CD       5  6  1000uF IC=160V
*
XD1      3  4  DIODE_WITH_SNUB
XD3      0  4  DIODE_WITH_SNUB
XD2      6  0  DIODE_WITH_SNUB
XD4      6  3  DIODE_WITH_SNUB
*
VS       1  0  SIN(0 170V {FREQ} 0 0 0)
*
.TRAN    50us   100ms   0s    50us   UIC
.PROBE
.FOUR    60.0  v(1) i(LS)   i(rdc)   v(5,6)
.END
```

Note: No blank lines between 3 parts.

Problem 5-14





$$I_s\,(\text{rms}) = I_d$$

From Fourier analysis:

$$I_{s1}\,(\text{rms}) = \frac{4}{\pi}\sqrt{2}\,I_d = 0.9\,I_d$$

DPF angle $\phi_1 = 0$

$$\%THD_{i_s} = 100\,\frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} = 100 \times \frac{\sqrt{1 - 0.9^2}}{0.9} = 48.4\%$$

$$DPF = 1.0$$

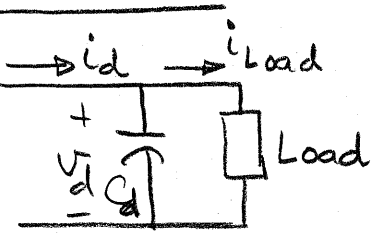
$$PF = \frac{I_{s1}}{I_s} \cdot DPF = 0.9$$

$$CF = \frac{I_{s,\text{peak}}}{I_s\,(\text{rms})} = 1$$

Problem 5-15

The results are quite similar to those in Example 5-3.

Problem 5-16



Representing the load by a constant instaneous power $p_d(t) = 1000\,W$ has the following consequences:

As v_d increases, i_{Load} goes down instantaneously. This corresponds to a negative incremental resistance (in a normal resistance, as the voltage across it goes up, the current also increases). Therefore, such a load representation introduces a negative damping. Loads such as switch-mode, regulated dc power supplies discussed in chapter 10 are examples of such loads.

With the load represented as above, the system with $C_d = 200\,\mu F$ in part (a) is operated and the dc voltage collapses to zero and the PSpice simulation ends with a convergence error. In part (b) with $C_d = 500\,\mu F$, the simulation becomes unstable with unsymmetric current waveforms during the negative half-cycles, compared to the positive half-cycles. This becomes very clear if the simulation is run upto 200 ms.

The results with C_d of 1000 μF and 1,500 μF are tablulated below based on the following observations from the PSpice output:

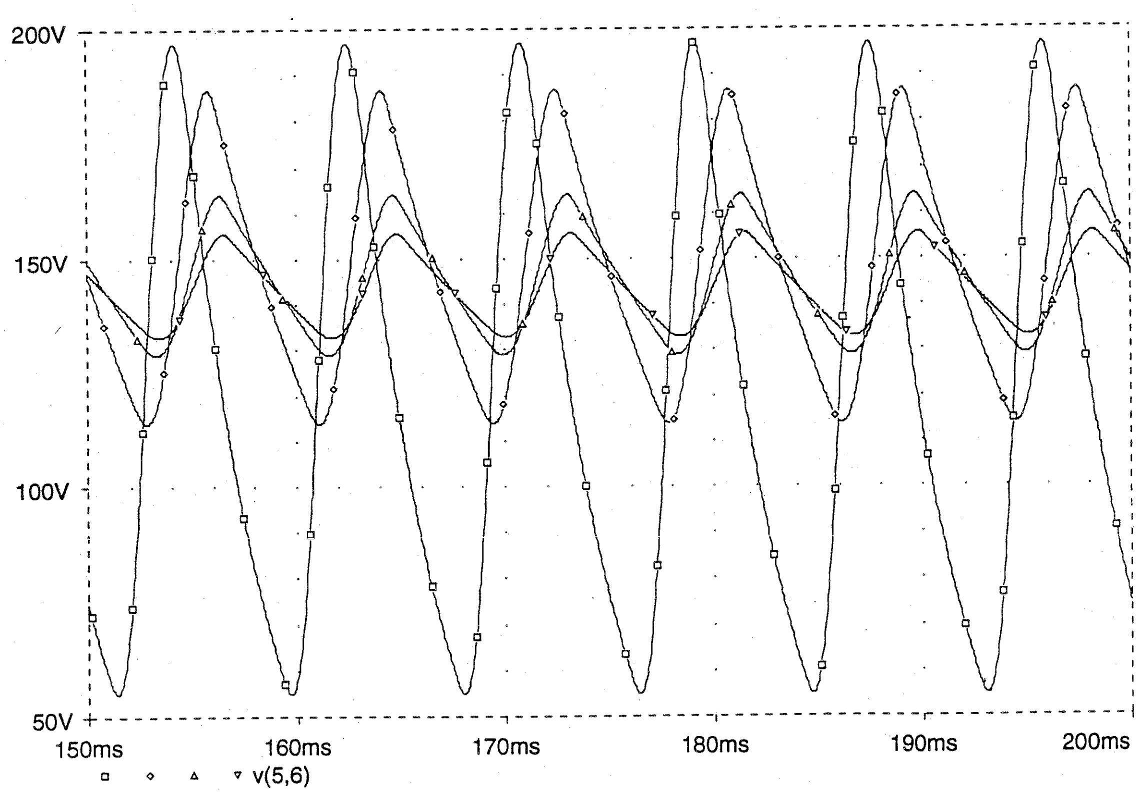
$C_d = 1000\,\mu F$	$C_d = 1{,}500\,\mu F$
$v_{d,min} = 131.1\,V$	$v_{d,min} = 133.8\,V$
$v_{d,max} = 164.5\,V$	$v_{d,max} = 156.0\,V$
$\phi_1 = -15.3^\circ$ (i_s lags v_s)	$\phi_1 = -17.81^\circ$
$THD_i = 72.3\%$ (based on harmonics upto the 25th)	$THD_i = 69.7\%$

[From Eq. 3-45, $PF = \dfrac{1}{\sqrt{1 + THD_i^2}} \cdot DPF$]

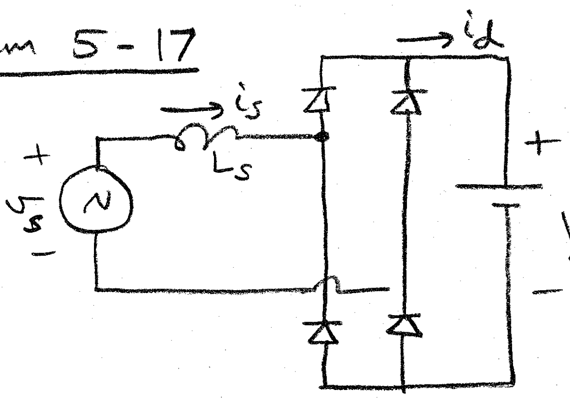
$DPF = 0.965$ (lag)	$DPF = 0.952$ (lag)
$\therefore$ $PF = 0.782$	$PF = 0.781$
$\Delta V_{d\,p\text{-}p} = 33.4\,V$	$\Delta V_{d\,pp} = 22.2\,V$

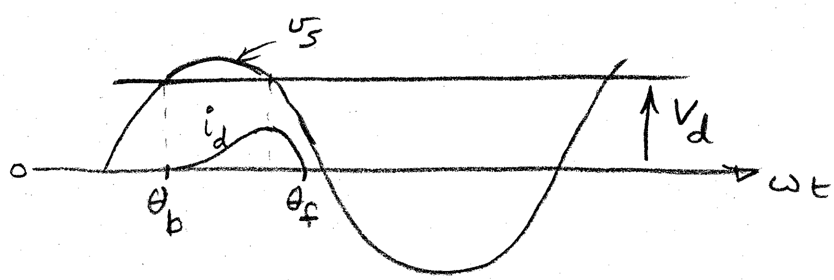
As a continuation of this problem, the load is represented in the attached PSpice listing by a 20 Ω resistor and the resulting dc-voltage v_d waveforms are plotted for the given 4 values of C_d.

```
Prob5_16A modified to represent load with a resistance
* Single-Phase, Diode-Bridge Rectifier
.LIB PWR_ELEC.LIB
.PARAM    FREQ = 60.0Hz
.PARAM    CVALUE = 1
*
LS        1  2  2mH
RS        2  3  0.4
*
rdc       4  5  1u
RLOAD     5  6  20
CD        5  6  {CVALUE} IC={160 + (CVALUE - 1000u)/100u}
*
XD1       3  4  DIODE_WITH_SNUB
XD3       0  4  DIODE_WITH_SNUB
XD2       6  0  DIODE_WITH_SNUB
XD4       6  3  DIODE_WITH_SNUB
*
VS        1  0  SIN(0 170V {FREQ} 0 0 0)
*
.TRAN     50us   200ms   0s    50us   UIC
.PROBE
.FOUR     60.0  25 v(1) i(LS)   i(rdc)   v(5,6)
.STEP PARAM CVALUE  LIST 200u,500u,1000u,1500u
.END
```



Problem 5-17





$$\theta_b < \theta < \theta_f$$

$$i_d(\theta) = \frac{1}{\omega L_s} \int_{\theta_b}^{\theta} \left[\sqrt{2}\, V_s \sin\omega t \cdot d(\omega t) - V_d\right] \cdot d(\omega t)$$

$$= \sqrt{2}\left(\frac{V_s}{\omega L_s}\right) \int_{\theta_b}^{\theta} \sin\omega t \cdot d(\omega t) - \frac{V_d}{V_s} \cdot \left(\frac{V_s}{\omega L_s}\right)(\theta - \theta_b)$$

$$\therefore \quad \frac{i_d(\theta)}{I_{\text{short circuit}}} = \sqrt{2}\left(\cos\theta_b - \cos\theta\right) - \frac{V_d}{V_s}(\theta - \theta_b)$$

where, $I_{\text{short circuit}} = \frac{V_s}{\omega L_s}$.

For a specific value of $\left(\frac{V_d}{V_s}\right)$, the following can be uniquely calculated:

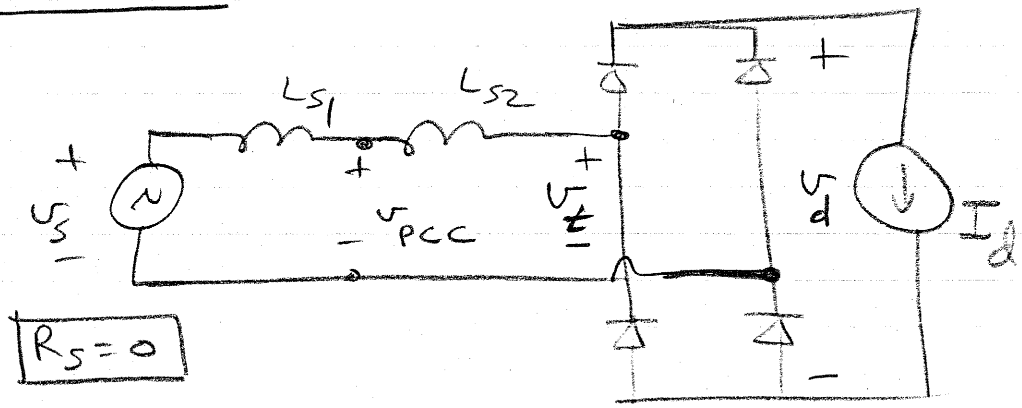
θ_b, θ_f, and from the above equation

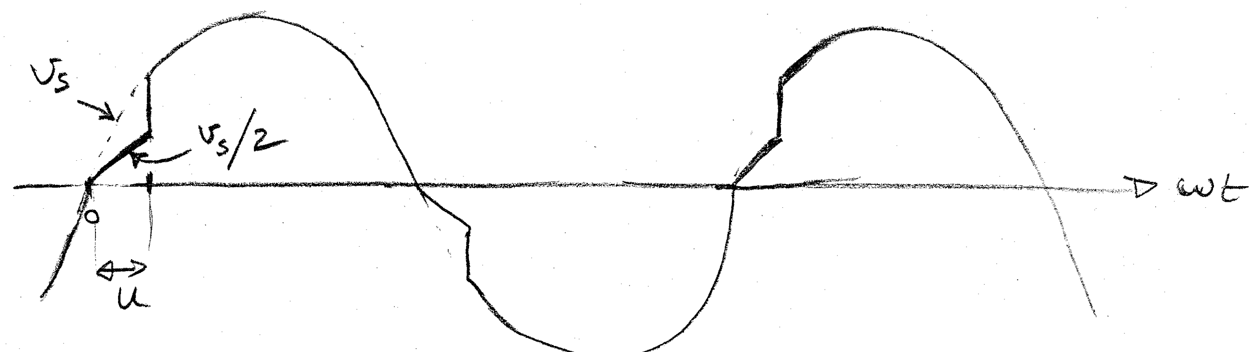
$\frac{i_d(\theta)}{I_{\text{short circuit}}}$ for $\theta_b < \theta < \theta_f$. Since $I_d = \left[\int_{\theta_b}^{\theta_f} i_d(\theta) \cdot d\theta\right] / \pi$,

$\frac{I_d}{I_{\text{short circuit}}}$ can also be uniquely calculated.

Since $i_d(\theta)$ is specified, PF, DPF, THD and CF can be uniquely obtained.

Problem 5-18





Procedure:

Given V_s, ω, $L_s (= L_{s1} + L_{s2})$ and I_d,

u can be calculated

During $0 < \omega t < u$

$$v_{pcc}(t) = \frac{v_s}{2}$$ as shown above.

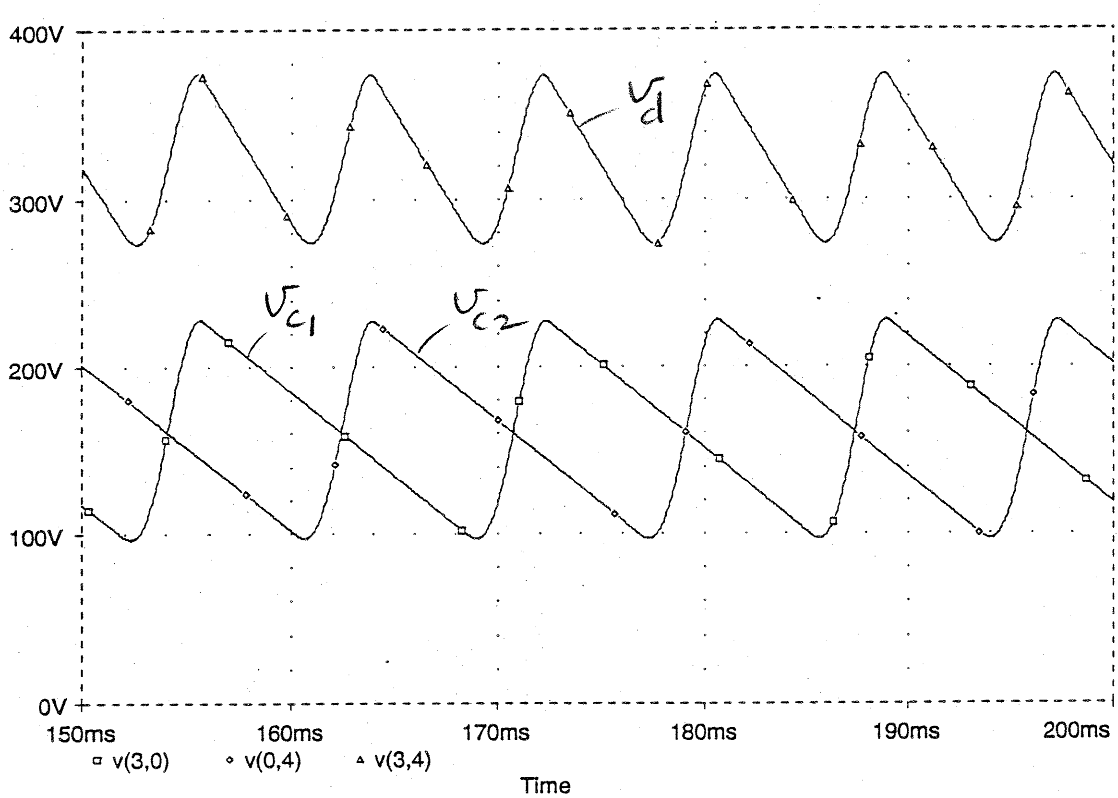
Using Fourier analysis in PSpice,

$$(THD)_{v_{pcc}} = 5.33\%$$

Problem 5-19

(a)

```
Prob5_19.cir
* Single-Phase, Voltage-Doubler Rectifier
* Power Electronics: Simulation, Analysis & Education.....by N. Mohan.
.LIB PWR_ELEC.LIB
.PARAM  FREQ = 60.0Hz
LS      1  2   1mH
*
ILOAD   3  4   10A
CD1     3  0   1000uF IC=145V
CD2     0  4   1000uF IC=180V
*
XD1     2  3   DIODE_WITH_SNUB
XD2     4  2   DIODE_WITH_SNUB
*
VS      1  0   SIN(0 170V {FREQ} 0 0 0)
*
.TRAN   50us   200ms   0s    50us    UIC
.PROBE
.END
```

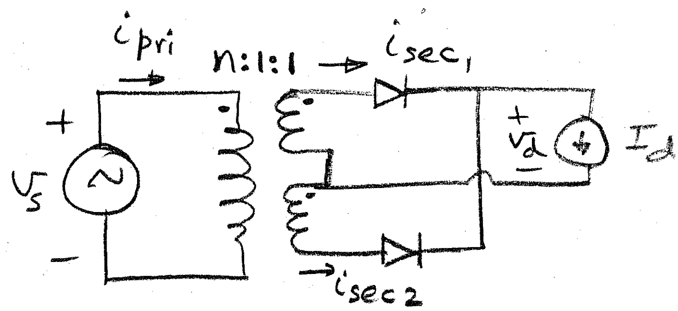


(b) $\Delta V_{d\,p\text{-}p} / V_d = 99.6\text{ V} / 321.7\text{ V} \simeq 31\%$

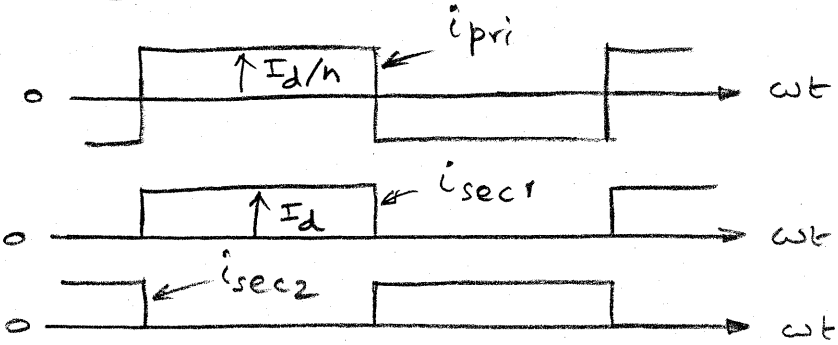
(c) Single-phase rectifier

$\Delta V_{d\,p\text{-}p} / V_d = 117.61 / 329.2\text{V} = 35.7\%$

Problem 5-20



V_s = rms voltage



Primary: V_s = rms voltage, $I_s = I_d/n$ = rms current

Secondary 1: V_s/n = rms voltage, $\frac{I_d}{\sqrt{2}}$ = rms current

Secondary 2: Same as secondary 1

$\therefore$ Winding volt-amp ratings

$$= V_s \frac{I_d}{n} + 2\left(V_s/n \cdot \frac{I_d}{\sqrt{2}}\right)$$

$$= \frac{V_s I_d}{n}\left(1+\sqrt{2}\right)$$

Transformer VA rating = Winding ratings$/2$

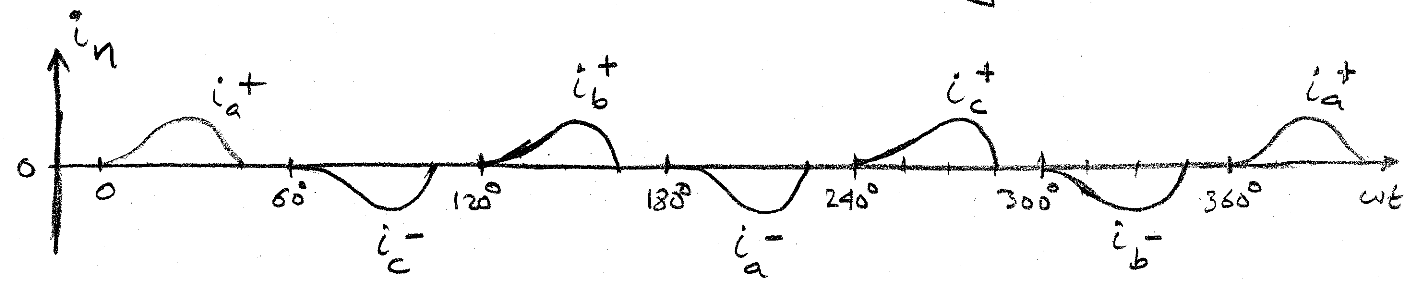
$$= \frac{V_s I_d}{n} \frac{1+\sqrt{2}}{2}$$

$V_d = 0.9\, V_s/n$ $\therefore$ Load Power $= \frac{0.9\, V_s}{n} I_d$

$$\therefore \frac{\text{Transformer VA}}{\text{Load Power}} = \frac{\frac{1+\sqrt{2}}{2}}{0.9} = 1.34$$

Problem 5-21

If the load currents flow for less than 60 degrees per half-cycle of the line-to-neutral voltages, then the neutral-current will have the following waveform.



where the superscripts + and − refer to the phase current waveforms in the positive and the negative half-cycles, respectively.

Considering only i_a, lets say that its rms value is I_{line}. If the area under the square of each pulse is A, then the rms value

$$I_{line} = \sqrt{\frac{2A}{2\pi}}$$ for phase A.

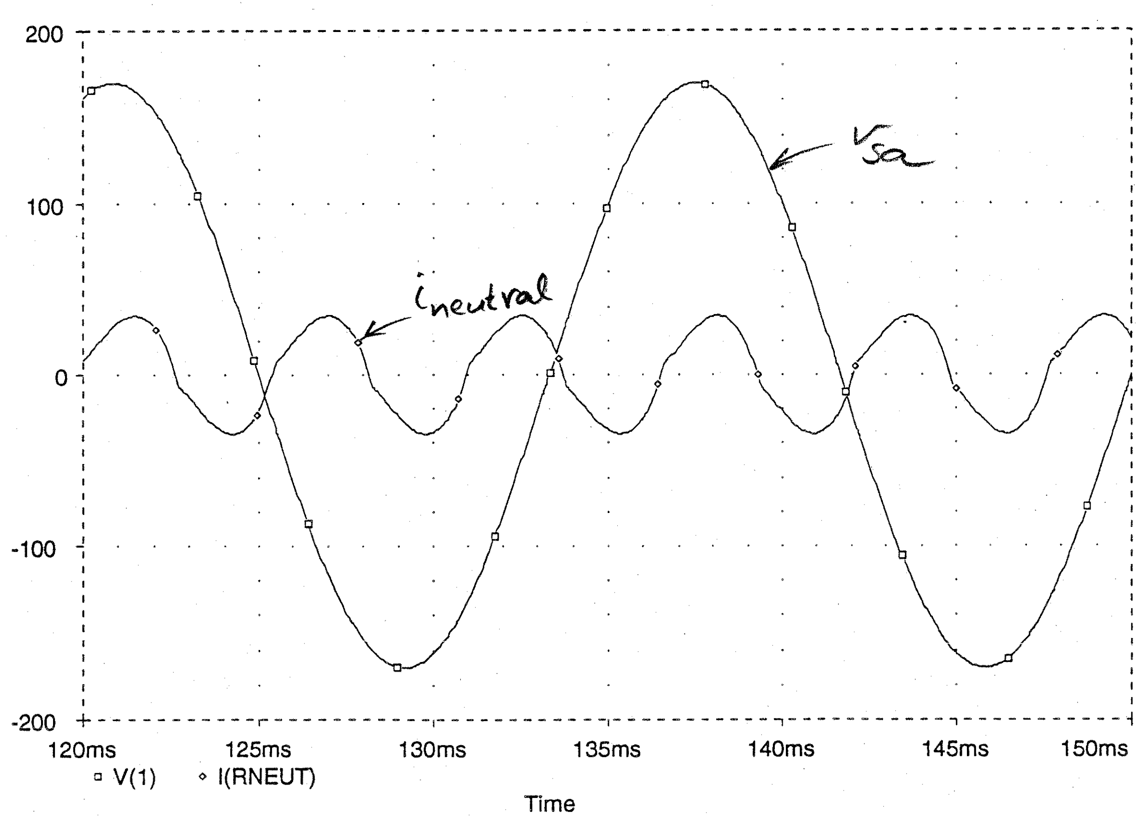
In the i_n waveform, the rms value will be

$$I_n = \sqrt{\frac{6A}{2\pi}}$$ because it has 6 pulses in 2π radians.

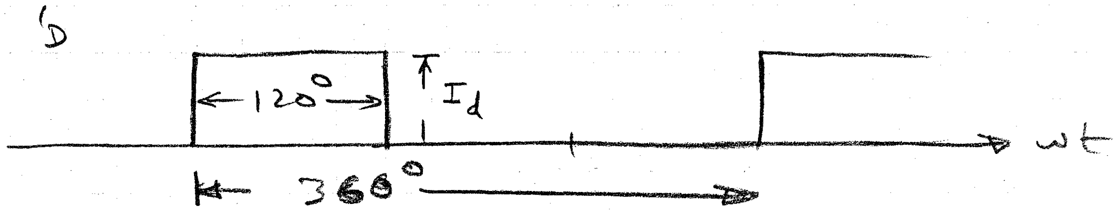
$\therefore \; I_n = \sqrt{3}\, I_{line}$.

Problem 5-22

```
Prob5_22.cir
* Neutral Current in a 3-Phase, 4-Wire Distribution System
* Power Electronics: Simulation, Analysis & Education.....by N. Mohan.
.PARAM FREQ = 60.0
*
RNEUT   10   0    1m
*
RMEASA    1    11   1m
RMEASB    2    12   1m
RMEASC    3    13   1m
*
XRECTA    11   10   1_PHASE_RECTIFIER    PARAMS:  IC_CAP = 160V  LOAD_RES = 20
XRECTB    12   10   1_PHASE_RECTIFIER    PARAMS:  IC_CAP = 140V  LOAD_RES = 20
XRECTC    13   10   1_PHASE_RECTIFIER    PARAMS:  IC_CAP = 180V  LOAD_RES = 20
*
VSA       1    0    SIN(0 170 {FREQ} 0 0 0)
VSB       2    0    SIN(0 170 {FREQ} 0 0 -120)
VSC       3    0    SIN(0 170 {FREQ} 0 0 -240)
*
.TRAN     50us   150ms   0s    50us     UIC
.PROBE  i(RNEUT) i(RMEASA) i(RMEASB) i(RMEASC) v(1)
*
*
.SUBCKT  1_PHASE_RECTIFIER  101 100 PARAMS:   IC_CAP = 150V  LOAD_RES = 20
--
--    <-- 1-phase rectifier subcircuit statements
.ENDS         should go in here.
*
.END
```



Problem 5-23



$$I_D\,(avg) = \frac{120°}{360°} \cdot I_d = \frac{I_d}{3}$$

$$I_D\,(rms) = \sqrt{\frac{I_d^2}{3}} = \frac{I_d}{\sqrt{3}}$$

Problem 5-24

(a) Volt-radian Area $A_u = \omega L_s I_d$

Assume $\dfrac{d\,v_{comm}}{d\,(\omega t)} \simeq \sqrt{2}\,V_{LL}$

or $v_{comm} = \left(\sqrt{2}\,V_{LL}\right)\omega t$

$$A_u = \omega L_s I_d = \int_0^u \frac{v_{comm}}{2} \cdot d(\omega t)$$

$$= \frac{\sqrt{2}\,V_{LL}}{2}\int_0^u \omega t \cdot d(\omega t)$$

$$= \frac{\sqrt{2}\,V_{LL}}{2\times 2}\,(\omega t)^2\Big|_0^u = \frac{V_{LL}}{2\sqrt{2}}\,u^2$$

$$\therefore\; u = \left(\frac{2\sqrt{2}\,\omega L_s I_d}{V_{LL}}\right)^{1/2}$$

(b) Substituting values, $u = 18.35°$

Problem 5-25

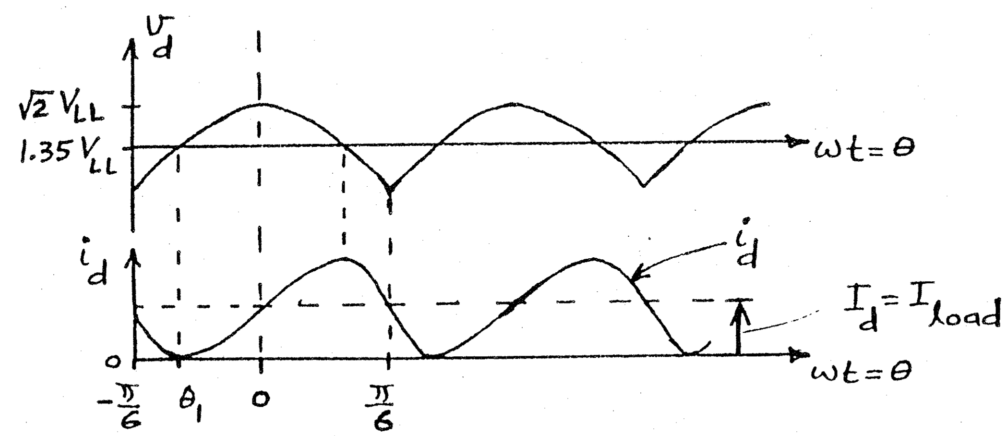
The following values are obtained from the PSpice output:

	$C_d = 220\mu F$	$C_d = 550\mu F$	$C_d = 1{,}100\mu F$	$C_d = 1500\mu F$	$C_d = 2{,}200\mu F$
$V_{d,min} =$	262.5V	271.9V	274.2V	274.8V	275.2V
$V_{d,max} =$	293.7V	281.1V	278.4V	277.7V	277.2V
$\therefore \Delta V_{d(p-p)} =$	31.2V	9.2V	4.2V	2.9V	2.0V
$\phi_1 =$	-14.4° (i_s lags v_s)	-13.67°	-13.23°	-13.12°	13.02°
$THD_i =$ (based on 25 harmonics)	73.8%	57.9%	53.85%	52.9%	52.2%
DPF =	0.968 (lag)	0.972 (lag)	0.973 (lag)	0.974 (lag)	0.974 (lag)

[From Eq. 3-45, $PF = \dfrac{1}{\sqrt{1 + THD_i^2}} \cdot DPF$]

$\therefore$ PF =	0.779	0.841	0.857	0.861	0.863

Problem 5-26



Derive equation for $i_d(\theta)$:

$V_d = 1.35\ V_{LL}$ (continuous conduction)

$$i_d(\theta) = \frac{1}{\omega L_d} \int (\sqrt{2}\ V_{LL} \cos\theta - 1.35 V_{LL})\, d\theta \qquad -\frac{\pi}{6} < \theta < \frac{\pi}{6}$$

$$i_d(\theta) = \frac{V_{LL}}{\omega L_d} (\sqrt{2}\ \sin\theta - 1.35\theta + K)$$, K is a constant of integration.

Calculate K:

At $\theta = \theta_1$, $i_d = 0$. To calculate (θ_1) defined in the figure above

$\sqrt{2}\ V_{LL} \cos(\theta_1) = 1.35\ V_{LL}$ or $\theta_1 = -0.302$ rad

Using $\theta = \theta_1$ in the expression for $i_d(\theta)$

$$i_d(\theta_1) = \frac{V_{LL}}{\omega L_d} (\sqrt{2}\ \sin(-0.302) - 1.35\ (-0.302) + K) = 0 \qquad \therefore K = 0.0129$$

and,

$$i_d(\theta) = \frac{V_{LL}}{\omega L_d} (\sqrt{2}\ \sin\theta - 1.35\ \theta + 0.0129)$$

$$-\frac{\pi}{6} < \theta < \frac{\pi}{6}$$

Calculate I_d:

$$I_d = \int_{-\pi/6}^{\pi/6} i_d(\theta)d\theta \Big/ \frac{\pi}{3}$$

$$= \frac{3}{\pi}\frac{V_{LL}}{\omega L_d}\int_{-\pi/6}^{\pi/6}(\sqrt{2}\ \sin\theta - 1.35\theta + 0.0129)d\theta$$

$$= \frac{3V_{LL}}{\pi\omega L_d}\left[-\sqrt{2}\ \cos\theta - \frac{1.35}{2}\ \theta^2 + 0.0129\theta\right]_{-\pi/6}^{\pi/6}$$

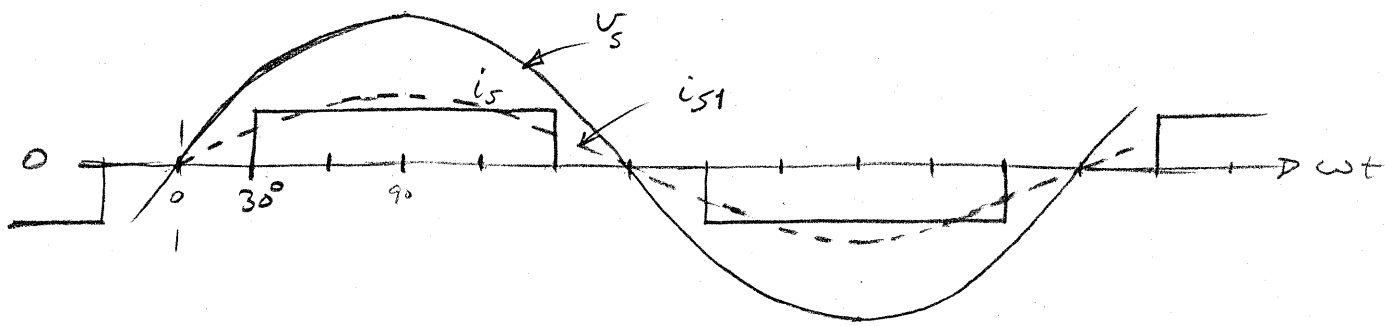
$$= \frac{3V_{LL}}{\pi\omega L_d}\ (0.0129(\pi/6 - (-\pi/6))) = \frac{3V_{LL}}{\pi\omega L_d}\ (0.0129\ \frac{\pi}{3})$$

$$I_d = 0.0129\ \frac{V_{LL}}{\omega L_d}$$

Therefore,

$$L_{d,min} \simeq \frac{0.013}{\omega I_d}\ V_{LL}$$

Problem 5-27



Using $\omega t = 0$ as shown above, i_s odd and quarter-wave symmetric. Therefore, from Table 3-1

$a_h = 0$ for all h, $b_h = 0$ for all even h and,

$$b_h = \frac{4}{\pi}\int_0^{90^\circ} i_s \cdot \sin h\omega t \cdot d(\omega t) \quad \text{for odd } h$$

for

$0 < \omega t < 30^\circ$, $i_s = 0$

$30^\circ < \omega t < 90^\circ$, $i_s = I_d$

$$\therefore \quad b_h = \frac{4 I_d}{\pi}\int_{30^\circ}^{90^\circ} \sin(h\omega t) \cdot d(\omega t) = \frac{4 I_d}{\pi h} \cos h\omega t \Big|_{90^\circ}^{30^\circ}$$

$$= 4\frac{I_d}{\pi h}\left[\cos(30h^\circ) - \cos(90h^\circ)\right] \qquad h = \text{odd}$$

Quantity within the bracket:

$$\left[\cos(30h^\circ) - \cos(90h^\circ)\right] = \frac{\sqrt{3}}{2} \quad \text{for } h = 1, 5, 7, 11, 13, \ldots$$

$$= 0 \quad \text{for all other odd } h$$

$$\therefore \quad b_h = \frac{4}{\pi}\frac{\sqrt{3}}{2}\frac{I_d}{h} = \frac{2\sqrt{3}}{\pi h} I_d \quad \text{for } h = 1 \text{ and } 6n \pm 1 \text{ where } n = 1, 2, 3, \ldots$$

From Eq. 3-21

$$i_s(t) = \frac{2\sqrt{3}}{\pi} I_d \left[\sum \frac{1}{h} \sin(h\omega t)\right]$$

$$\therefore \quad i_{s1}(t) = \frac{2\sqrt{3}}{\pi} I_d \sin \omega t \qquad \text{for } h = 1$$

$\therefore \ \phi_1 = 0$ and DPF $= 1.0$ Eq 5-72

$$I_{s1} = \frac{1}{\sqrt{2}}\left(\frac{2\sqrt{3}}{\pi} I_d\right) = \frac{\sqrt{6}}{\pi} I_d$$ Eq 5-70

$$\frac{I_{sh}}{I_{s1}} = \frac{1}{h}$$ Eq. 5-71

$$I_s = \sqrt{I_{s1}^2 + \sum I_{sh}^2}$$

$$= \sqrt{\left(\frac{\sqrt{6}}{\pi} I_d\right)^2 \left[1 + \sum_{h=5} \left(\frac{1}{h}\right)^2\right]}$$

$$= \frac{\sqrt{6}}{\pi} I_d \sqrt{1 + \sum_{h=5} \frac{1}{h^2}} = 0.816\, I_d$$ Eq 5-69

$$PF = DPF \cdot \frac{I_{s1}}{I_s} = 0.955$$ Eq 5-73

Problem 5-28

In the single-phase rectifier:

$V_{d,max} = 138.6\,V$ $\quad \Delta V_{d\,p\text{-}p} = 31.3\,V$

$V_{d,min} = 107.3\,V$

$V_{d\,(avg)} = 122.2\,V$ $\quad \% \Delta V_{d\,p\text{-}p} = \frac{31.3}{122.2} \times 100 = 25.6\,\%$

$\phi_1 = -24.31^\circ$, $THD_i = 45.34\%$

$\therefore$ DPF $= 0.911$ (lagging) $\quad$ PF $= 0.83$ [using Eq 3-45]

Energy Storage (based on $V_{d(avg)}$) $= \frac{1}{2} C V_{d\,avg}^2 = 8.21$ W-s

Three-Phase Rectifier

By trial-and-error, it is determined that $C_d = 275\,\mu F$ results in approximately the same energy stored as in the 1-phase case. However, the simulation becomes unstable with a constant power load. Therefore, it is represented by $R_{Load} = 14.8\,\Omega$.

$V_{d,max} = 285.9\,V$

$V_{d,min} = 257.8\,V$ $\quad \Delta V_{d\,p-p} = 28.1\,V$

$V_{d,avg} = 271.4\,V$ $\quad \% \Delta V_{d\,p-p} = 10.35\,\%$

$\phi_1 = -15.64^\circ$ (lag), $\quad THD_i = 43.5\,\%$

DPF = 0.963 (lagging)

PF = 0.883

Comparing the two results, the 3-phase rectifier results in substantially less dc-voltage ripple and also operates at a better power factor.

Problem 5-29

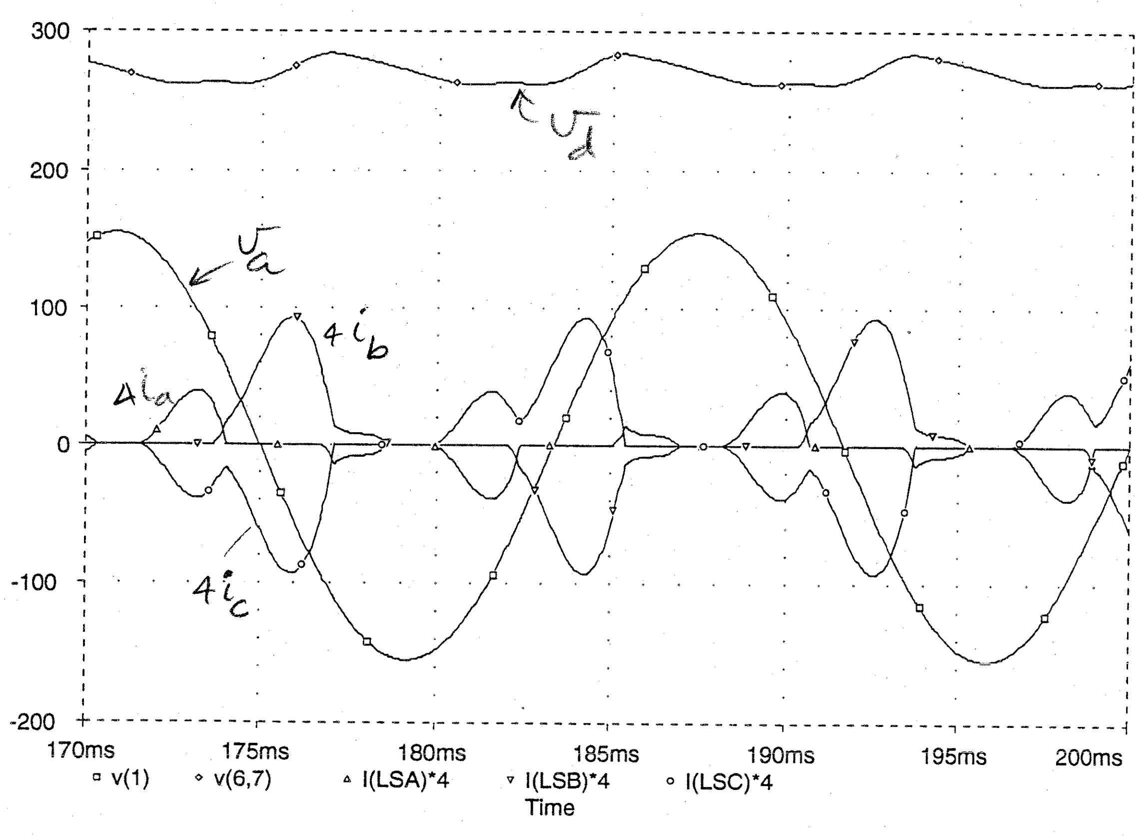
From PSpice simulations:

$(THD)_{i_a} = 109.4\%$

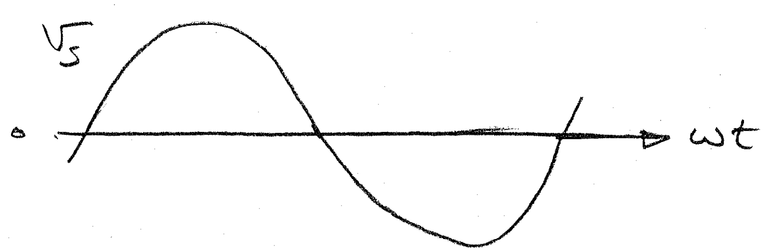
$(THD)_{i_b} = 74.1\%$

$(THD)_{i_c} = 59.4\%$

Compared to the balanced input voltages in Example 5-7 which resulted in a THD of 54.9%, the THD in currents here is much higher.

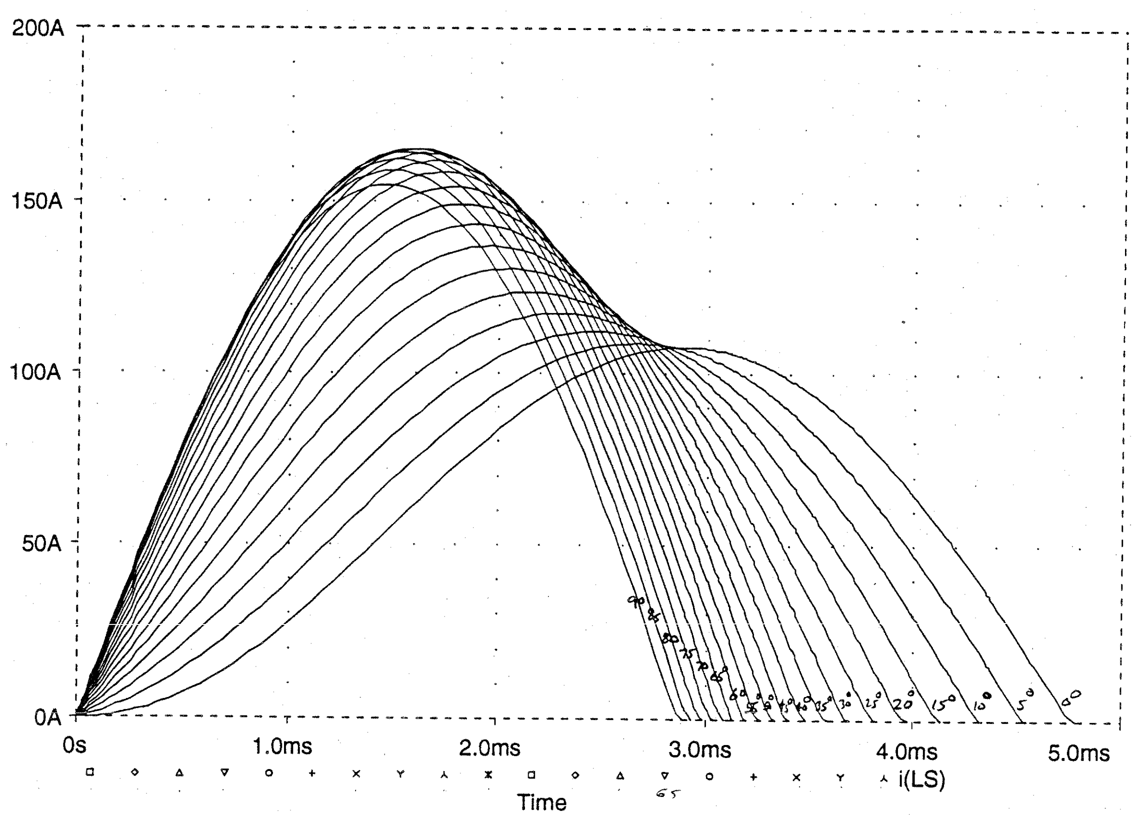


Problem 5-30



It is clear that the switching instant being sought lies between $\omega t = 0$ and $\omega t = 90°$. This is simulated in PSpice by applying a voltage source $v_s = \hat{V}_s \sin(\omega t + \alpha)$ at time $t = 0$, where α is varied from $0°$ to $90°$ in steps of $5°$.

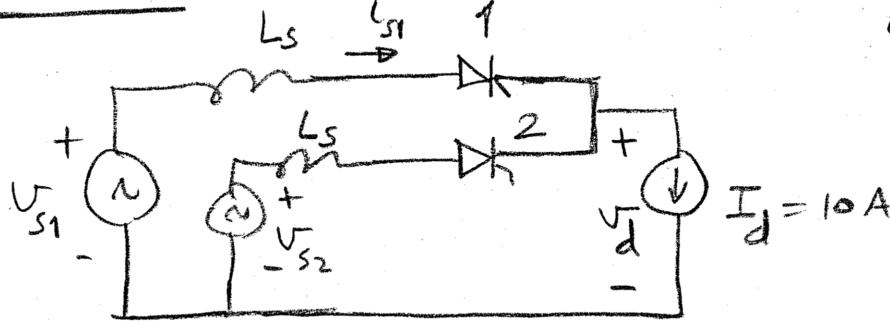
Using a color display, it is clear that it corresponds to $\alpha \cong 70°$ with the peak current of 165.3A.



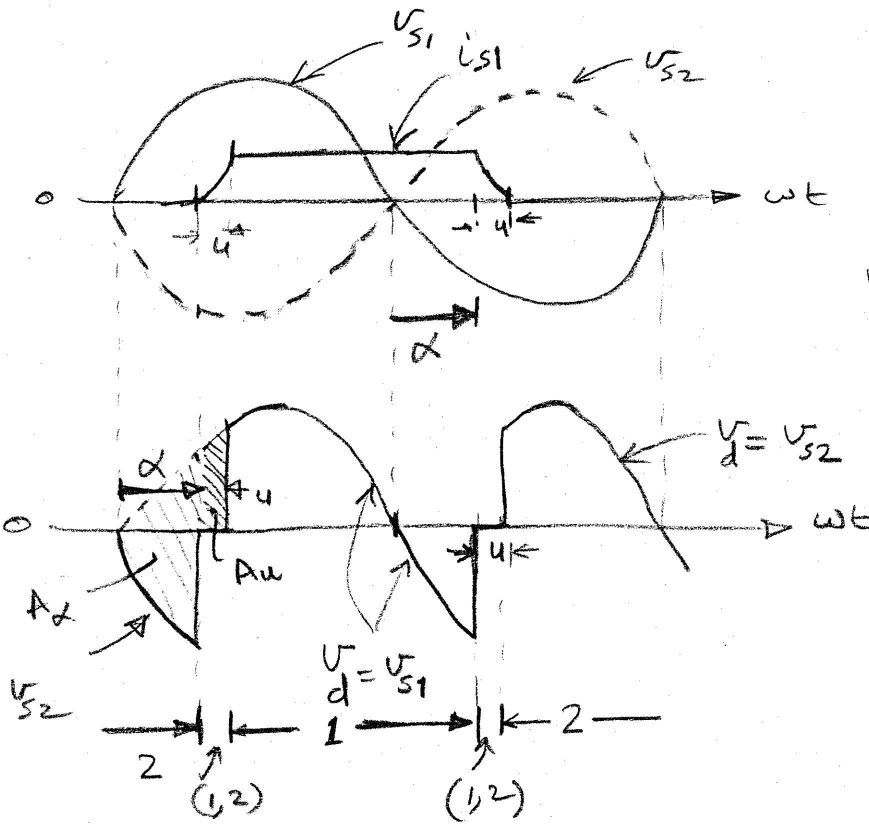
Problem 5-31

A similer procedure as in Problem 5-32 is followed. The peak current is approximately 32 A and it occurs at $\alpha = 90^\circ$

Problem 6-1



$L_s = 5\,mH$



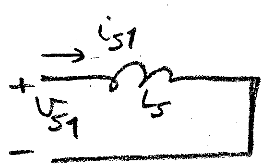
$v_d = v_{s1}$ 1 Cond.

$= v_{s2}$ 2 Cond.

$= 0$ 1, 2 Cond.

Considering L_s in series with Thyristor 1 during Commutation —

$i_{s1}(\alpha) = 0, \quad i_{s1}(\alpha+u) = I_d$



$v_d = 0$ during Commutation

$L_s \frac{di_{s1}}{dt} = v_{s1}$

$\therefore \sqrt{2}\, V_s \int_{\alpha}^{\alpha+u} \sin \omega t \, d(\omega t) = \omega L_s \int_0^{I_d} d\, i_{s1}$

$\therefore \sqrt{2}\, V_s \left[\cos\alpha - \cos(\alpha+u)\right] = \omega L_s I_d = A_u$

$\therefore \quad \cos(\alpha+u) = \cos\alpha - \frac{\omega L_s I_d}{\sqrt{2} V_s} \Rightarrow$ Calculate u

$V_{do}\,(\alpha=0,\ L_s=0) = 0.9\,V_s, \quad V_{d\alpha} = 0.9\,V_s \cos\alpha$

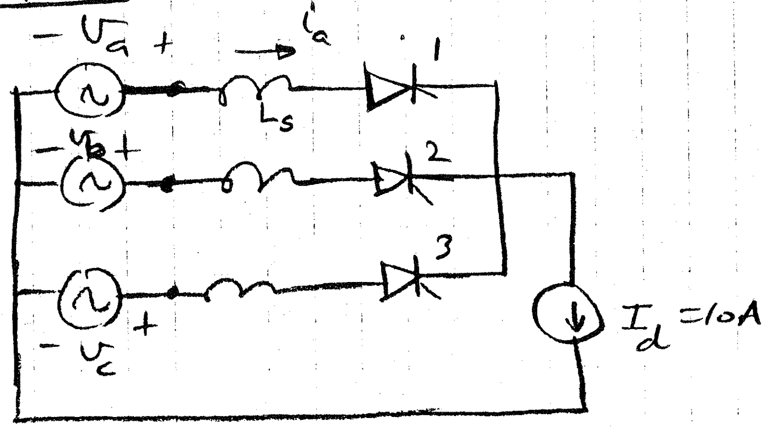
$V_d = 0.9\,V_s \cos\alpha - \frac{\omega L_s}{\pi} I_d.$

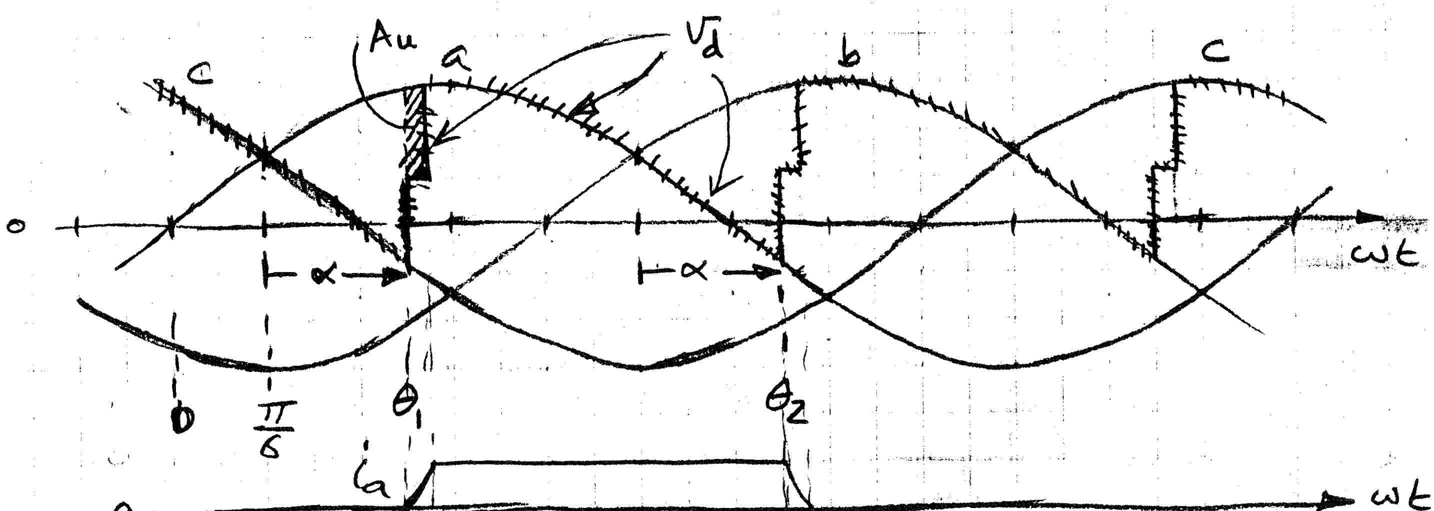
$\therefore$

(a) at $\alpha = 45^o$ $u = 8.41^o$, $V_d = 70.37V$

(b) at $\alpha = 135^o$ $u = 9.9^o$, $V_d = -82.37V$

Problem 6-2





Procedure:

$$V_{d\alpha} = \left[\sqrt{2}\, V_s \int_{\theta_1}^{\theta_2} \sin\omega t \cdot d(\omega t)\right] \Big/ \left(\frac{2\pi}{3}\right)$$

$(L_s = 0)$ 120°

where $\theta_1 = (30^o + \alpha^o) \cdot \frac{\pi}{180}$ rad

and $\theta_2 = \theta_1 + \left(\frac{2\pi}{3}\right)$ rad

120°

$A_u = \omega L_s \times$ [change in i_a during commutation] $= \omega L_s I_d$

$$\therefore \quad V_d = V_{d\alpha} - \frac{A_u}{\frac{2\pi}{3}}$$

where

$$V_{d\alpha} = \frac{\sqrt{2}\,V_s}{\left(\frac{2\pi}{3}\right)} \left[\cos\theta_1 - \cos\theta_2\right]$$

from the 1st equation.

To calculate the procedure leading up to Eq. 6-62 in the text applies here as well. Therefore,

$$\cos(\alpha + u) = \cos\alpha - \frac{2\omega L_s}{\sqrt{2}\,V_{LL}} I_d$$

where V_{LL} = rms value of the line-line voltage

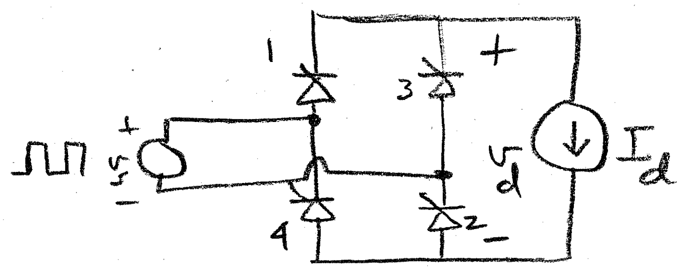
$= \sqrt{3} \times V_{phase}(rms)$

Substituting value

(a) at $\alpha = 45^\circ$, $\quad u = 9.63^\circ$, $\quad V_d = 99.24\,V$

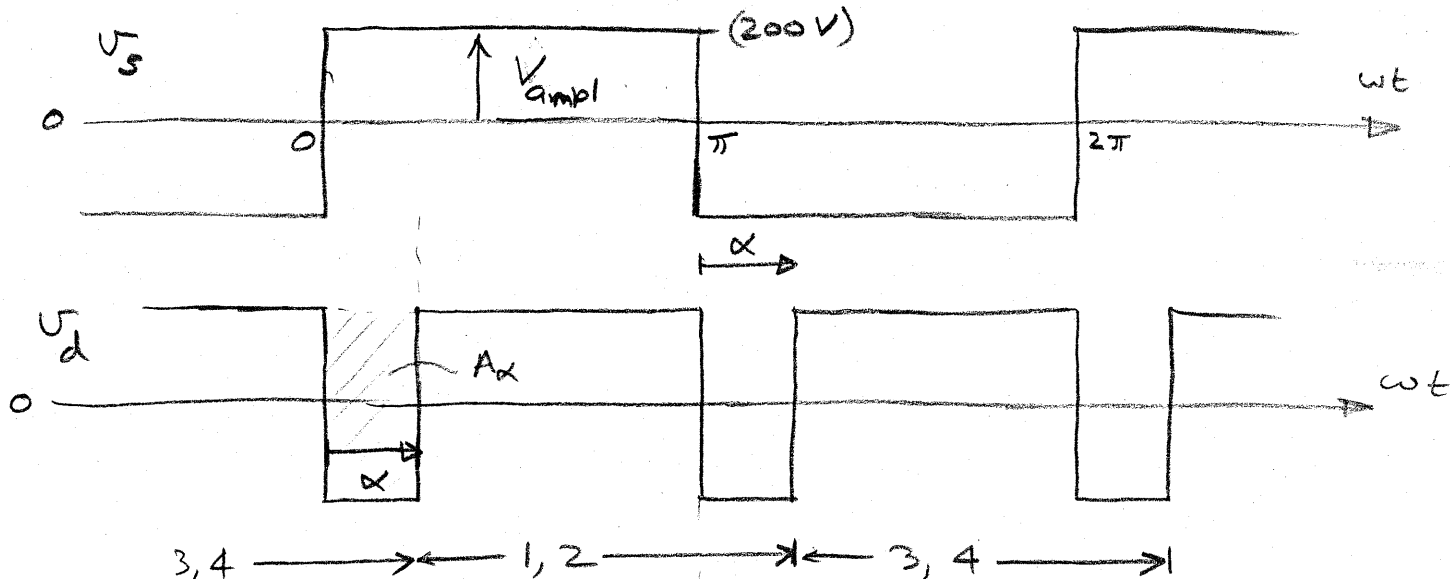
(b) at $\alpha = 135^\circ$; $\quad u = 11.65^\circ$, $\quad V_d = -99.24\,V$

Problem 6-3



1, 2 Cond. $v_d = v_s$

3, 4 Cond. $v_d = -v_s$



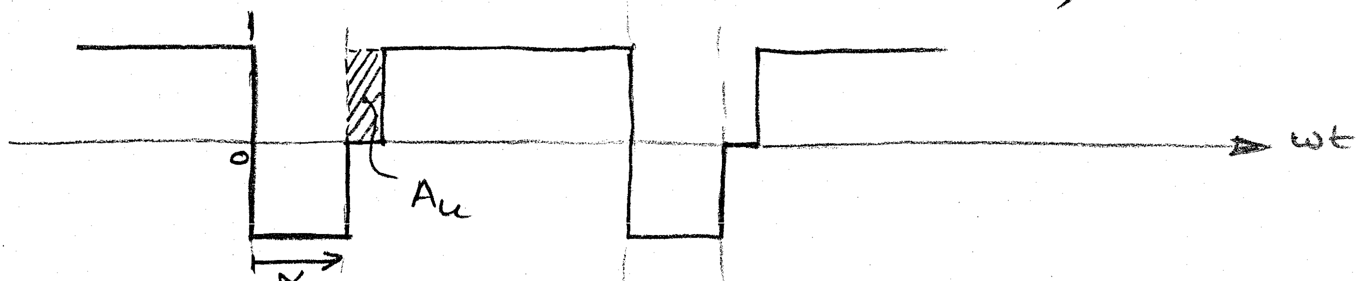
$$V_{d\alpha} = V_{do} - \frac{A_\alpha}{\pi} = V_{ampl} - 2V_{ampl} \cdot \frac{\alpha}{\pi}$$ ← in radians

$$= V_{ampl.}\left(1 - \frac{2\alpha}{\pi}\right);$$ at $\alpha = 45^\circ$, $V_{d\alpha} = 100\,V$

at $\alpha = 135^\circ$, $V_{d\alpha} = -100\,V$

Problem 6-4

(a) In the above circuit in the presence of L_s,



$$A_u = \omega L_s[I_d - (-I_d)] = 2\omega L_s I_d$$

$$\therefore V_d = V_{d\alpha} - \frac{A_u}{\pi}.$$

($V_{d\alpha}$ in problem 6-3)

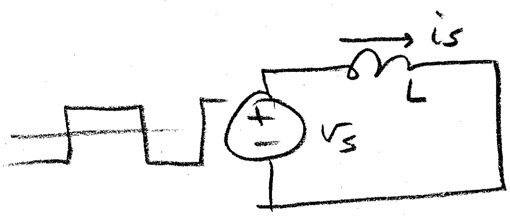
u is independent of α here because the commutation voltage is the same, independent of α.

$\therefore$

$$V_d = V_{ampl}\left(1 - \frac{2\alpha}{\pi}\right) - \frac{2\omega L_s}{\pi} I_d .$$

in radians

To calculate u:



Circuit during the commutation interval. $v_s = V_{ampl.}$

$$L_s \frac{di_s}{dt} = v_s$$

$$\therefore \quad V_{ampl.} \int_{\alpha}^{\alpha+u} d(\omega t) = \omega L_s \int_{-I_d}^{I_d} di_s$$

$$\therefore \quad V_{ampl}\left[(\alpha+u) - \alpha\right] = 2\omega L_s I_d$$

or $$u = \frac{2\omega L_s I_d}{V_{ampl.}}$$

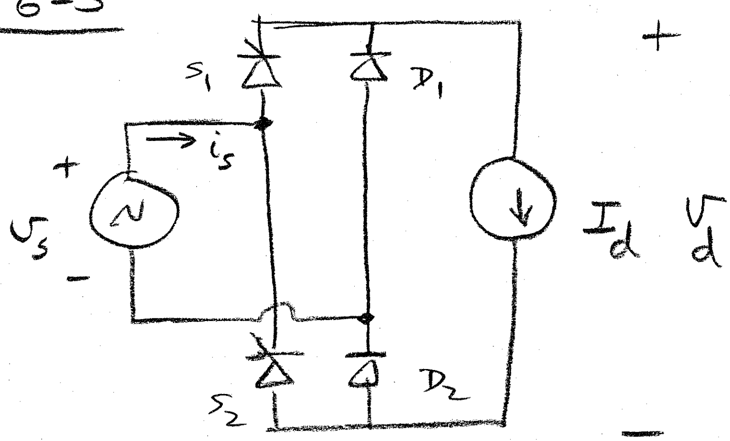
(b)

Substituting values

$\alpha = 45°$, $\quad V_d = 92.8\,V$, $\quad u = 6.48°$

$\alpha = 135°$, $\quad V_d = -107.2\,V$, $\quad u = 6.48°$

Problem 6-5

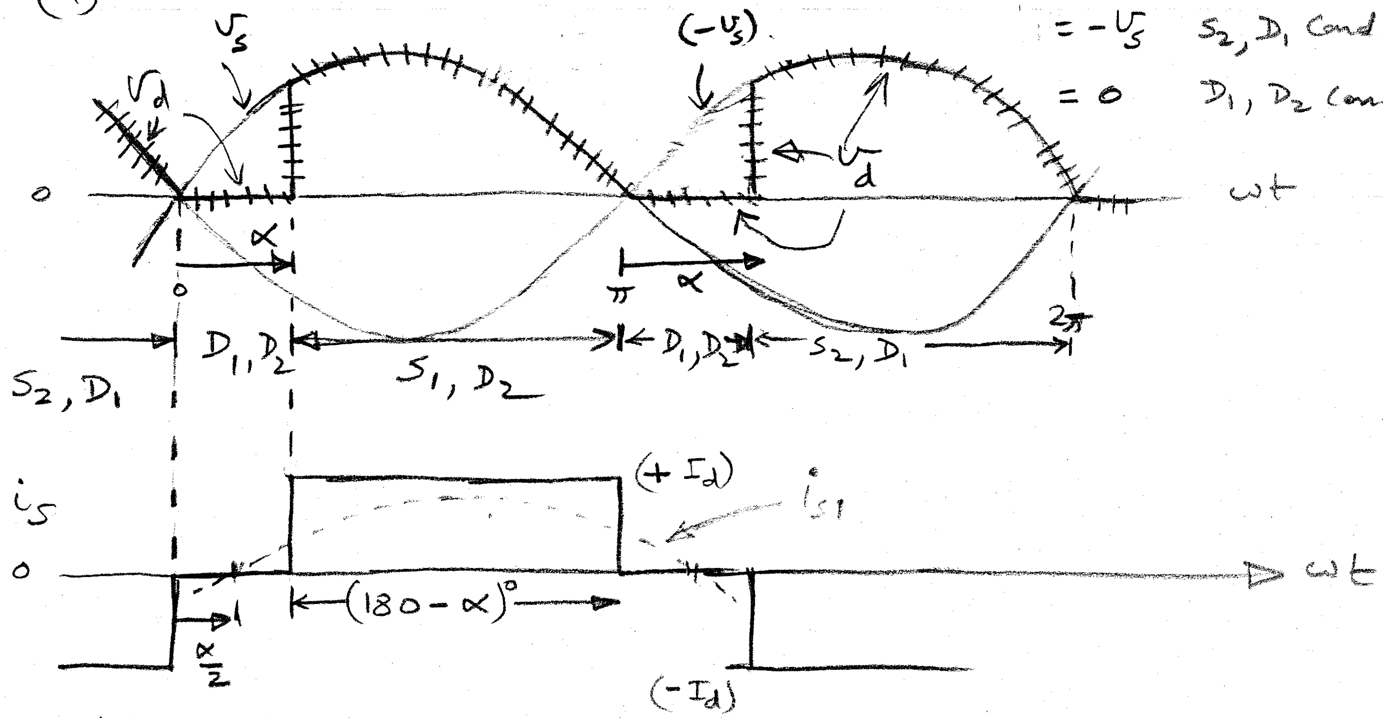


$v_d = v_s$ S_1, D_2 cond.

$\quad = -v_s$ S_2, D_1 cond.

$\quad = 0$ D_1, D_2 cond.

(a)



$V_{do} = 0.9\, V_s$

$$V_{d\alpha} = \sqrt{2}\, V_s \int_{\alpha}^{\pi} \sin\omega t \cdot d(\omega t) \Big/ \pi = \frac{\sqrt{2}\, V_s}{\pi}\left[\cos\alpha + 1\right]$$

$DPF = \cos\left(\frac{\alpha}{2}\right)$ by observation of i_s waveform

rms value

$I_s = I_d\sqrt{\dfrac{180^\circ - \alpha^\circ}{180^\circ}}$;   rms I_{s1} can be calculated by equating power:

$$V_s I_{s1} \cdot DPF = V_{d\alpha} I_d$$

$$\therefore \quad I_{s1} = \frac{V_{d\alpha}\, I_d}{V_s \underbrace{\cos\frac{\alpha}{2}}_{DPF}}$$

(b) $V_{d\alpha} = \frac{1}{2} V_{do}$

$$\therefore \frac{\sqrt{2} V_s}{\pi}(1+\cos\alpha) = \frac{1}{2} \times 0.9 V_s$$

or $$\cos\alpha = \frac{\pi \times 0.9}{2\sqrt{2}} - 1$$

$$\therefore \alpha = 90^\circ$$

$$\therefore DPF = \cos\left(\frac{\alpha}{2}\right) = 0.707$$

$$I_s = I_d \sqrt{\frac{180-90}{180}} = 0.707 I_d$$

$$I_{s1} = \frac{0.5 \times 0.9 V_s}{V_s \cos\left(\frac{90^\circ}{2}\right)} I_d = 0.636 I_d$$

$$\therefore PF = \frac{I_{s1}}{I_s} \cdot DPF = 0.636$$

$$\% THD_i = \frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} \% = \frac{\sqrt{0.707^2 - 0.636^2}}{0.636} \times 100$$

$$= 48.55\%$$

(c) In a full-bridge converter,

$$V_{d\alpha} = V_{do} \cos\alpha \qquad [\text{Eq } 6-6]$$

$\therefore$ for $V_{d\alpha} = 0.9 V_o$, $\alpha = 60^\circ$

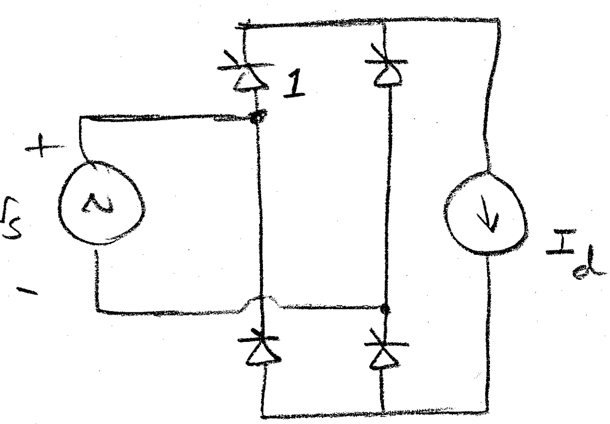
$$DPF = \cos\alpha = 0.5 \qquad [\text{Eq } 6-16]$$

$$PF = 0.9 \cos\alpha = 0.45 \qquad [\text{Eq } 6-17]$$

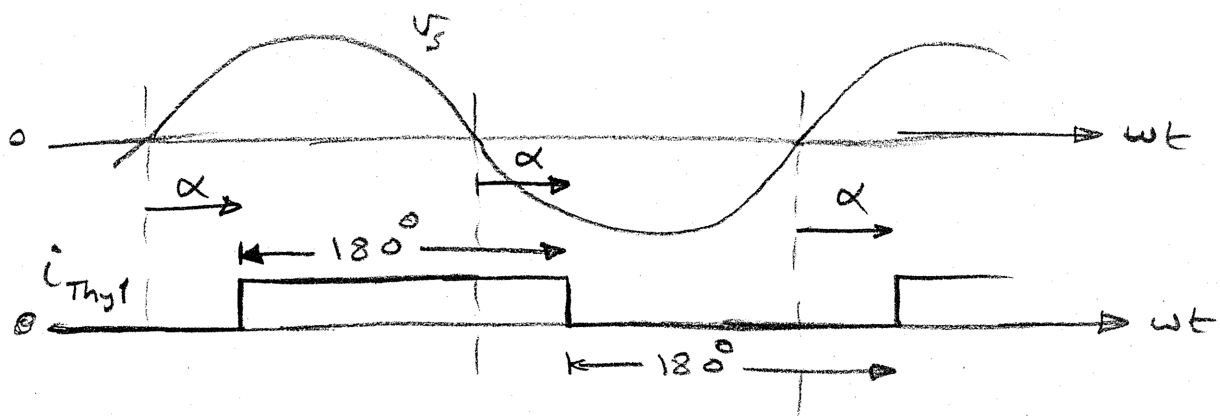
$\% THD_i = 48.43\%$

(d) Comparison of (b) and (c) shows that the Power Factor is better in the half-controlled converter.

Problem 6-6



peak inverse voltage $= \sqrt{2}\, V_s$



$$\frac{I_{thy}\,(avg)}{I_d} = \frac{1}{2}\,, \qquad \frac{I_{thy}\,(rms)}{I_d} = \frac{1}{\sqrt{2}}$$

Problem 6-7

$P_d = 1\,kW$, $I_d = 10\,A$, $i_d \simeq I_d$

$V_s = 115\,V\ (+5\%, -10\%)$

Calculate L_s:

$\omega = 2\pi \times 60 = 377$ rad/s

Transformer $VA = 1500\ VA$

, rated $V_{pri} = 120\ V$

rated $I_{pri} = 1500/120 = 12.5\ A$

$Z_{base} = V_{pri}/I_{pri}$ (rated) $= 9.6\,\Omega$

$\omega L_s = (8\%)\ Z_{base}$

or $L_s = 0.08 \times 9.6/377$

$\simeq 2\,mH$

α cannot go below $0°$. Therefore, to calculate the minimum transformer turns ratio a $\left(\text{where } a = \dfrac{V_{pri}}{V_{sec}}\right)$, in Eq. 6-26

$$V_d = \frac{0.9}{a} V_s \cos\alpha - \frac{2}{\pi}\omega (L_s) I_d$$

In a rigorous calculation, this should be $\left(\frac{L_s}{a^2}\right)$

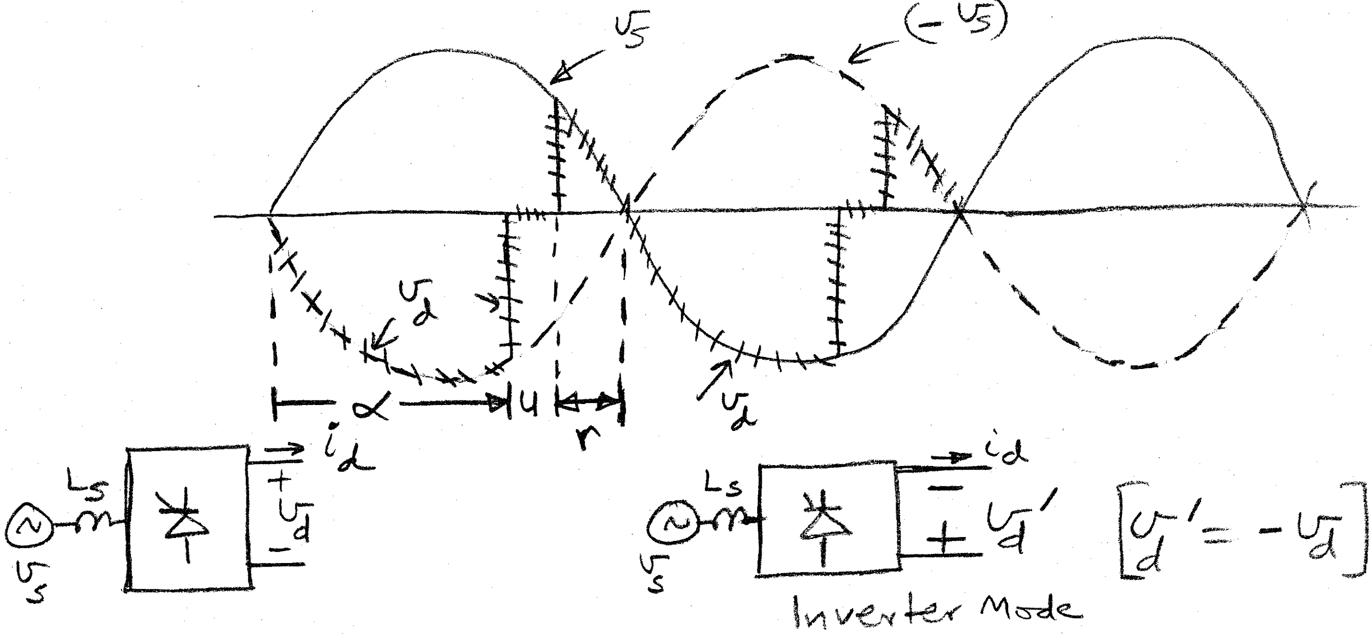
Substitute $V_d = 100\,V$, $\alpha = 0°$ and $V_s^{min} = 115\,V(-10\%)$
$= 103.5\,V$

$\therefore$ From the above Eq: $a_{minimum} \simeq 0.89$

With $a_{minimum}$ and $V_s = 115\ V(+5\%) = 120.75\,V$,

the above Eq yields $\alpha = 31°$ for $V_d = 100\,V$.

Problem 6-8



To represent the inverter mode of operation, often the output dc voltage polarity is defined as shown in the diagram to the right by v_d' where $v_d' = -v_d$.

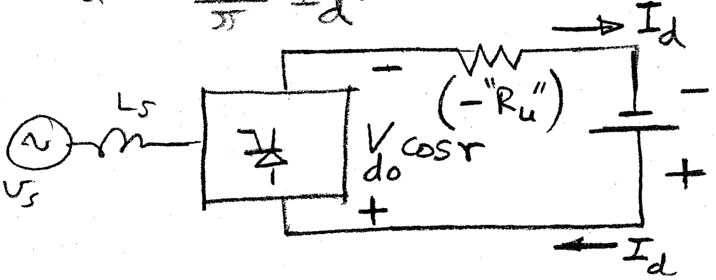
If $L_s = 0$ (and hence $u = 0$) but r is the same as in the waveforms above, then we can calculate

$$V_d'(\text{with } L_s = 0) = 0.9 V_s \cos r$$

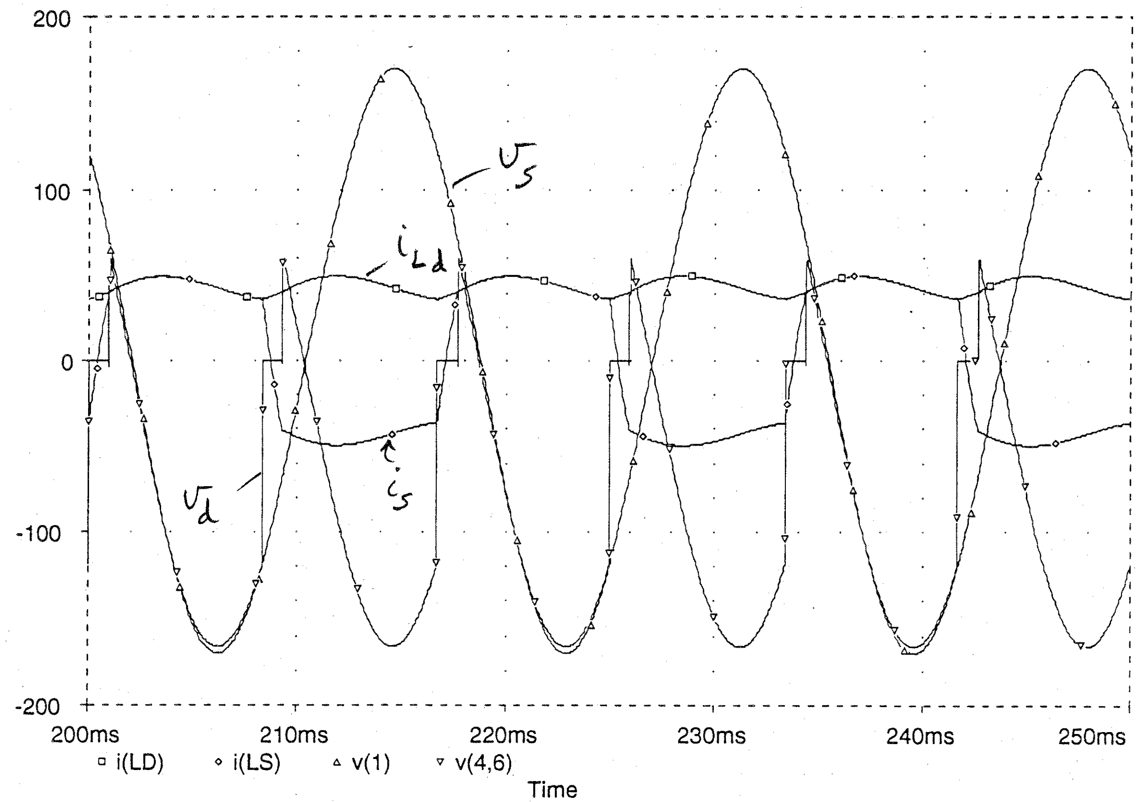
Because of a finite L_s and a finite u,

$$V_d' = \underbrace{0.9 V_s}_{V_{do}} \cos r - \frac{2\omega L_s}{\pi} I_d = V_{do} \cos r - \text{"}R_u\text{"} I_d$$

where "R_u" $= \frac{2\omega L_s}{\pi} I_d$. This can be represented as below:



Problem 6-9



Problem 6-10

For α less than $135°$, the current becomes so large (for the given L_s) that the inverter operation is not possible. For α in a range of $135°$ to $160°$, I_d is tabulated below:

α	$135°$	$140°$	$145°$	$150°$	$155°$	$160°$	$165°$
I_d	44A	24A	7.6A	5.86A	4.5A	3.3A	2.4A

Problem 6-11

$$V_d = V_{do}\cos\alpha - \frac{3\omega L_s}{\pi} I_d \qquad \text{Eq 6-55}$$

$$\cos(\alpha+u) = \cos\alpha - \frac{2\omega L_s}{\sqrt{2}V_{LL}} I_d \qquad \text{Eq 6-62}$$

$$V_{do} = \frac{3\sqrt{2}}{\pi} V_{LL} \qquad \text{Eq 6-36}$$

Replacing $(\omega L_s I_d)$ in Eq 6-55 by Eq 6-62,

$$\therefore V_d = V_{do}\cos\alpha - \underbrace{\frac{3}{\pi}\frac{\sqrt{2}}{2}V_{LL}}_{(V_{do}/2)}\left[\cos\alpha - \cos(\alpha+u)\right]$$

$$\therefore V_d = \frac{V_{do}}{2}\left[\cos\alpha + \cos(\alpha+u)\right]$$

$$\therefore \text{dc power } P_d = V_d I_d = \frac{V_{do} I_d}{2}\left[\cos\alpha + \cos(\alpha+u)\right]$$

On the ac side; ac power is

$$P_{ac} = \sqrt{3}\, V_{LL} I_{s1}\cos\phi_1$$

Approximation: $$I_{s1} \simeq \underbrace{\frac{\sqrt{6}}{\pi}}_{0.78} I_d \qquad \text{Eq 6-44}$$

The above equation is approximate because the above equation is correct only if $L_s = 0$ (and hence $u = 0$).

$$\therefore \quad P_{ac} \simeq \sqrt{3}\, V_{LL} \frac{\sqrt{6}}{\pi} I_d \cos\phi_1$$

Equating $\quad P_{ac} = P_{dc}$

$$\sqrt{3}\, V_{LL} \frac{\sqrt{6}}{\pi} I_d \cos\phi_1 \simeq \underbrace{\frac{3\sqrt{2}}{\pi} V_{LL}}_{V_{do}} \frac{I_d}{2} \left[\cos\alpha + \cos(\alpha+u)\right]$$

$$\therefore \quad DPF = \cos\phi_1 \simeq \frac{1}{2}\left[\cos\alpha + \cos(\alpha+u)\right]$$

Problem 6-12

$V_{LL} = 460V$, $f = 60$ Hz, $\omega = 377 \frac{rad}{s}$, $L_s = 25\ \mu H$, $V_d = 525$ V , and $P_d = 500$ kW .

$$I_d = \frac{P_d}{V_d} = \frac{500x1000}{525} = 952.4 \text{ A}$$

$$V_d = 1.35\, V_{LL} \cos\alpha - \frac{3\omega L_s}{\pi} I_d$$

$$\therefore \quad \cos\alpha = \frac{525 + (\frac{3 \times 377}{\pi} \times 25 \times 10^{-6} \times 952.4)}{1.35 \times 460}$$

$\therefore\ \alpha = 30.77^{o}$

$$\cos(\alpha+u) = \cos\alpha - \frac{2\omega L_s}{\sqrt{2}\, V_{LL}} I_d$$

$$= 0.859 - \frac{2 \times 377 \times 25 \times 10^{-6}}{\sqrt{2} \times 460} \times 952.4$$

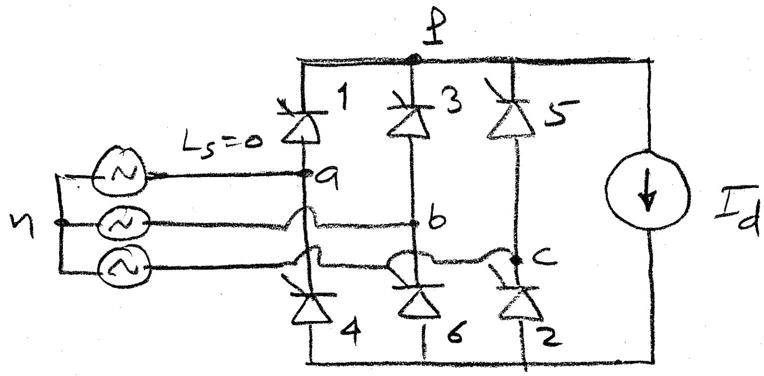
$$= 0.831$$

or $\quad (\alpha+u) = 33.76^{o}$

Therefore, the commutation angle u is

$$u = 33.76^{o} - 30.77^{o} \simeq 3^{o}$$

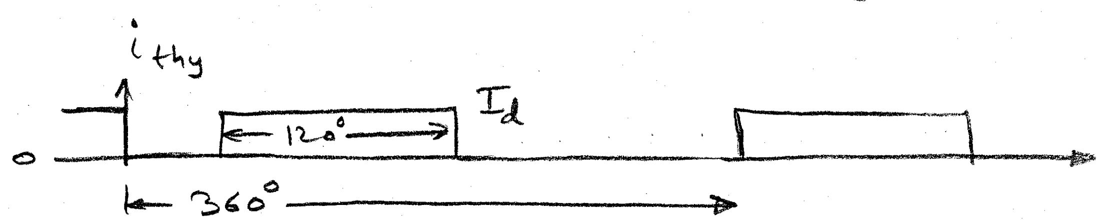
Problem 6-13



When the thyristor 3 is conducting, the peak (maximum) inverse voltage appearing across thyristor 1 is the peak of v_{ab}, which is $\hat{V}_{LL}$. Therefore, the peak inverse voltage (PIV) across any thyristor is

$$PIV = \hat{V}_{LL} = \sqrt{2}\, V_{LL} = \sqrt{2}\sqrt{3}\, V_{ph}(rms).$$

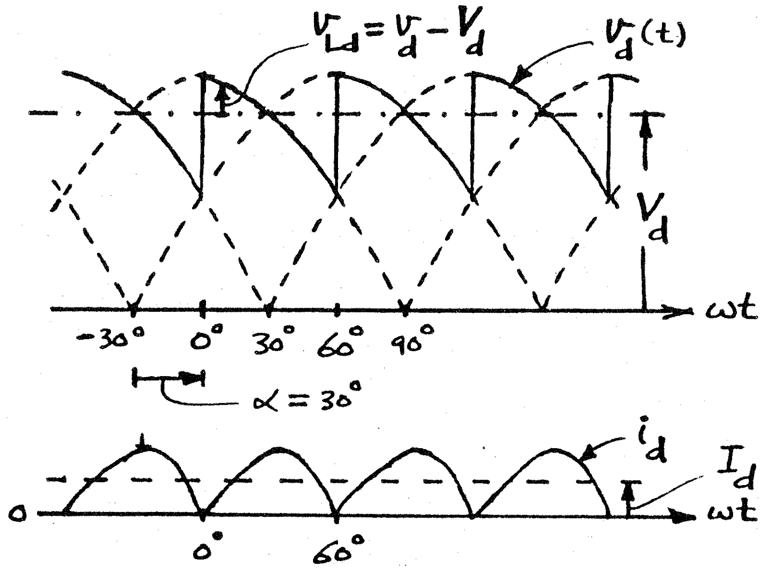
In the absence of L_s, the current through each thyristor flows for 120° during each cycle.



$$\therefore \quad I_{thy}(avg) = I_d/3$$

$$I_{thy}(rms) = \frac{I_d}{\sqrt{3}}$$

Problem 6-14



Redefine $\omega t = 0$ to be at the start of the commutation ($\alpha = 30°$). Then,

$v_d(\omega t) = \sqrt{2}\, V_{LL} \cos\omega t \qquad 0° < \omega t < 60°$

$V_d = 1.35\, V_{LL} \cos 30° = 1.169\, V_{LL}$

$$i_d(\omega t) = \frac{1}{\omega L_d} \int v_{Ld}\, d\omega t \qquad 0° < \omega t < 60°$$

$$= \frac{1}{\omega L_d} (\sqrt{2}\, V_{LL} \sin\omega t - 1.169\, V_{LL}\, \omega t + K) \quad , \quad k = \text{a constant of integration}$$

Calculate K:

$i_d(0) = 0$ in the figure above

$\therefore$ $K = 0$ and,

$$i_d(\omega t) = \frac{\sqrt{2}\, V_{LL}}{\omega\, L_d} (\sin\omega t - 0.8266\, \omega t) \qquad 0 < \omega t < \frac{\pi}{3}$$

Calculate the average current I_d:

$$I_d = \frac{1}{\pi/3} \int_0^{\pi/3} i_d\, d\omega t$$

$$= \frac{3\sqrt{2}\,V_{LL}}{\pi\omega L_d}\left[-\cos\omega t - \frac{0.8266}{2}(\omega t)^2\right]_0^{\pi/3}$$

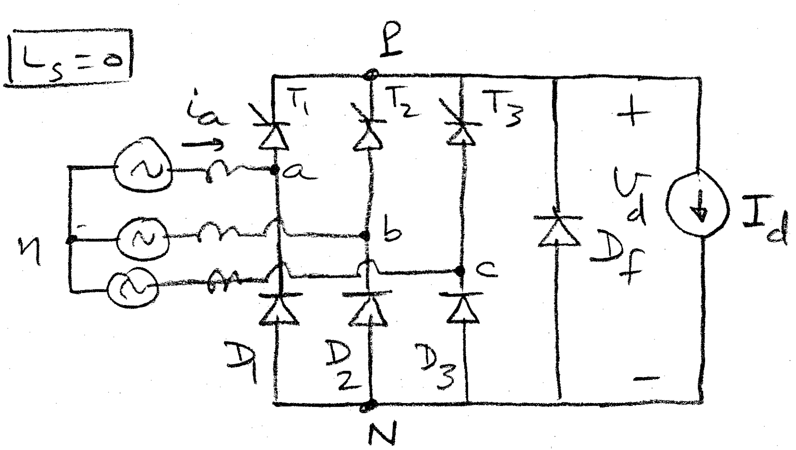
$$= \frac{3\sqrt{2}\,V_{LL}}{\pi\omega L_d}\left[-\frac{1}{2} + 1 - 0.4532\right]$$

Therefore, the minimum dc current I_{dB} at $\alpha = 30^o$ is

$$I_{dB} = 0.1403\,\frac{\sqrt{2}}{\pi}\frac{V_{LL}}{\omega L_d} = 0.0632\,\frac{V_{LL}}{\omega L_d}\,.$$

Problem 6-15

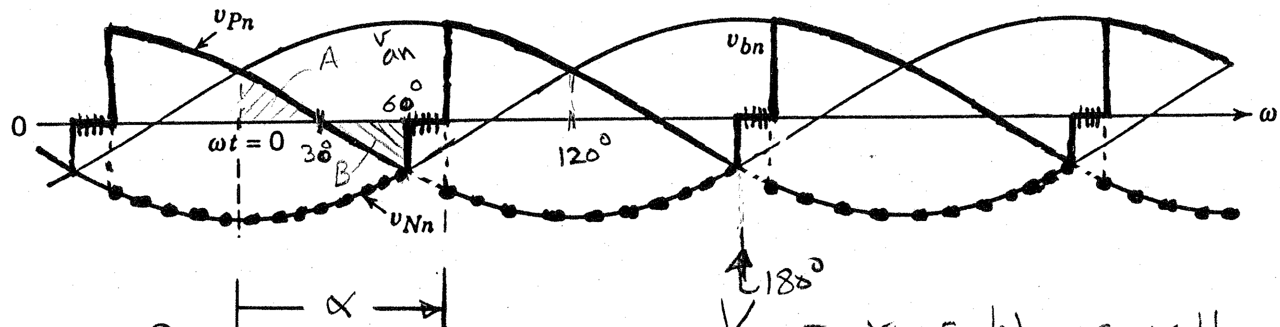
$\boxed{L_s = 0}$



$$V_d = v_{Pn} - v_{Nn}$$

D_f will not conduct, so long that $v_d(t) > 0$. Considering the waveforms shown in Fig. 6-20a, so long as $\alpha < 60°$, $v_{Pn}(t)$ will always be greater than v_{Nn}. Hence for $0 < \alpha < 60°$, D_f will not conduct.

For a value of α greater than $60°$, the v_d will become equal to zero for the interval $\omega t = 60°$ and α, as shown below:



V_{ph} = rms phase voltage

$\underline{\alpha > 60°}$

v_{Pn} waveform repeats every $120°$. To calculate V_{Pn} (avg), consider $\omega t = 0$ to $\omega t = 120°$ interval. Areas A and B cancel each other. With the time origin $\omega t = 0$ shown as above,

$$V_{Pn} = \left[\sqrt{2}\, V_{ph} \int_{\alpha}^{120°} \sin(\omega t + 30°) \cdot d(\omega t)\right] \Big/ \left(\frac{2\pi}{3}\right)$$

$$= \frac{\sqrt{2}\, V_{ph}}{\frac{2\pi}{3}} \left[\cos(\alpha + 30°) - \cos 150°\right]$$

The waveform v_{Nn} also repeats every $120°$. To calculate, consider the interval from α to $\alpha + 120°$.

$$\therefore\ V_{Nn} = \left[\sqrt{2}\, V_{ph} \int_{\alpha}^{180°} \sin(\omega t + 150°) \cdot d(\omega t)\right] \Big/ (2\pi/3)$$

$$= \frac{\sqrt{2}\, V_{ph}}{\left(\frac{2\pi}{3}\right)} \left[\cos(\alpha + 150°) - \cos(330°)\right]$$

$$V_d = V_{Pn} - V_{Nn} = \frac{3 V_{LL}}{\sqrt{2}\pi}(1 + \cos\alpha) \qquad \text{where, } V_{LL} = \sqrt{3}\, V_{ph}$$

$$V_{do} = \frac{3\sqrt{2}}{\pi} V_{LL} \qquad \text{Eq 6-36}$$

$$\therefore \; V_d = 0.5\, V_{do}(1 + \cos\alpha)$$

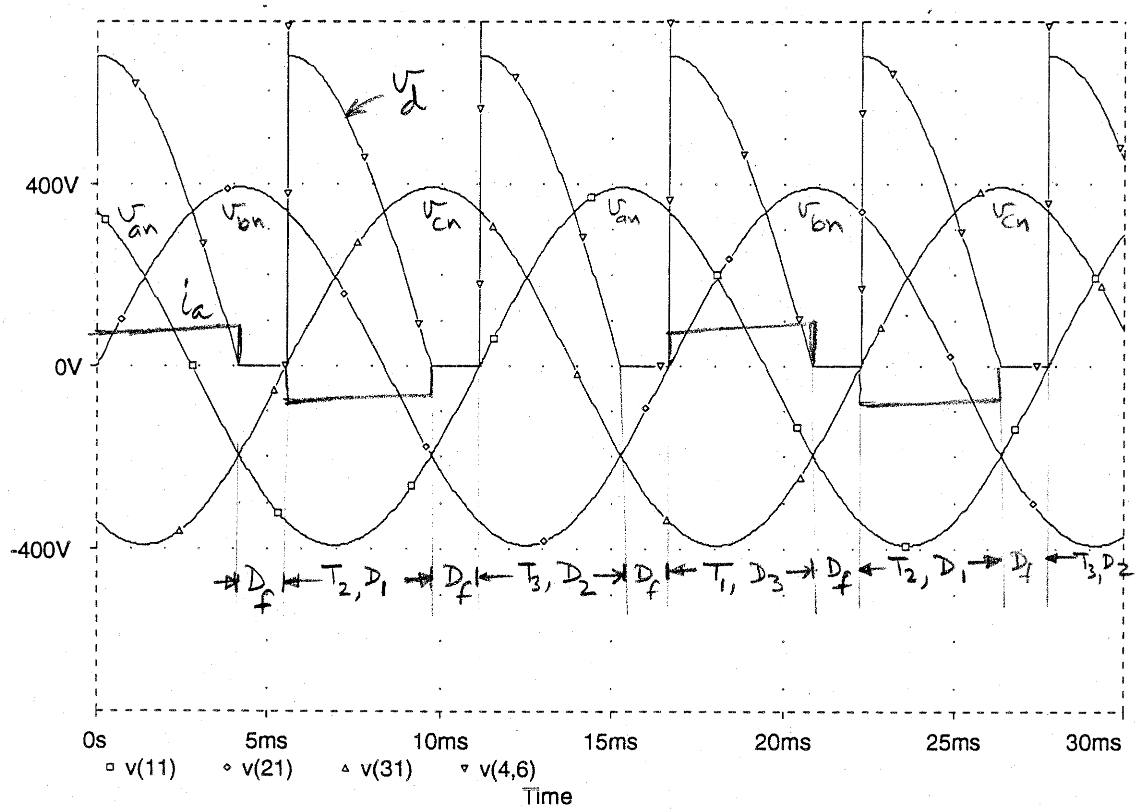
If $V_d = 0.5\, V_{do}$, then

$$0.5\, V_{do}(1 + \cos\alpha) = 0.5\, V_{do}$$

or

$$(1 + \cos\alpha) = 1$$

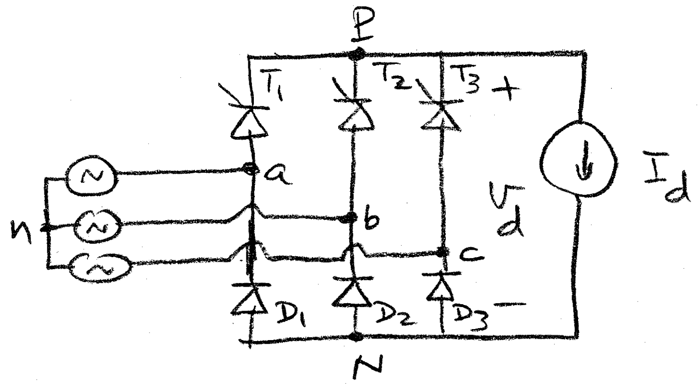
$$\therefore \; \alpha = 90^\circ$$



$\phi_1 = -15^\circ$ (lagging) $\therefore$ DPF $= 0.966$ (lagging)

$THD_i = 78\%$ $\therefore$ PF $= 0.76$

Problem 6-16



We will compare this to the full-controlled bridge designated by subscript FB. Also, $V_{do} = \frac{3\sqrt{2}}{\pi} V_{LL}$

Here, $V_{Pn} = V_{Pn(FB)} = \frac{1}{2} V_{do} \cos\alpha$

$$V_{Nn} = V_{Nn(FB)} \text{ (with } \alpha = 0) = -\frac{1}{2} V_{do}$$

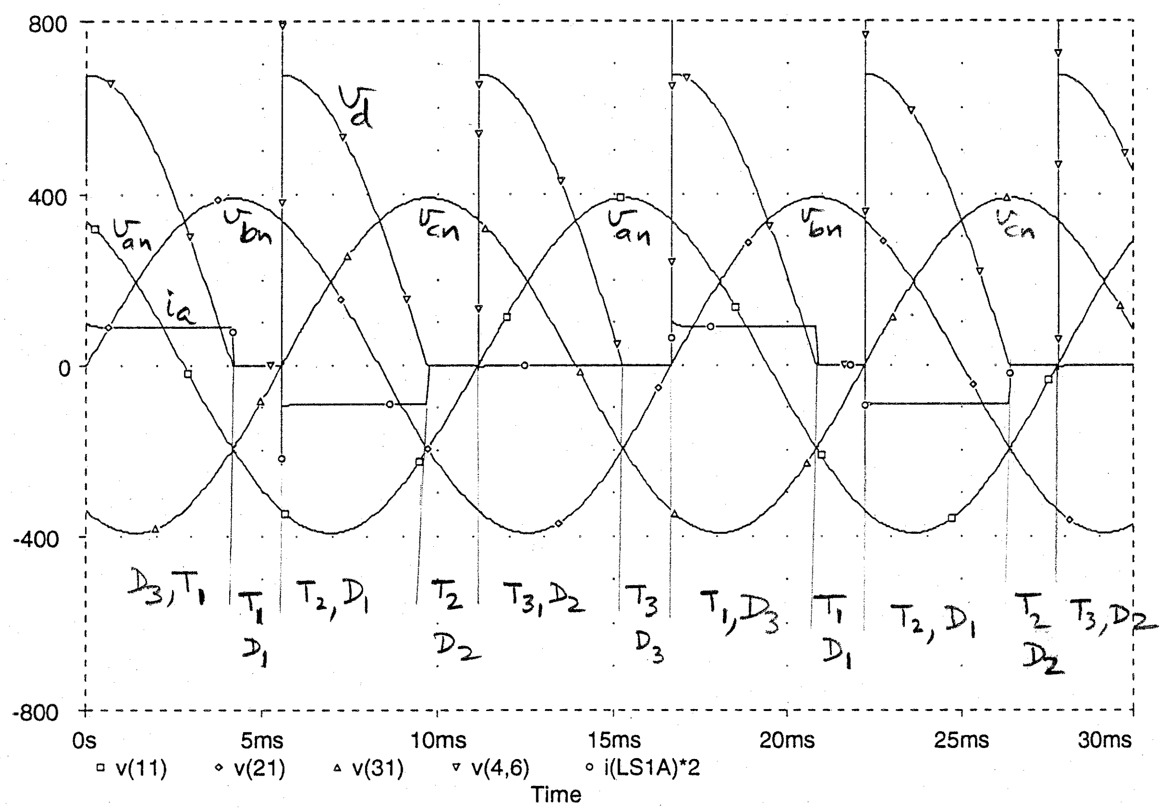
$$\therefore \quad V_d = V_{Pn} - V_{Nn} = \frac{1}{2} V_{do} (1 + \cos\alpha)$$

For $V_d = \frac{1}{2} V_{do} (1 + \cos\alpha) = \frac{1}{2} V_{do}$

$$\therefore \quad (1 + \cos\alpha) = 0$$

and $\alpha = 90^\circ$

Note that v_d waveform is the same as in Prob 6-15. Therefore, V_d is also the same. Also, i_a waveform is identical with exactly the same DPF, THD$_i$ and PF, as in Prob 6-15. However, the devices conducting are different as shown in the figure below.



Problem 6-17

Transformer per phase:

ratings: 5 kVA, $V_{pri} = 120V$ at 60 Hz, $X_{\ell_s} = 8\%$

$$Z_{base} = \frac{V^2}{(VA)} = \frac{120^2}{5000} = 2.88\,\Omega$$

$$\therefore \quad L_s = 0.08 \times \frac{Z_{base}}{\omega\,(=377)} = 0.61\,mH$$

Minimum transformer turns ratio = a,

where $\frac{V_{pri}}{V_{sec}} = a$.

Therefore,

$$V_d = \frac{1.35}{a} V_{LL} \cos\alpha - \frac{3\omega L_s}{\pi} I_d$$

In rigorous calculations, we should use $\left(\frac{L_s}{a^2}\right)$

Substitute $V_d = 300V$, $\alpha = 0^\circ$ and $V_{LL}^{min} = 208V\ (-10\%)$
$= 187.2V$,

Also, $I_d = \frac{12,000}{300} = 40A$

$$\therefore \quad a_{min} = 0.818$$

With $a_{min} = 0.818$ and $V_{LL} = 208V\,(+5\%) = 218.4V$,

$\alpha = 31^\circ$ for $V_d = 300V$.

Problem 6-18

$$V_d = 1.35 V_{LL} \cos\alpha - \frac{3\omega L_s}{\pi} I_d = -550\,V$$

$$P_d = E_d I_d = 550\, I_d = 55 \times 10^3$$

$$\therefore I_d = 100\,A$$

$\therefore$ In the above equation:

$$1.35 V_{LL} \cos\alpha = -550 + 18 = -532\,V$$

$$\therefore \alpha \simeq 145^\circ$$

$$\cos(\alpha + u) = \cos\alpha - \frac{2\omega L_s}{\sqrt{2}\, V_{LL}} I_d$$

$$= -0.857 - 0.058 = -0.915$$

$$\therefore \alpha + u = 156.2^\circ$$

$$\therefore \gamma = 180 - (\alpha + u)$$

$$= 23.8^\circ$$

Problem 6-19

$$I = \sqrt{I_1^2 + \sum_{h=5} I_h^2}$$

$$\text{Ratio} = I_1 / I$$

$$\% THD_i = \frac{\sqrt{I^2 - I_1^2}}{I_1} \times 100$$

$\therefore$ $(\text{Ratio})_{typical} = 0.98$

$(THD)_{typical} = 20.5\%$

$(\text{Ratio})_{ideal} = 0.96$

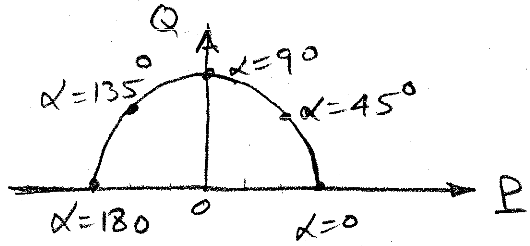
$(THD)_{ideal} = 28.6\%$ based on up to 25th harmonic.

Problem 6-20

$L_s = 0$

$$P = \sqrt{3}\, V_{LL}\, I_{s1} \cos\alpha$$

$$Q = \sqrt{3}\, V_{LL}\, I_{s1} \sin\alpha$$



Problem 6-21

transformer: 3ϕ, 500 KVA, 480 V_{LL}, leakage reactance $X_{L_{s1}} \approx$ impedance = 6%

Find L_{s1} ___

$$X_{L_{s1}} = 0.06 \frac{(\frac{480V}{\sqrt{3}})^2}{(\frac{500\ KVA}{3})} = 27.65\ m\Omega$$

$$L_{s1} = X_{L_{s1}}/\omega = \frac{27.65\ m\Omega}{377} = 73.34\ \mu h$$

Find L_{s2} ___

$$L_{s2} = 200ft \times 0.1 \frac{\mu h}{ft} = 20\ \mu h$$

find α ___

$$L_s = L_{s1} + L_{s2} = 93.34\ \mu H$$

$$V_d = 525V$$

$$V_{LL} = 460\ V$$

$$I_d = \frac{25kW}{525V} = 47.6A$$

$$V_d = 1.35\ V_{LL} \cos\alpha - \frac{3\omega L_s}{\pi} I_d$$

$$\therefore\ \alpha = 32^o$$

Using Eq. 6-67

$$u = \frac{2\omega L_s I_d}{\sqrt{2}\ V_{LL} \sin\alpha} = 971.8 \times 10^{-5} \text{rad}$$

$$\therefore\ t_{notch} = \frac{u}{\omega} \simeq 25.78\ \mu s$$

Using Eq. 6-68

$$\rho = \frac{L_{s1}}{L_{s1} + L_{s2}} = \frac{73.3}{73.3 + 20} = 0.786 = 78.6\%$$

Using Eq. 6-65

deep notch area $A_n = 2\omega(L_{s1} + L_{s2})I_d = 3.35$ V-rad = 8886V-μs

At the point of common coupling, the deep-notch area is

$$A_{ncc} = \rho A_n \simeq 6980 \text{ V-}\mu\text{s}$$

Comparing the results with the limits given in Table 6-2, $\rho = 78.6\%$ exceeds the limits for all class of applications. However, the line-notch area of 6980 V-μs is within the limits for all class of applications.

Problem 6-22

Find the leakage impedance of the 480V, 40 kVA, 3-phase transformer --

$$x_{Ll} = 0.03 \times \frac{(\frac{480}{\sqrt{3}})^2}{(\frac{40 \times 10^3}{3})} = 0.1728\ \Omega$$

$$\therefore\ L_l = \frac{0.1728}{\omega} \simeq 458\ \mu H$$

From problem 6 - 21

$$L_{s2} = 20\ \mu H + 458\ \mu H = 478\ \mu H$$

and $L_s = L_{s1} + L_{s2} = 73.34 + 478 = 551.34\ \mu H$

From Eq. 6-55

$$525 = 1.35 \times 460 \times \cos\alpha - \frac{3 \times 377 \times 551.34 \times 10^{-6}}{\pi} \times 47.6$$

$$\therefore\ \alpha = 30.6^o$$

$$u = \frac{2\omega L_s I_d}{\sqrt{2}\, V_{LL} \sin\alpha} = 59.75 \times 10^{-3} \text{ rad}$$ From Eq. 6-67

$$t_{notch} = \frac{u}{\omega} = 158.5\ \mu s$$

$$\rho = \frac{L_{s1}}{L_{s1} + L_{s2}} = \frac{73.34}{551.34} = 13.3\%$$

deep notch area $A_n = 2\omega(L_s)I_d$ From Eq. 6-65

$$= 19.79\ \text{V-rad} = 52{,}487\ \text{V-}\mu\text{s}$$

At the point of common coupling

$A_{n_{cc}} = \rho A_n = 0.133 \times 52{,}487 \simeq 6980$ V-μs (same as in problem

Comparing the results with the limits in Table 6-2, the factor ρ now (as compared to the system in Problem 6-21) is acceptable for dedicated and general systems. As before, the line-notch area is acceptable for all systems.

Problem 6-23

The rms value of the distortion voltage component V_{dis} can be approximately calculated by considering the six voltage notches in one cycle of the voltage waveform.

All notch widths are equal to t_{notch}. There are two deep notches, each with a depth of V_{notch}. There are 4 shallow notches, each with a depth (or height) of $\frac{V_{notch}}{2}$.

Therefore, the rms value of the voltage distortion can be approximated as

$$V_{dis} \approx \left[\frac{2 \times V^2_{notch}\, t_{notch} + 4 \times (\frac{V_{notch}}{2})^2\, t_{notch}}{1/f_1} \right]^{1/2}$$

where f_1 is the line frequency of 60 Hz. (see Reference 4 of Chapter 6)

$$\therefore V_{dis} = \sqrt{3\, V^2_{notch}\, t_{notch}\, f_1}$$ in line-line voltage.

It is reasonable to assume that the fundamental frequency line-to-line voltage at the point of common coupling equals 460V (rms).

(a) In problem 6-21, at the point of common coupling

$$V_{notch} = \frac{A_{n_{cc}}}{t_{notch}} = \frac{6980}{25.78} = 270.75\ V$$

$$\therefore\ V_{dis} = \sqrt{3 \times 270.75^2 \times 25.78 \times 10^{-6} \times 60}$$

$$= \sqrt{340.2} = 18.4\ V$$

$$\therefore \%THD = \frac{V_{dis}}{V_{LL1}} \times 100 = \frac{18.4}{460} \times 100 \simeq 4\ \%$$

(b) In problem 6-22, at the point of common coupling

$$V_{notch} = \frac{A_{ncc}}{t_{notch}} = \frac{6980}{158.5} \simeq 44\ V$$

$$\therefore\ V_{dis} = \sqrt{3 \times 44^2 \times 158.5 \times 10^{-6} \times 60} = 7.4\ V$$

$$\therefore \%THD = \frac{V_{dis}}{V_{LL1}} \times 100 = \frac{7.4}{460} \times 100 = 1.61\%$$

Problem 6-24

In Fig. 6-35, the voltage at the point of common coupling on a per-phase basis is

$$(V_{pcc})_h = (\omega h L_{s1}) I_h$$

$$THD = \frac{\left[\sum_{h \neq 1} (V_{pcc})_h^2\right]^{1/2}}{V_1}$$

$$THD = \frac{I_1 \omega L_{s1}}{V_1} \left[\sum_{h \neq 1} (h\, I_h/I_1)^2\right]^{1/2}$$

$$= \frac{I_1}{V_1}\, 27.6 \times 10^{-3}\, [1.82]^{1/2} = 0.0373\, \frac{I_1}{V_1}$$

$$V_1 = \frac{460}{\sqrt{3}} = 266\ V$$

$$I_1 = \frac{\sqrt{6}}{\pi} I_d = \frac{\sqrt{6}}{\pi} (47.6) = 37.1\ A\ (rms)$$ (from Eq. 6-44)

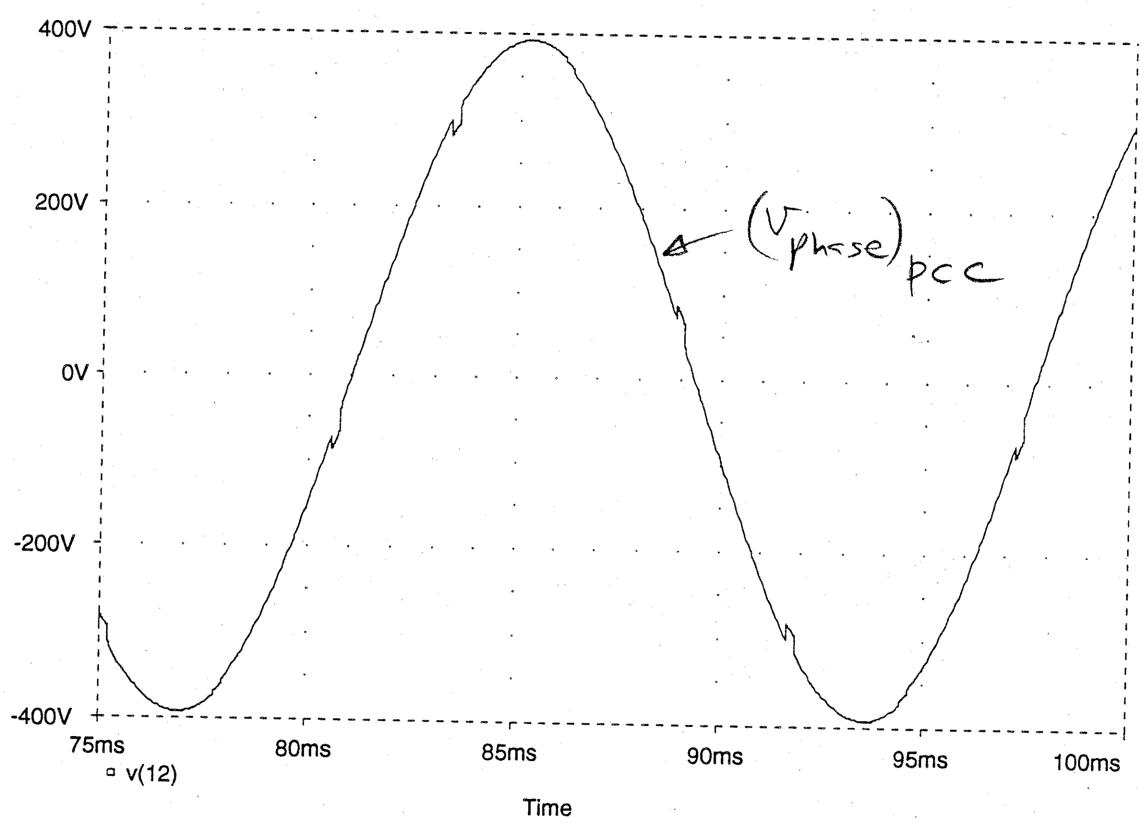
(approximate here)

$$\therefore THD = \frac{37.1}{266}\, 0.0373 = 5.2 \times 10^{-3} = 0.52\%$$

Note that the THD = 0.52% in this problem is much smaller than 4 % calculated in Problem 6-23(a) and 1.61% in Problem 6-23(b). This suggests that the typical harmonics given in Table 4-1 correspond to a system with a larger value of L_{s2} compared to that in Problem 6-21 or Problem 6-22.

Problem 6-25

The total harmonic distortion is 1.3%



Problem 7-1

$V_o = 5$ V, $V_d = 10$ V to 40 V, $P_o \geq 5$ W, $f_s = 50$ kHz

Find the minimum inductance to keep the converter in the continuous conduction mode under all conditions.

Solution: For a given load and output voltage, the likelihood that the inductor current will fall to zero is increased by lowering the duty ratio and thus increasing the OFF time. The duty ratio is lowest when V_d = 40V. $P_o/V_o = 5W/5V = I_o = 1A$

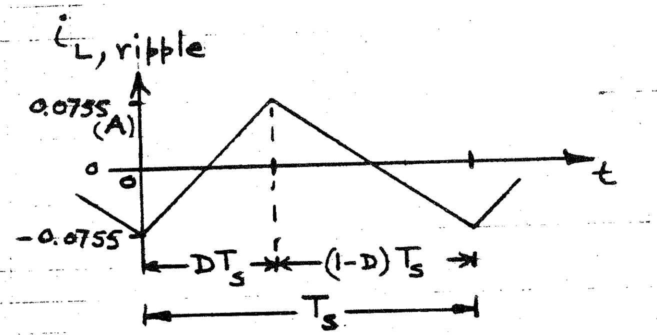
For continuous conduction from Eq. 7-5, $I_o \geq \dfrac{D}{2f_sL}[V_d - V_o]$

$$D = \frac{5}{40} = 0.125 \; ; \; L = \frac{D}{2f_sI_o}[V_d - V_o] = \frac{0.125}{2 \cdot 50{,}000 \cdot 1}[40-5]$$

$$\boxed{L = 43.75\ \mu F}$$

Problem 7-3

Find the RMS ripple current through L.



Solution: $V = L[\frac{di_L}{dt}]$; During t_{on}, $\frac{di_L}{dt} = \frac{12.6-5}{.001} = 7600$ A/s

During t_{off}, $\frac{di_L}{dt} = \frac{-5}{.001} = -5{,}000$ A/s; $D = \frac{5}{12.6} = 0.397$. Therefore, the peak-to-peak ripple current is

$$\Delta i_L = 0.397(50\mu s)(7600\ A/s) = 0.151A$$ Note: $T_s = 50\mu s$

$i_{L,ripple}(t) = -0.0755 + 7600t$ for $0 < t < 19.84\ \mu s$ Note: $DT_s = 19.84\ \mu s$

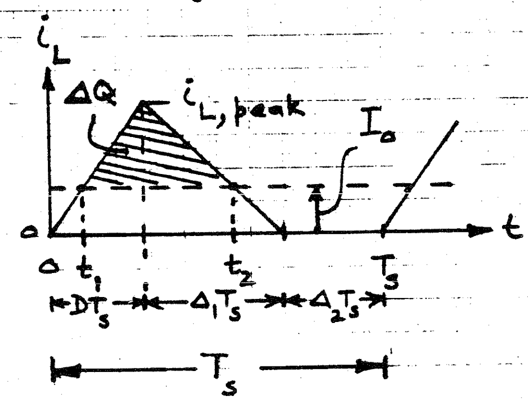
$= 0.1747 - 5000t$ for $19.84\ \mu s < t < 50\ \mu s$.

$$\sqrt{\frac{1}{T_s}\int_0^{T_s} [i_{L,ripple}]^2 \cdot dt} = I_{L,ripple},\ RMS$$

$\boxed{\text{RMS ripple current} = 43.66\ \text{mA}}$

Problem 7-4

Derive an expression for ΔV_o in the discontinuous mode.



Solution: $\frac{di}{dt} = \frac{V}{L}$; $[\frac{V_d - V_o}{L}]t_1 = I_o;\ t_1 = \frac{LI_o}{V_d - V_o}$ where t_1 is defined in the figure

$$i_{L,peak} = \frac{DT_s}{L}[V_d - V_o]$$

$$t_2 = DT_s + \frac{L(i_{L,peak}-I_o)}{V_o};\quad t_2 - t_1 = \frac{-LI_o}{V_d - V_o} + DT_s + \frac{L(i_{L,peak}-I_o)}{V_o}$$

$$t_2 - t_1 = \frac{DT_s(V_d-V_o)V_o - LI_oV_o + L(V_d-V_o)[\frac{DT_s}{L}(V_d-V_o) - I_o]}{V_o(V_d-V_o)}$$

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{1}{C}\cdot\frac{1}{2}(i_{L,peak}-I_o)(t_2-t_1)$$

$$= \frac{1}{2C}[\frac{DT_s}{L}(V_d-V_o) - I_o](t_2-t_1)$$

$$\therefore \boxed{\Delta V_o = \frac{[DT_s(V_d-V_o)-LI_o][DT_s(V_d-V_o)V_o-LI_oV_o+(V_d-V_o)(DT_s(V_d-V_o)-LI_o)]}{2LC\,V_o(V_d-V_o)}}$$

Problem 7-5

Calculate the ripple voltage in problem 7-2 if the load current is reduced to $I_{oB}/2$.

Solution: $I_o = 0.0377$ A; $\frac{V_o}{V_d} = \frac{5}{12.6} = \frac{D}{D + \Delta_1}$; $\therefore \Delta_1 = 1.52\,D$

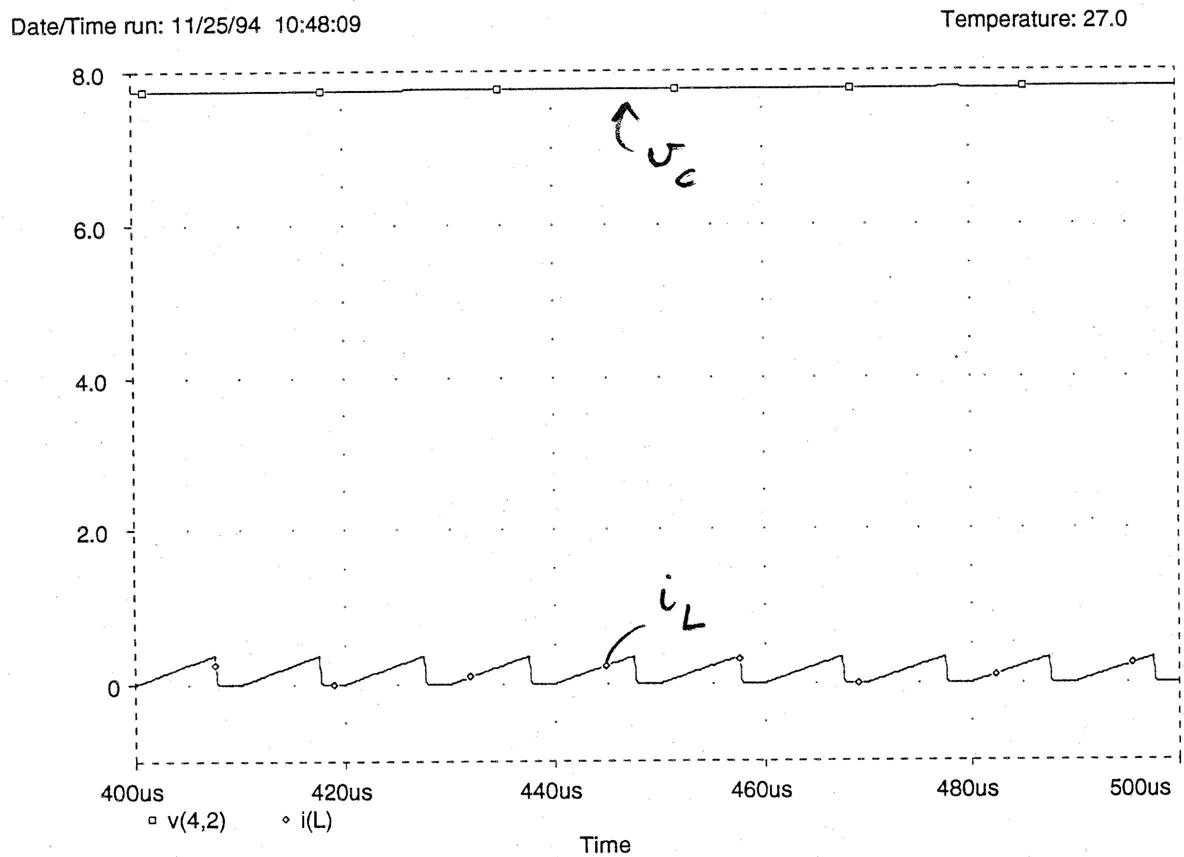
Using Eq. 7-14,

$$I_o = 0.0377 = \frac{V_dT_sD\Delta_1}{2L} = \frac{12.6\,D\Delta_1}{2\cdot 20{,}000\cdot 0.001};\quad \therefore \Delta_1 = \frac{0.1197}{D}$$

$0.1197 = 1.52\,D^2$; $D = 0.281$. Using the equation derived in problem 7-4,

$$\boxed{\Delta V_o = 1.66\ \text{mV}}$$

Problem 7-6



STEP-UP CONVERTERS

Problem **7-7**

$V_d = 8V$ to $16V$, $V_o = 24V$, $f_s = 20$ kHz , $C = 470\ \mu F$, $P_o \geq 5W$,

$I_o = \frac{5W}{24V} = 0.2083A.$

Solution: Case 1: $V_d = 8V; \; \frac{24V}{8V} = 3 = \frac{1}{1 - D}; \; \therefore \; D = \frac{2}{3} = 0.667$

From Eq. 7-29,

$$0.2083 = \frac{24 \cdot 0.667(1-0.667)^2}{2 \cdot 20{,}000 \, L} \; ; \; \therefore \; L = 0.213 \text{ mH}$$

Case 2: $V_d = 16V \; ; \; \therefore \; D = \frac{1}{3} = 0.333$

From Eq. 7-29

$$0.2083 = \frac{24 \cdot 0.333(1-0.333)^2}{2 \cdot 20{,}000 \, L} \; ; \; \therefore \; \boxed{L = 0.427 \text{ mH}}$$

Problem 7-8

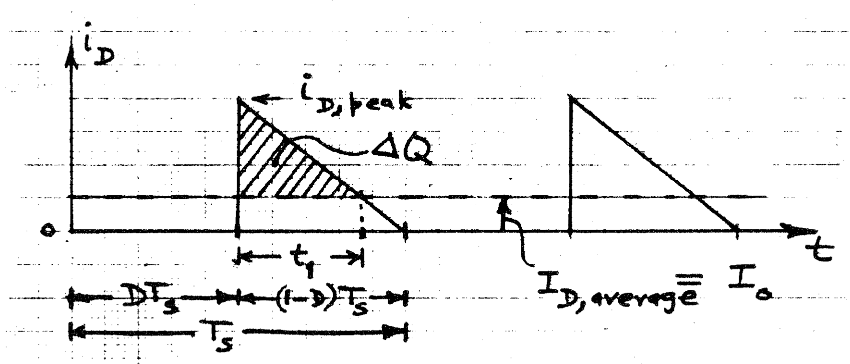
$V_d = 12V$, $V_o = 24V$, $I_o = 0.5A$, $L = 150\mu H$, $C = 470\mu F$, $f_s = 20kHz$.

Find ΔV_o.

Solution: Initially, assume that the converter is in a continuous-conduction mode.

$\frac{24}{12} = \frac{1}{1 - D} \; \therefore \; D = 0.5$; From Eq.7-29 $I_{oB} = \frac{24 \cdot 0.5(0.5)^2}{2 \cdot 20{,}000 \cdot 150 \cdot 10^{-6}} = 0.5 \text{ A}$

Boundary case since $I_o = I_{oB}$.



$$i_{D,peak} = i_{L,peak} = \frac{V_d}{L} DT_s = \frac{12 \cdot 0.5}{150 \text{x} 10^{-6} \cdot 20{,}000} = 2A$$

during off time: $\frac{di_D}{dt} = \frac{V_d - V_o}{L} = \frac{12 - 24}{150 \text{x} 10^{-6}} = -80{,}000 \text{ A/s}$

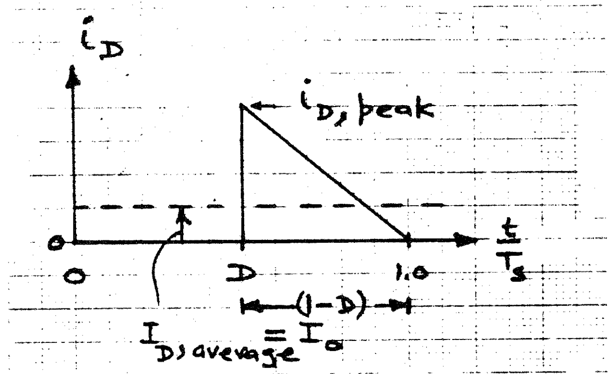
Therefore, $\left(-\frac{di_D}{dt}\right) = \frac{i_{D,peak} - I_o}{t_1} = 80{,}000 \; ; \; \therefore \; t_1 = \frac{2-0.5}{80{,}000} = 18.75 \text{x} 10^{-6}$

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{1}{2} \frac{(i_{D,peak} - I_o)t_1}{C} = \frac{1}{2} \cdot \frac{(2-0.5)18.75 \text{x} 10^{-6}}{470 \text{x} 10^{-6}} = \boxed{\Delta V_o = 29.92 \text{ mV}}$$

Note that the expression for ΔV_o given by Eqs. 7-39 and 7-40 is valid only if the minimum value of i_L is greater than I_o in the continuous-conduction mode of operation (as shown in Fig. 7-17a).

Problem 7-9

From problem 7-8 we know that this converter is operating at the conduction boundary between continuous and discontinuous. Therefore $i_{D,peak}$ is as shown below.



$$I_{D,rms} = \sqrt{(1-D)}\ \frac{i_{D,peak}}{\sqrt{3}} = \sqrt{0.5}\ \frac{(2.0)}{\sqrt{3}} = 0.816\text{A}$$

($\sqrt{3}$ = RMS FACTOR FOR TRIANGULAR WAVES).

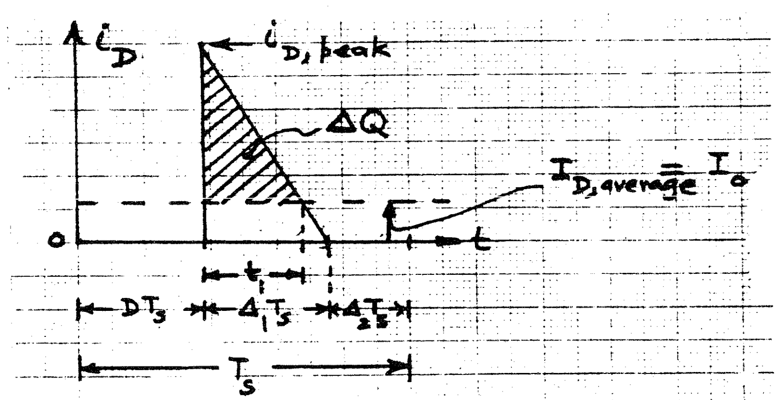
$$i_d = i_{D,average} + i_{ripple}$$

Since $I_{D,average} = I_o$,

$$I_{ripple,rms} = [I_{D,rms}^2 - I_o^2]^{1/2} = \sqrt{0.816^2 - 0.5^2}$$

$$\boxed{I_{ripple,rms}} = 0.645 \text{ A}$$

Problem 7-10



$$i_{D,peak} = i_{L,peak} = \frac{V_d}{L} DT_s$$

during t_{off}: $\frac{di_D}{dt} = \frac{V_d - V_o}{L}$

$$-\frac{di_D}{dt} = \frac{i_{D,peak} - I_o}{t_1} = \frac{V_o - V_d}{L}$$

$$\therefore\ t_1 = \frac{L(i_{D,peak}-I_o)}{V_o - V_d} = \frac{L(\frac{V_d}{L} DT_s - I_o)}{V_o - V_d}$$

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{1}{2C}(i_{D,peak}-I_o)t_1 = \frac{1}{2LC}\frac{(V_d DT_s - LI_o)^2}{(V_o - V_d)}$$

Problem 7-11

$V_d = 12\,V$, $V_o = 24\,V$, $L = 150\,\mu H$, $C = 470\,\mu F$, $f_s = 20\,kHz$

From the solution of Problem 7-8

$$I_{oB} = 0.5\,A$$

$$\therefore\ I_o = \frac{I_{oB}}{2} = 0.25\,A$$

From Eq. 7-31

$$I_{oB,max} = 0.074\,\frac{T_s V_o}{L} = 0.592\,A$$

From Eq. 7-38

$$D = \left[\frac{4}{27}\left(\frac{V_o}{V_d}\right)\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oB,max}}\right]^{1/2}$$

$$= \left[\frac{4}{27}(2)(1)\frac{0.25}{0.592}\right]^{1/2} = 0.354$$

From the expression derived in Problem 7-10

$$\Delta V_o = \frac{1}{2\times 150\times 10^{-6}\times 470\times 10^{-6}}\frac{(12\times 0.354\times 50\times 10^{-6} - 150\times 10^{-6}\times 0.25)^2}{(24-12)}$$

$$= 18.1\,mV$$

BUCK-BOOST CONVERTERS

Problem 7-12

$$V_d = 8V\text{-}40V,\ V_o = 15V,\ f_s = 20\ \text{kHz},\ \ C = 470\ \mu F,\ P_o \geq 2W$$

$$I_o = \frac{P_o}{V_o} = \frac{2W}{15V} = 0.133A = I_{oB}$$

From Eq. 7-47,

$$I_{oB} = \frac{T_s V_o}{2L}(1\text{-}D)^2 \ \therefore\ L_{min} = \frac{T_s V_o}{2 I_{oB}}(1\text{-}D)^2$$

Smallest D results in largest L_{min}

$$\frac{D}{1 - D} = \frac{V_o}{V_d}\ ;\ D = \frac{V_o}{V_o + V_d}\ ,\ \ D(\text{smallest}) = \frac{15}{15 + 40} = 0.273$$

$$\therefore\ L_{min} = \frac{15\ (1\text{-}0.273)^2}{2x0.133x20{,}000} = 1.49\ \text{mH}$$

Problem 7-13

$$V_d = 12V,\ \ V_o = 15V,\ \ I_o = 0.25A,\ \ L = 150\mu H,\ \ C = 470\mu F,\ \ f_s = 20kHz$$

From Eq. 7-47, (assume continuous conduction, $\therefore D = 0.556$)

$$I_{oB} = \frac{T_s V_o}{2L}(1\text{-}D)^2 = \frac{15\ (1\ \text{-}0.556)^2}{2x\ \ 150x10^{-6}x20x10^3} = 0.493A,$$

$$R = \frac{V_o}{I_o} = 60\Omega$$

Since $I_o < I_{oB}$: Discontinuous Conduction Mode
Calculation for ΔV_o will require a new derivation: see Problem 7-15.

From Eq. 7-49,

$$I_{oB,max} = \frac{T_s V_o}{2L} = \frac{15}{2x150x10^{-6}x20x10^3} = 2.5A$$

From Eq. 7-55,

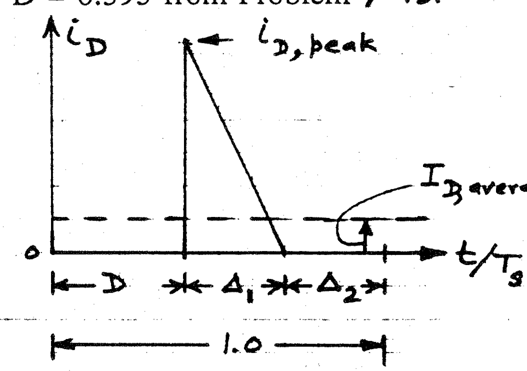
$$D = \frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oB,max}}} = \frac{15}{12}\sqrt{\frac{0.25}{2.5}} = 0.395,$$

From Prob. 7-15,

$$\Delta V_o = \frac{1}{470\text{x}10^{-6}} \left[\left(1 - \frac{0.395 \cdot 12}{15}\right) \frac{0.25}{20\text{x}10^3} + \frac{150\text{x}10^{-6}\text{x}0.25}{2\text{x}60}\right] = 18.86 \text{ mV}$$

Problem 7-14

D = 0.395 from Problem 7-13.



$$i_{D,peak} = \frac{V_d}{L} DT_s = \frac{12}{150\text{x}10^{-6}} \text{ x } 0.395 \text{ x } 50 \text{ x } 10^{-6} = 1.58\text{A}$$

Calculate Δ_1:

$$\Delta_1 T_s = \frac{L i_{D,peak}}{V_o} \quad \therefore \quad \Delta_1 = \frac{L i_{D,peak}}{V_o T_s} = \frac{150\text{x}10^{-6}\text{x}1.58}{15\text{x}50\text{x}10^{-6}} = 0.316$$

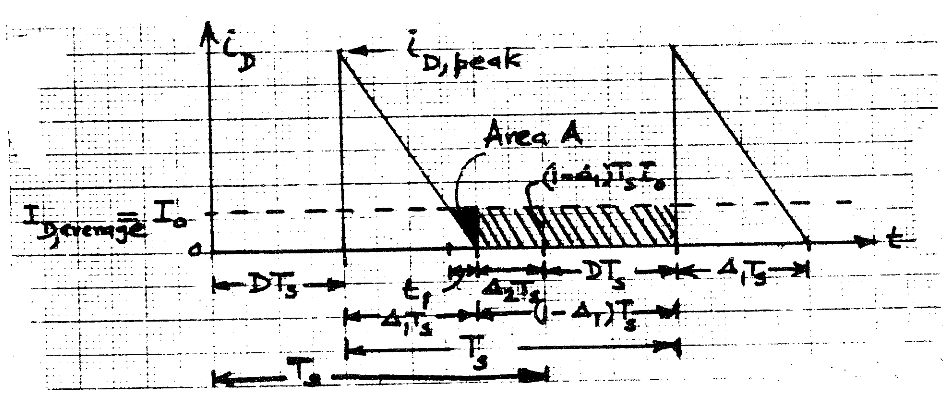
From the figure above; using the definition of rms (same as in Prob. 5-9)

$$I_D(\text{rms}) = i_{D,peak}\sqrt{\frac{\Delta_1}{3}} = 1.58\sqrt{\frac{0.316}{3}} = 0.513\text{A}$$

$$i_D = I_{D,average} + i_{ripple} = I_o + i_{ripple}$$

$$\therefore \; I_{ripple}(\text{rms}) = [\, I_D^2(\text{rms}) - I_o^2 \,]^{1/2} = \sqrt{0.513^2 - 0.25^2} = 0.448 \text{ A}$$

Problem 7-15



$$\Delta Q = (1-\Delta_1)T_sI_o + \text{Area A}$$

From the figure above,

$$\frac{t_1}{\Delta_1 T_s} = \frac{I_o}{i_{D,peak}} \; ; \; t_1 = \frac{\Delta_1 T_s I_o}{i_{D,peak}}$$

$$i_{D,peak} = i_{L,peak} = \frac{V_d}{L} DT_s \; ; \; \text{From Eq. 7-52, } \Delta_1 = \frac{V_d}{V_o} \cdot D$$

$$\therefore t_1 = V_o \frac{I_o}{L} = \frac{LI_o}{V_o} = \frac{L}{R} \quad \text{where R = load resistance}$$

$$\therefore \text{Area A} = \frac{1}{2} I_o t_1 = \frac{1}{2} \frac{LI_o}{R}$$

$$\therefore \Delta V_o = \frac{\Delta Q}{C} = \frac{1}{C}\left[(1 - D\frac{V_d}{V_o})T_sI_o + \frac{1}{2} L \frac{I_o}{R}\right]$$

Problem 7-16

From Eq. 7-55, $D = \frac{V_o}{V_d}\sqrt{\frac{I_o}{I_{oB,max}}}$

$I_{oB} = 0.493A$ (Problem 7-13)

$\therefore I_o = \frac{I_{oB}}{2} = 0.2465A$

$I_{oB,max} = 2.5$ (Problem 7-13)

$$\therefore D = \frac{15}{12}\sqrt{\frac{0.2465}{2.5}} = 0.3925 \; ; \; R = \frac{V_o}{I_o} = \frac{15}{0.2465} = 60.85\Omega$$

From the derivation of ΔV_o in problem 7-15

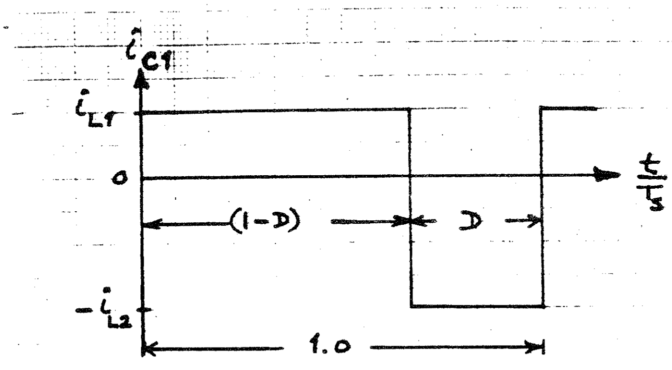
$$\Delta V_o = \frac{1}{470\text{x}10^{-6}}\left[(1-\frac{0.3925\text{x}12}{15})50\text{x}10^{-6}\text{x}0.2465 + \frac{1}{2}150\text{x}10^{-6}\frac{0.2465}{60.85}\right]$$

$$= 18.64 \text{ mV}$$

Ćuk CONVERTER

Problem 7-17

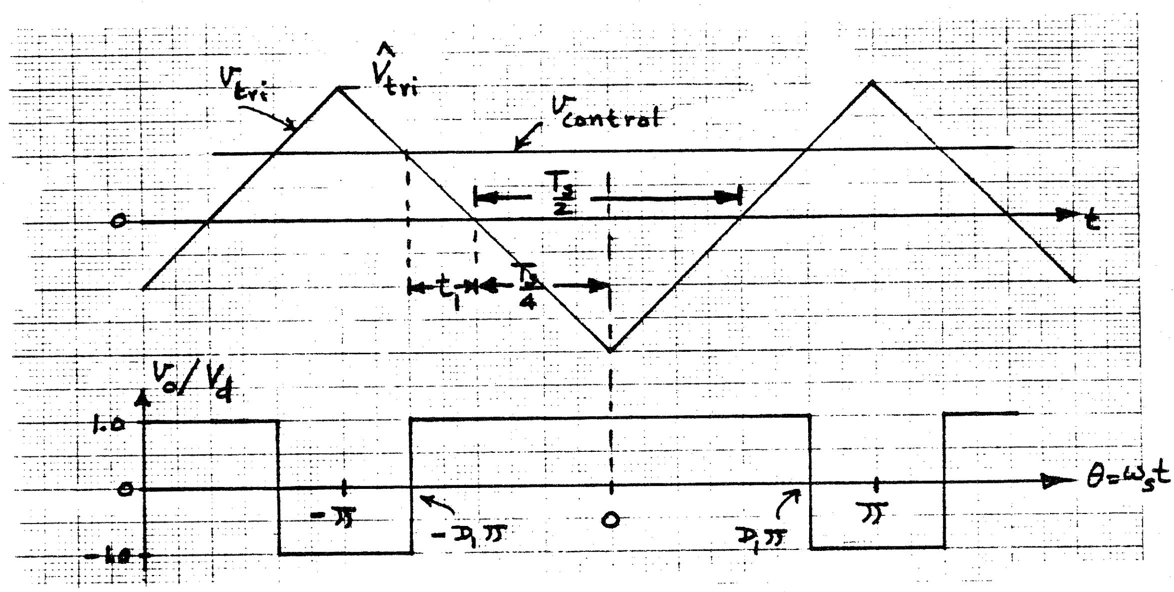
When the switch is off: $i_{c1} = i_{L1}$
When the switch is on: $i_{c1} = -i_{L2}$
Assume a constant $i_{L1} = I_d = 0.5A$, and a constant $i_{L2} = I_o = 1A$.
$D = 0.333$ (Example 7-3)



$$\therefore I_{c1}(rms) \approx \sqrt{\frac{(0.5^2 x 0.667) + 1^2 \times 0.333}{1}} = 0.707A$$

FULL-BRIDGE CONVERTER

Problem 7-18



Even function: f(t) = -f(t)

From Eqs. 7-71 and 7-72

$t_1 + \frac{T_s}{4} = \frac{D_1 T_s}{2} \quad \therefore \omega_s(t_1 + \frac{T_s}{4}) = \pi D_1$

For an even function:

$$f(t) = \frac{1}{2} a_o + \sum_{n=1}^{\infty} a_n \cos n\omega_s t$$

$$a_n = \frac{4}{T_s} \int_0^{T_s/2} f(t) \cdot \cos(n\omega_s t) \cdot dt$$

$$= \frac{4}{\omega_s T_s} \int_0^{\omega T_s/2} f(\omega_s t) \bullet \cos(n\omega_s t) \bullet d(\omega_s t) = \frac{2}{\pi} \int_0^{\pi} f(\theta) \bullet \cos(n\theta) \bullet d\theta$$

where $\omega_s T_s = 2\pi$ and $\omega_s t = \theta$. D_1 is the duty-ratio as defined in Eq. 7-72.

$$\therefore a_o = \frac{2}{\pi}\left[\int_0^{D_1\pi} 1 \bullet d\theta + \int_{D_1\pi}^{\pi} (-1) \bullet d\theta \right] = \frac{2}{\pi}[D_1\pi - \pi + D_1\pi] = 2[2D_1 - 1]$$

From Eq. 7-72, $D_1 = 0.75$

$$\therefore \frac{V_o}{V_d} = \frac{1}{2} a_o = 0.5$$

$$a_n = \frac{2}{\pi}\left[\int_0^{D_1\pi} 1 \bullet \cos(n\theta) \bullet d\theta + \int_{D_1\pi}^{\pi} (-1) \bullet \cos(n\theta) \bullet d\theta \right]$$

$$= \frac{2}{n\pi} [\sin(D_1 \bullet n\pi) - \sin(n\pi) + \sin(D_1 n\pi)] = \frac{2}{n\pi} [2 \bullet \sin(D_1 n\pi)]$$

$$= \frac{4}{n\pi} \sin(D_1 n\pi)$$

$$\therefore \frac{\hat{V}_{o,n}}{V_d} = \frac{4}{n\pi} \sin D_1 n\pi \qquad n = 1,2,3,\ldots$$

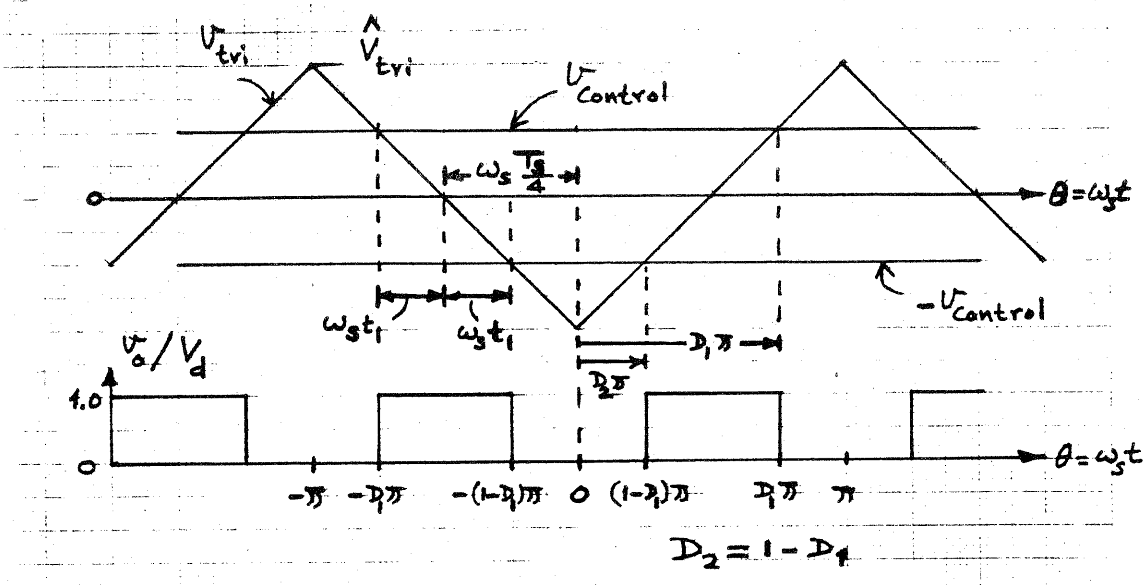
(n represents the nth multiple of the switching frequency f_s and $\hat{V}_{o,n}$ is the peak amplitude)

n	$\frac{\hat{V}_{o,n}}{V_d}$ ($D_1 = 0.75$)
1	0.9
2	0.64
3	0.3
4	0
5	0.18
6	0.212

Note: We are asked to assume that $i_o(t) \simeq I_o$. Therefore, $[i_d(t)/I_o]$ waveform will be identical to the $[v_o(t)/V_d]$ waveform shown earlier.

Also, the Fourier components of $[i_d(t)/I_o]$ will be identical to those shown in the Table on the left.

Problem 7-19



even function:

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(\theta)\cdot\cos(n\theta)\cdot d\theta$$

From Eq. 7-78, $D_1 = 0.75\pi$

$$a_o = \frac{2}{\pi}\left[\int_{0.25\pi}^{0.75\pi} (1)d\theta\right] = \frac{2}{\pi}[0.5\pi] = 1$$

$$\therefore \frac{V_o}{V_d} = \frac{1}{2}a_o = 0.5$$

Note: We are asked to assume that $i_o(t) \simeq I_o$. Therefore, $[i_d(t)/I_o]$ waveform will be identical to the $[v_o(t)/V_d]$ waveform shown above.

Also, the Fourier components of $[i_d(t)/I_o]$ will be identical to those shown in the Table below.

$$a_n = \frac{2}{\pi}\left[\int_{0.25\pi}^{0.75\pi} 1\cdot\cos(n\theta)\cdot d\theta\right] = \frac{2}{n\pi}[\sin(0.75n\pi) - \sin(0.25n\pi)]$$

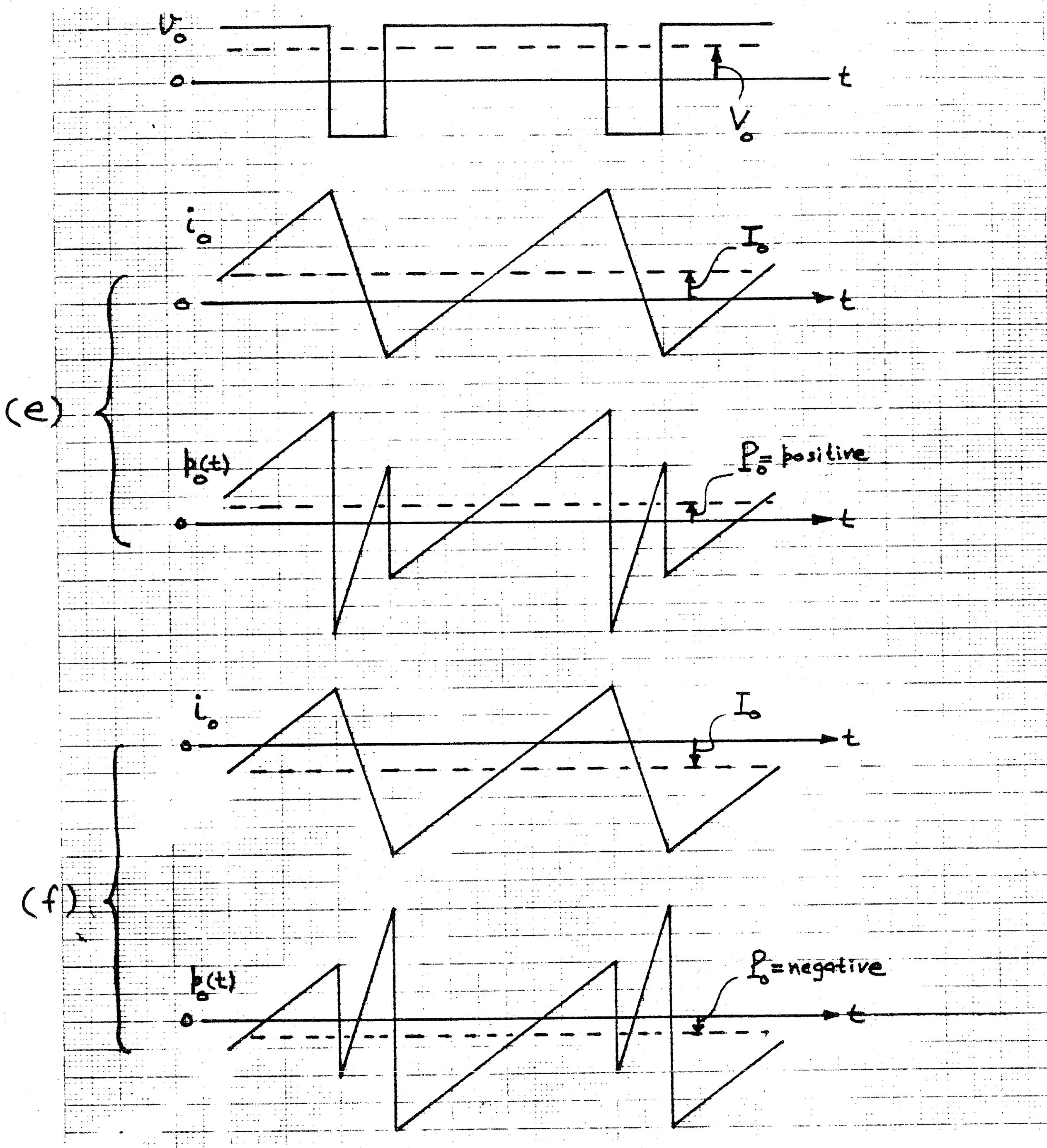
$$= \frac{4}{n\pi}\left[\sin(0.25n\pi) - 2\sin^3(0.25n\pi)\right]$$

Note: $[\sin 3A = 3\sin A - 4\sin^3 A]$

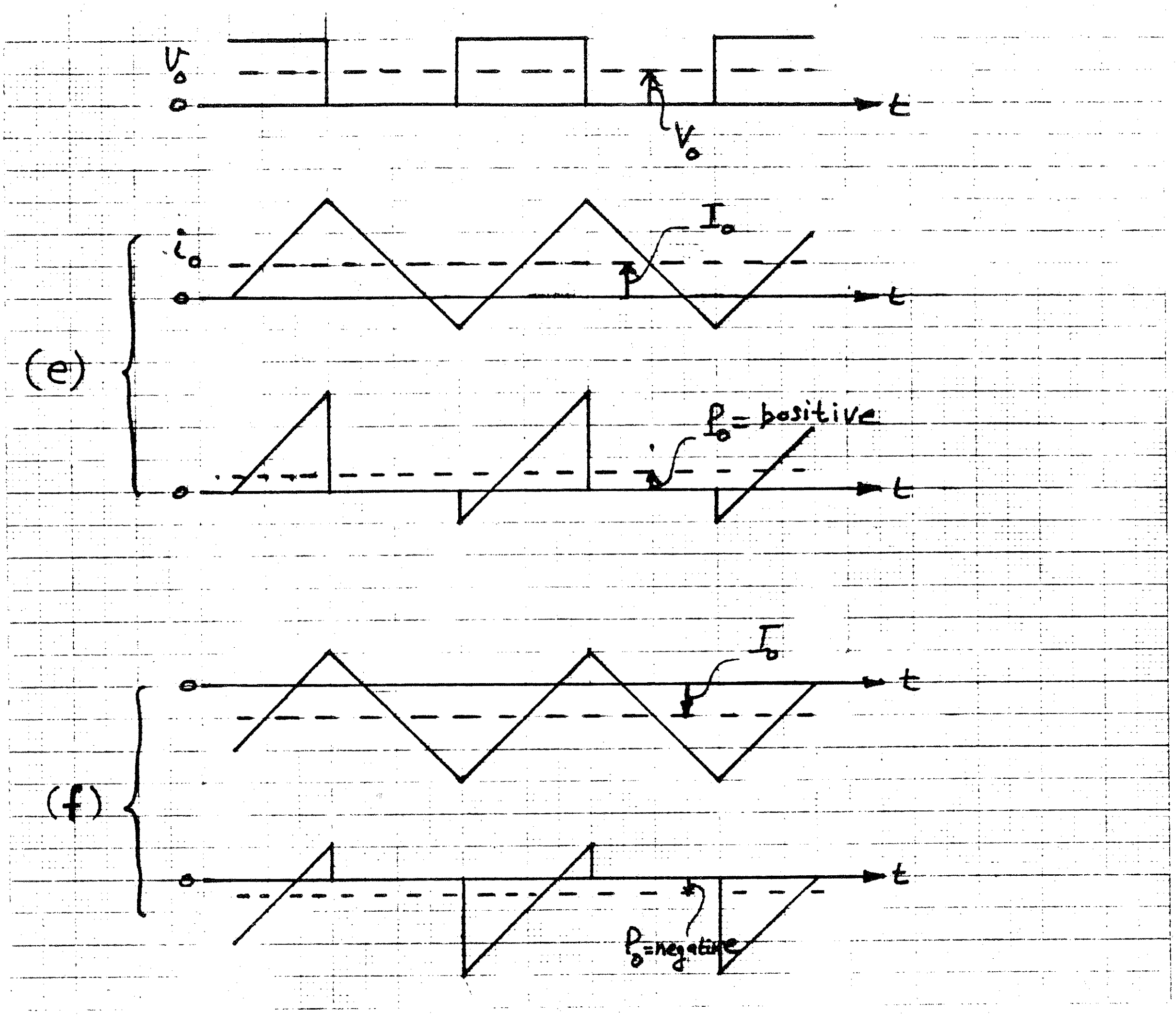
n	1	2	3	4	5	6
$\hat{V}_{on}/V_d$	0	0.64	0	0	0	0.212

Problem 7-20

The waveforms are drawn with $R_a = 0$ in Fig. 7-27. The average power is obtained as $P_o = V_o \cdot I_o$.



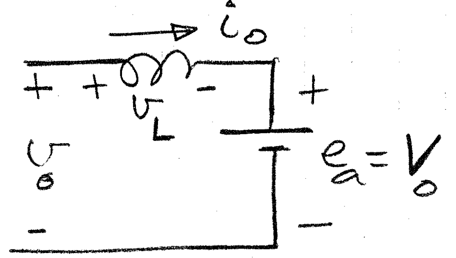
Problem 7-21

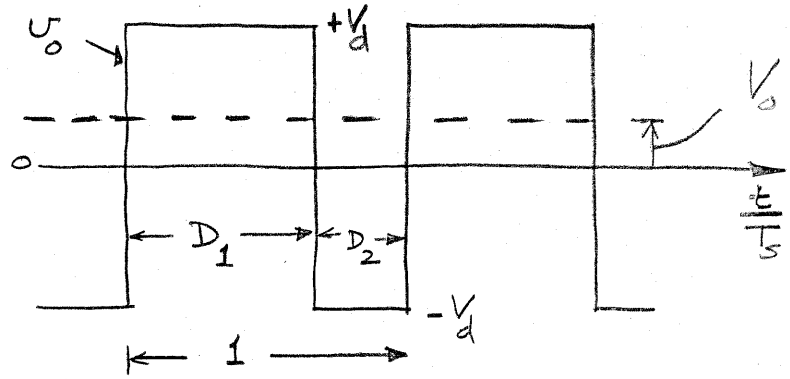


Problem 7-22

In the circuit of Fig. 7-27, $R_a \simeq 0$.

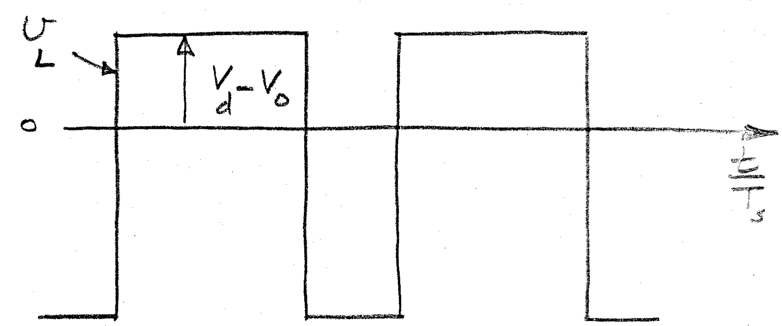
Consider the waveforms in Fig. 7-28 under steady state conditions.





during D_1: T_A^+, T_B^- ON

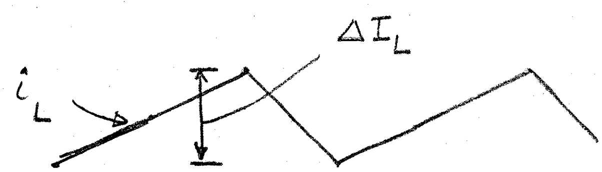
during D_2: T_A^-, T_B^+ ON



$$\Delta I_L = \frac{V_d - V_o}{L_a} D_1 T_s$$

From Eq. 7-74

$$D_1 = \frac{1}{2}\left(1 + \frac{V_o}{V_d}\right)$$



$$\therefore \Delta I_L = \frac{T_s V_d}{2 L_a}\left(1 + \frac{V_o}{V_d}\right)\left(1 - \frac{V_o}{V_d}\right)$$

$$\frac{\partial(\Delta I_L)}{\partial\left(\frac{V_o}{V_d}\right)} = \frac{T_s V_d}{2 L_a}\left(-2\frac{V_o}{V_d}\right) \underset{(\text{set to})}{=} 0 \quad \therefore \text{ maximum occurs at}$$

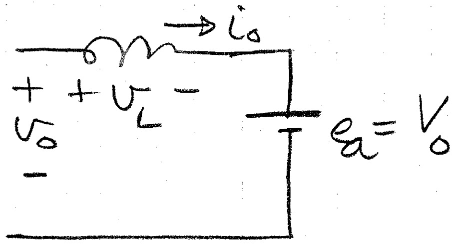
$$\frac{V_o}{V_d} = 0.$$

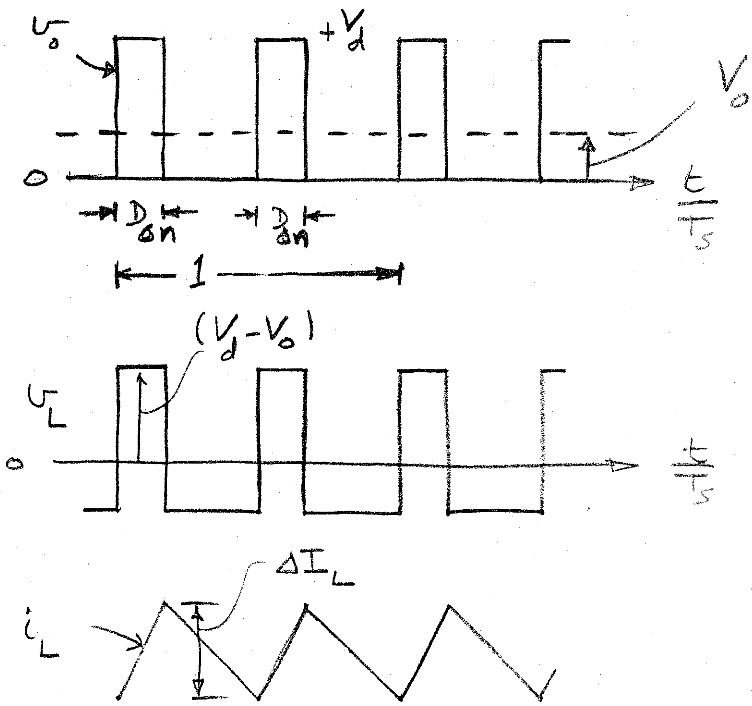
Using $\frac{V_o}{V_d} = 0$ in the equation for ΔI_L,

$$(\Delta I_L)_{maximum} = \frac{T_s V_d}{2 L_a} = \frac{V_d}{2 L_a f_s}.$$

Problem 7-23

In the circuit of Fig 7-27, $R_a \simeq 0$. Consider the waveforms in Fig. 7-29 under steady state.





during D_{on}: T_{A+}, T_{B-} ON

$$\Delta I_L = \frac{V_d - V_o}{L_a} D_{on} T_s$$

From the Waveform of v_o:

$$2 V_d D_{on} = V_o$$

$$\therefore D_{on} = \frac{1}{2}\frac{V_o}{V_d}$$

$$\therefore \Delta I_L = \frac{T_s V_d \left(1 - \frac{V_o}{V_d}\right)}{2 L_a} \frac{V_o}{V_d}$$

$$\frac{\partial(\Delta I_L)}{\partial\left(\frac{V_o}{V_d}\right)} = \frac{T_s V_d}{2 L_a}\left[-\frac{V_o}{V_d} + \left(1 - \frac{V_o}{V_d}\right)\right] \underset{\text{(set to)}}{=} 0$$

$\therefore$ maximum occurs at $\frac{V_o}{V_d} = \frac{1}{2}$.

Using $\frac{V_o}{V_d} = \frac{1}{2}$ in the equation for ΔI_L,

$$(\Delta I_L)_{maximum} = \frac{T_s V_d}{2 L_a}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{T_s V_d}{8 L_a}$$

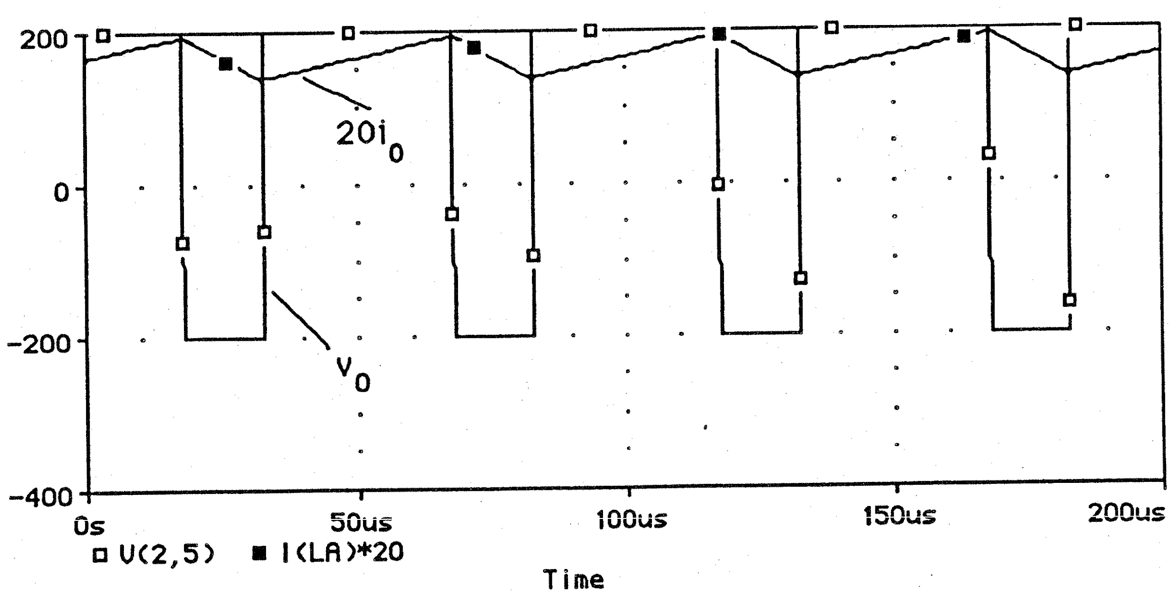
$$= \frac{V_d}{8 L_a f_s}.$$

Problem 7-24

(a) Bi-polar voltage switching

Input Circuit File Listing:

```
FBBSDCDC.CIR
* Full-Bridge, Bipolar-Switching, DC-DC Converter
* Power Electronics: Simulation, Analysis & Education.....by N. Mohan.
.LIB        PWR_ELEC.LIB
.PARAM      RISE=24.99us, FALL=24.99us, PW=0.01us, PERIOD=50us
*
VCONTL      50    0     0.416V
XLOGIC      50    0     52     53     PWM_TRI
*
XSWA1       1     2     52     0      SWITCH
XDA1        2     1     SW_DIODE_WITH_SNUB
XSWA2       2     0     53     0      SWITCH
XDA2        0     2     SW_DIODE_WITH_SNUB
*
XSWB1       1     5     53     0      SWITCH
XDB1        5     1     SW_DIODE_WITH_SNUB
XSWB2       5     0     52     0      SWITCH
XDB2        0     5     SW_DIODE_WITH_SNUB
*
RA          2     3     0.37
LA          3     4     1.5mH  IC=8.33A
*
VEMF        4     5     79.5V
VD          1     0     200V
*
.TRAN       0.1us   200.0us     0s     0.5us    uic
.PROBE
.END
```



(b) Unipolar Voltage Switching

```
FBUSDCDC.CIR
* Full-Bridge, Unipolar-Switching, DC-DC Converter
* Power Electronics: Simulation, Analysis & Education.....by N. Mohan.
.LIB PWR_ELEC.LIB
.PARAM VcntlA = 0.416V, VcntlB = -0.416V
.PARAM RISE=24.99us, FALL=24.99us, PW=0.01us ,PERIOD=50us
*
VCONTLA  49  0  0.416V
VCONTLB  50  0 -0.416V
*
XLOGICA  49  0 52 53 PWM_TRI
XLOGICB  50  0 54 55 PWM_TRI
*
XSWA1     1  2 52  0 SWITCH
XDA1      2  1 SW_DIODE_WITH_SNUB
XSWA2     2  0 53  0 SWITCH
XDA2      0  2 SW_DIODE_WITH_SNUB
*
XSWB1     1  5 54  0 SWITCH
XDB1      5  1 SW_DIODE_WITH_SNUB
XSWB2     5  0 55  0 SWITCH
XDB2      0  5 SW_DIODE_WITH_SNUB
*
RA        2  3 0.37
LA        3  4 1.5mH  IC=8.33A
*
VEMF      4  5 79.5V
VD        1  0 200V
*
.TRAN   0.1us  200.0us   0s   0.5us   uic
.PROBE
.END
```