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Solutions Manual
to Accompany

Analysis of Electric Machinery

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Solutions Manual to Accompany
ANALYSIS OF ELECTRICAL MACHINERY

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Chapter 1.

1-1 $$R = \frac{l}{\mu_r \mu_0 A} = 79577 \ (1/H)$$ $$\frac{1}{۲۵۰۰ \times ۴\pi \times ۱۰^{-۷} \times ۰/۰۰۴}$$

$$L_{m1} = \frac{N_1^2}{R} = 31.4 \ (mH)$$

$$L_{m2} = 125.7 \ (mH)$$

1-2 $$R_{iron} = 79418 \ (1/H)$$

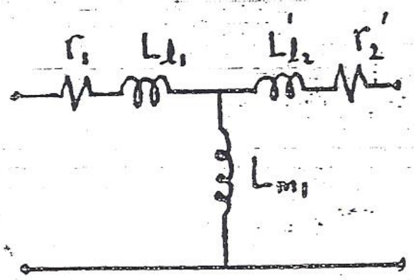
$$R_{air} = 6.3662 \times 10^5 \ (1/H)$$

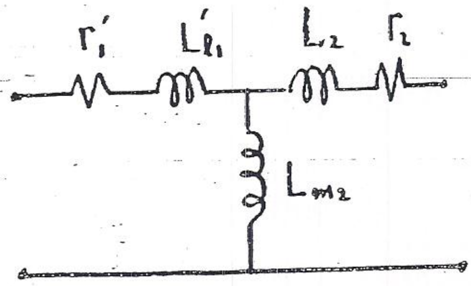
$$R = R_{iron} + R_{air} = 7.1604 \times 10^5 \ (1/H)$$

$$L_{m1} = 3.49 \ (mH)$$

$$L_{m2} = 13.97 \ (mH)$$

1-3





$$r_2' = 10 \ (\Omega)$$

$$L_{m1} = 90 \ (mH)$$

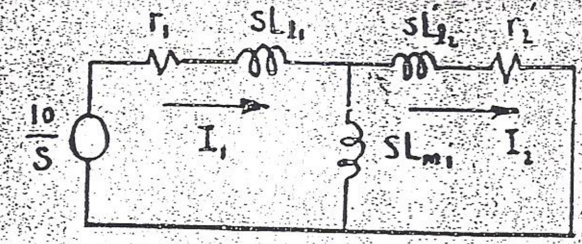
$$L_{l2}' = L_{l1} = 10 \ (mH)$$

$$r_1' = 2.5 \ (\Omega)$$

$$L_{m2} = 22.5 \ (mH)$$

$$L_{l1}' = L_{l2} = 2.5 \ (mH)$$

1-4



$$I_1(s) = \frac{10(r'_2 + sL'_{22})}{S[(L_{m_1}L_{l_1} + L_{m_1}L'_{l_2} + L_{l_1}L'_{l_2})S^2 + (r_1L'_{22} + r'_2L_{11})S + r_1r'_2]}$$

$$= \frac{S + 100.0}{.0019S^3 + 2.0S^2 + 100.0S}$$

$$= \frac{1.0}{S} - \frac{0.5}{S+52.63} - \frac{0.5}{S+1000.0}$$

$$i_1(t) = 1.0 - 0.5e^{-52.63t} - 0.5e^{-1000.0t} \quad (A)$$

$$I_2(s) = \frac{10L_{m_1}\cancel{S}}{\cancel{S}[(L_{m_1}L_{l_1} + L_{m_1}L'_{l_2} + L_{l_1}L'_{l_2})S^2 + (r_1L'_{22} + r'_2L_{11})S + r_1r'_2]}$$

$$= \frac{1.0}{.0019S^2 + 2.0S + 100.0}$$

$$= \frac{0.556}{S+52.63} - \frac{0.556}{S+1000.0}$$

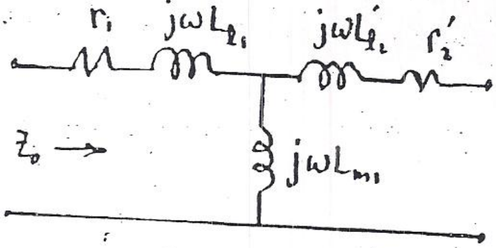
$$i_2(t) = 0.556(e^{-52.63t} - e^{-1000.0t}) \quad (A)$$

$$i_1(\infty) = I_1(t) = 1.0 \quad (A)$$

$$i_2(\infty) = I_2(t) = 0 \quad (A)$$

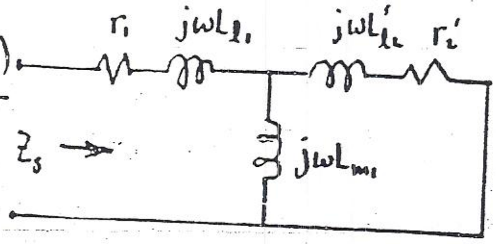
1-5 (a) $Z_0 = r_1 + j\omega L_{11}$

$= 10 + j\,37.7$

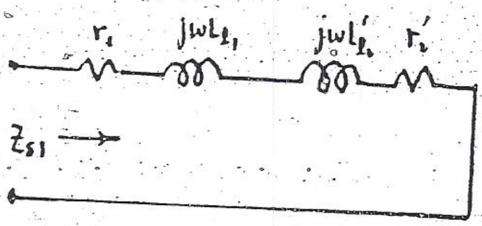


(b) $Z_s = r_1 + j\omega L_{\ell 1} + \dfrac{j\omega L_{m1}\,(r'_2 + j\omega L'_{\ell 2})}{r'_2 + j\omega L'_{22}}$

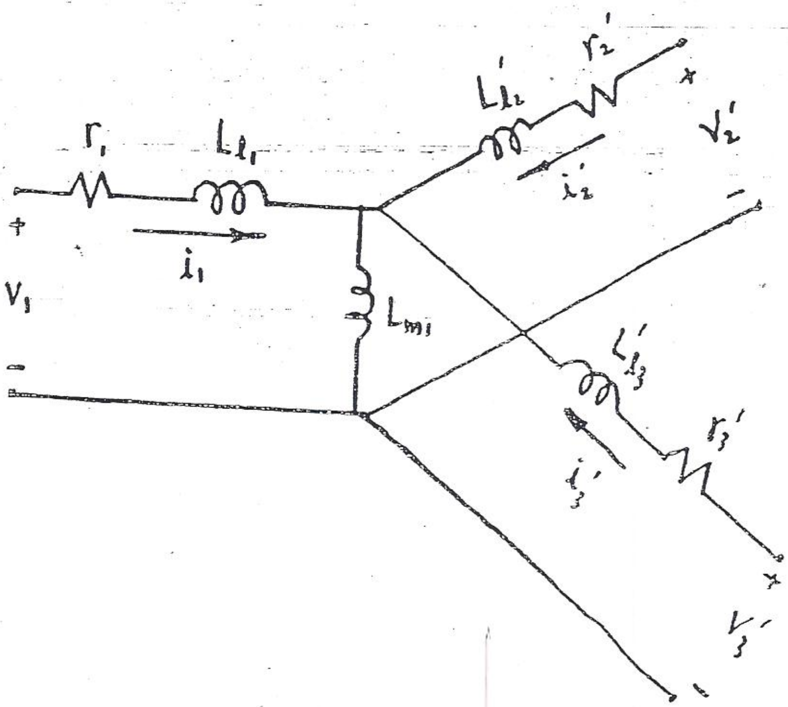
$= 17.57 + j\,9.17$



(c) $Z_{s1} = 20.0 + j\,7.54$



1-6



1-8 $$i = 10\, e^{-60t} \qquad W_{es}(t) = \frac{1}{2} L i^2 = 12.5\, e^{-120t} \ (\mathrm{J})$$

$$W_{eL}(t) = r \int_0^t i^2 dt = 1500 \int_0^t e^{-120t} dt = 12.5\,(1.0 - e^{-120t}) \ (\mathrm{J})$$

1-10 (a) $$\lambda(i, x) = i^{2/3} x^2$$

$$W_f = \int i\, d\lambda = \frac{2}{3} \int x^2 i^{2/3} di = \frac{2}{5} x^2 i^{5/3} \ (\mathrm{J})$$

$$W_c = \int \lambda\, di = \int x^2 i^{2/3} di = \frac{3}{5} x^2 i^{5/3} \ (\mathrm{J})$$

(b) $$\lambda(i, x) = \left(K \sin \frac{x}{a}\pi - x\right) i$$

$$W_f = W_c = \frac{1}{2}\left(K \sin \frac{x}{a}\pi - x\right) i^2 \ (\mathrm{J})$$

1-11 $$\lambda_1 = L_{11} i_1 + L_{12} i_2 \qquad \lambda_2 = L_{12} i_1 + L_{22} i_2$$

$$W_f = \frac{1}{2} B_{11} \lambda_1^2 + B_{12} \lambda_1 \lambda_2 + \frac{1}{2} B_{22} \lambda_2^2$$

$$= \left(\frac{1}{2} B_{11} L_{11}^2 + B_{12} L_{11} L_{12} + \frac{1}{2} B_{22} L_{12}^2\right) i_1^2$$

$$+ \left[B_{11} L_{11} L_{12} + (L_{11} L_{22} + L_{12}^2) B_{12} + B_{22} L_{12} L_{22}\right] i_1 i_2$$

$$+ \left(\frac{1}{2} B_{11} L_{12}^2 + B_{12} L_{12} L_{22} + \frac{1}{2} B_{22} L_{22}^2\right) i_2^2$$

$\because \quad W_f = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + L_{22} i_2^2$

$$\therefore \begin{cases} L_{11} = L_{11}^2 B_{11} + 2 L_{11} L_{12} B_{12} + L_{12}^2 B_{22} \\ L_{12} = L_{11} L_{12} B_{11} + (L_{11} L_{22} + L_{12}^2) B_{12} + L_{12} L_{22} B_{22} \\ L_{22} = L_{12}^2 B_{11} + 2 L_{12} L_{22} B_{12} + L_{22}^2 B_{22} \end{cases}$$

Solve above equation for B_{11}, B_{12} and B_{22},

$$B_{11} = \frac{L_{22}}{L_{11}L_{22} - L_{12}^2} \qquad B_{22} = \frac{L_{11}}{L_{11}L_{22} - L_{12}^2}$$

$$B_{12} = -\frac{L_{12}}{L_{11}L_{22} - L_{12}^2}$$

1-12 $$W_f = \int i_1 \frac{\partial \lambda_1}{\partial i_1} di_1 + \int \left[i_1 \frac{\partial \lambda_1}{\partial i_2} di_2 + i_2 \frac{\partial \lambda_2}{\partial i_2} di_2 \right]$$

$$= \int 2X^2 i_1^2 \, di_1 + \int [X i_1 + 2X^2 i_2^2] \, di_2$$

$$= \frac{2}{3} X^2 i_1^3 + X i_1 i_2 + \frac{2}{3} X^2 i_2^3 \qquad (J)$$

$$W_c = \int [\lambda_1 di_1 + \lambda_2 di_2] \Big|_{i_2=0} + \int [\lambda_1 di_1 + \lambda_2 di_2] \Big|_{i_1=\text{cons}}$$

$$= \int X^2 i_1^2 \, di_1 + \int (X^2 i_2^2 + X i_1) \, di_2$$

$$= \frac{1}{3} X^2 i_1^3 + X i_1 i_2 + \frac{1}{3} X^2 i_2^3 \qquad (J)$$

1-13 (a) From 1-10, $W_c = \frac{3}{5} x^2 i^{5/3}$

$$f_e = \frac{\partial W_c}{\partial x} = \frac{6}{5} x i^{5/3}$$

(b) From 1-10, $W_c = \frac{1}{2}(K \sin \frac{x}{a}\pi - x) i^2$

$$f_e = \frac{\partial W_c}{\partial x} = \frac{i^2}{2}\left(\frac{K\pi}{a} \cos \frac{x}{a}\pi - 1\right)$$

1-14 From 1-12, $W_c = \frac{1}{3}x^2 i_1^3 + x i_1 i_2 + \frac{1}{3} x^2 i_2^3$

$$f_e = \frac{\partial W_c}{\partial x} = \frac{2}{3} x (i_1^3 + i_2^3) + i_1 i_2$$

1-16 $W_{es} = \frac{1}{2} l i^2 = 0$ because $l = 0$

$\because$ $x = 2.5$ mm , $L(x) = \frac{k}{x} = 2.517 \times 10^{-2}$ H

$W_f = \frac{1}{2} L i^2 = \frac{1}{2} L (\frac{V}{r})^2 = 3.146 \times 10^{-3}$ (J)

$W_{ms} = \frac{1}{2}(x - x_0)^2 K = \frac{1}{2}(0.5 \times 10^{-3})^2 \cdot 2667 = 3.334 \times 10^{-4}$ (J)

1-19 $T_e = -i_1 i_2 M \sin\theta_r$ [from (1B-7)]

$$= -I_{s_1} I_{s_2} \cos w_1 t \cos(w_2 t + \varphi_2) \cdot M \sin(\theta_r(0) + w_r t)$$

$$= -\frac{1}{2} I_{s_1} I_{s_2} M \left[\cos\left((w_1+w_2)t + \varphi_2\right) + \cos\left((w_1-w_2)t - \varphi_2\right) \right] \sin(w_r t + \theta_r(0))$$

(a) $w_1 = w_2 = 0$

$$T_e = -I_{s_1} I_{s_2} M \cos\varphi_2 \sin(w_r t + \theta_r(0))$$

If $w_r \neq 0$ Average $T_e = 0$.

If $w_r = 0$ and $\theta_r(0) = K\pi$, $(K = 0, 1, 2 \cdots)$ Average $T_e = 0$;

$w_r = 0$ and $\theta_r(0) \neq K\pi$, $(K = 0, 1, 2 \cdots)$ Average $T_e \neq 0$.

(b) $w_1 = w_2 \neq 0$

$$T_e = -\frac{1}{2} I_{s_1} I_{s_2} M \left[\cos(2w_1 t + \varphi_2) + \cos\varphi_2 \right] \sin(w_r t + \theta_r(0))$$

$$= -\frac{1}{4} I_{s_1} I_{s_2} M \Big[\sin(w_r t + \theta_r(0) + 2w_1 t + \varphi_2)$$

$$+ \sin(w_r t + \theta_r(0) - 2w_1 t - \varphi_2) + 2\cos\varphi_2 \sin(w_r t + \theta_r(0)) \Big]$$

If $w_r = 0$ and $\theta_r(0) \neq K\pi$ $(K = 0, 1, 2 \cdots)$ and $\varphi_2 \neq \frac{K\pi}{2}$ $(K = 1, 2, \cdots;)$,

Average $T_e \neq 0$

If $\omega_r = 2\omega_1$ and $\varphi_2 \neq \theta_r(0)$; Average $T_e \neq 0$.

If $\omega_r = -2\omega_1$ and $\varphi_2 \neq -\theta_r(0)$; Average $T_e \neq 0$.

(c) $\omega_1 \neq 0$, $\omega_2 = 0$

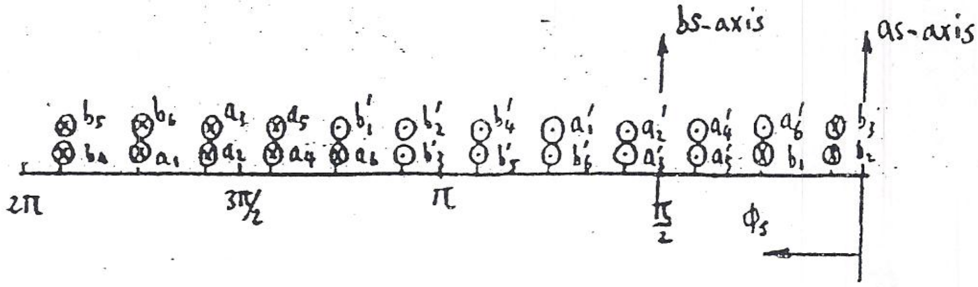
$$T_e = -\frac{1}{2} I_{S_1} I_{S_2} M \cos\varphi_2 \left[\cos\omega_1 t \sin\left(\omega_r t + \theta_r(0)\right) \right]$$

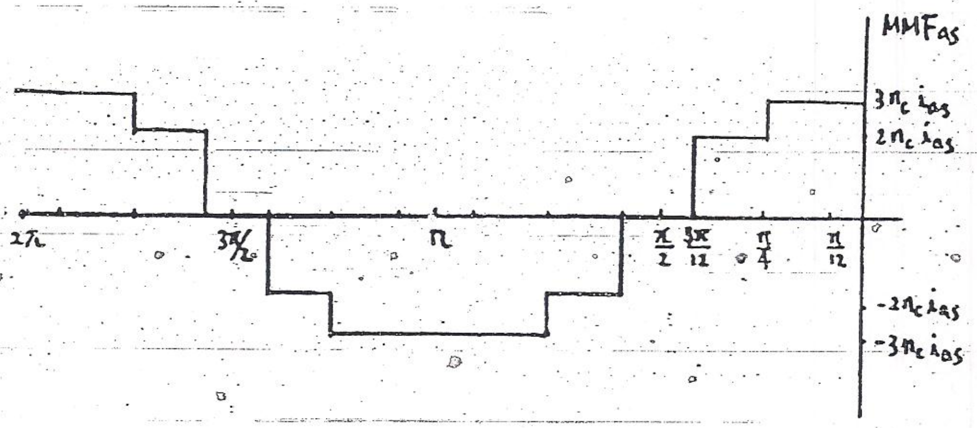
$$= -\frac{1}{4} I_{S_1} I_{S_2} M \cos\varphi_2 \left[\sin\left((\omega_r+\omega_1)t + \theta_r(0)\right) + \sin\left((\omega_r-\omega_1)t + \theta_r(0)\right) \right]$$

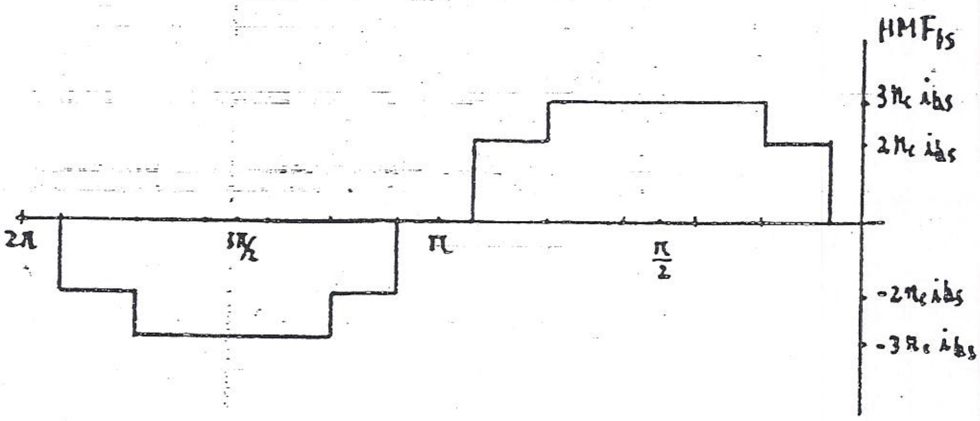
If $\varphi_2 \neq \frac{k\pi}{2}$ $(k=1,2\cdots)$ and $\omega_r = \pm\omega_1$ and $\theta_r(0) \neq k\pi$ $(k=0,1,2\cdots)$

Average $T_e \neq 0$

1-20


bs-axis
as-axis
b_5 b_6 a_3 a_5 b_1' b_2' b_4' a_1' a_2' a_4' a_6' b_3
b_4 a_1 a_2 a_4 a_6 b_3' b_5' b_6' a_3' a_5' b_1 b_2
2π
$3\pi/2$
π
$\frac{\pi}{2}$
ϕ_s


MMF_{as}
$3n_c i_{as}$
$2n_c i_{as}$
2π
$3\pi/2$
π
$\frac{\pi}{2}$
$\frac{5\pi}{12}$
$\frac{\pi}{4}$
$\frac{\pi}{12}$
$-2n_c i_{as}$
$-3n_c i_{as}$


MMF_{bs}
$3n_c i_{bs}$
$2n_c i_{bs}$
2π
$3\pi/2$
π
$\frac{\pi}{2}$
$-2n_c i_{bs}$
$-3n_c i_{bs}$

1-21 (a)

$$N_{as} = \begin{cases} N_p \sin \Phi_s & 0 \le \Phi_s \le \pi \\ -N_p \sin \Phi_s & \pi \le \Phi_s \le 2\pi \end{cases}$$

where $N_p = N_s/2$

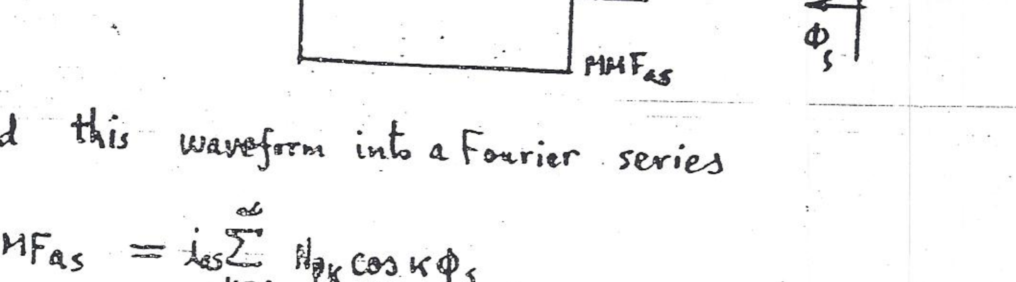
$$N_{bs} = \begin{cases} N_p \cos \Phi_s & 0 \le \Phi_s \le \frac{\pi}{2} \text{ and } \frac{3\pi}{2} \le \Phi_s \le 2\pi \\ -N_p \cos \Phi_s & \frac{\pi}{2} \le \Phi_s \le \frac{3\pi}{2} \end{cases}$$

(b) $$MMF_{as} = \frac{N_s}{2} i_{as} \cos \Phi_s$$

$$MMF_{bs} = \frac{N_s}{2} i_{bs} \sin \Phi_s$$

(c) $$MMF_s = \frac{N_s}{2} \left[i_{as} \cos \Phi_s + i_{bs} \sin \Phi_s \right]$$

1-22



Expand this waveform into a Fourier series

$$MMF_{as} = i_{as} \sum_{k=1}^{\infty} N_{pk} \cos k\Phi_s$$

$$i_{as} N_p = i_{as} N_{p1} = \frac{1}{\pi} \int_0^{2\pi} MMF_{as} \cos \Phi_s \, d\Phi_s = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} MMF_{as} \cos \Phi_s \, d\Phi_s$$

$$= \frac{4}{\pi} \left[\int_0^{\frac{\pi}{3}} 2n_c i_{as} \cos \Phi_s \, ds + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} n_c i_{as} \cos \Phi_s \, d\Phi_s \right]$$

$$N_p = \frac{4n_c}{\pi}\left[2\sin\frac{\pi}{3} + 1 - \sin\frac{\pi}{3}\right]$$

$$= \frac{4n_c}{\pi}\left[1 + \frac{\sqrt{3}}{2}\right] = 2.38\, n_c$$

$$N_s = 2N_p = 4.75\, n_c$$

1-23 (a) $I_{as} = \sqrt{2}\, I_s \cos W_e t \qquad I_{bs} = \sqrt{2}\, I_s \sin W_e t$

$$MMF_s = \frac{N_s}{2}\cdot\sqrt{2}\, I_s\left(\cos W_e t \cos\phi_s + \sin W_e t \sin\phi_s\right)$$

$$= \frac{\sqrt{2}\, N_s I_s}{2}\cos\left(W_e t - \phi_s\right)$$

(b) $I_{as} = \sqrt{2}\, I_s \cos W_e t \qquad I_{bs} = -\sqrt{2}\, I_s \sin W_e t$

$$MMF_s = \frac{\sqrt{2}\, N_s I_s}{2}\cdot\left(\cos W_e t \cos\phi_s - \sin W_e t \sin\phi_s\right)$$

$$= \frac{\sqrt{2}\, N_s I_s}{2}\cos\left(W_e t + \phi_s\right)$$

(c) $I_{as} = \sqrt{2}\, I_a \cos W_e t \qquad I_{bs} = \sqrt{2}\, I_b \sin W_e t$

$$MMF_s = \frac{\sqrt{2}\, N_s}{2}\left(I_a \cos W_e t \cos\phi_s + I_b \sin W_e t \sin\phi_s\right)$$

$$= \frac{\sqrt{2}\, N_s}{2}\left[\frac{I_a + I_b}{2}\left(\cos W_e t \cos\phi_s + \sin W_e t \sin\phi_s\right)\right.$$

$$\left. + \frac{I_a - I_b}{2}\left(\cos W_e t \cos\phi_s - \sin W_e t \sin\phi_s\right)\right]$$

$$= \frac{\sqrt{2}\, N_s}{2}\left[\frac{I_a + I_b}{2}\cos\left(W_e t - \phi_s\right) + \frac{I_a - I_b}{2}\cos\left(W_e t + \phi_s\right)\right]$$

1-24. $I_{bs} = -I_{as} = -\sqrt{2} I \cos w_e t$

$$MMF_{as} = \frac{N_s}{4}\sqrt{2} I \cos w_e t \cos 2\varphi_s$$

$$MMF_{bs} = -\frac{N_s}{4}\sqrt{2} I \cos w_e t \cos\left(2\varphi_s - \frac{2\pi}{3}\right) \qquad MMF_{cs} = 0$$

$$\therefore MMF_s = \frac{N_s}{4}\sqrt{2} I \cos w_e t \left[\cos 2\varphi_s - \cos\left(2\varphi_s - \frac{2\pi}{3}\right)\right]$$

$$= \frac{\sqrt{6} N_s}{4} I \cos w_e t \cos\left(2\varphi_s + \frac{\pi}{6}\right)$$

$$= \frac{\sqrt{6} N_s}{8} I \left[\cos\left(w_e t + 2\varphi_s + \frac{\pi}{6}\right) + \cos\left(w_e t - 2\varphi_s - \frac{\pi}{6}\right)\right]$$

1-26. $V_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$

$$V_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$V_{fd} = r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt}$$

where $\lambda_{as} = L_{asas} i_{as} + L_{asbs} i_{bs} + L_{asfd} i_{fd}$,

$$\lambda_{bs} = L_{bsas} i_{as} + L_{bsbs} i_{bs} + L_{bsfd} i_{fd} ,$$

$$\lambda_{fd} = L_{fdas} i_{as} + L_{fdbs} i_{bs} + L_{fdfd} i_{fd} .$$

And (see problem # 25)

$$L_{asas} = L_{ls} + L_A - L_B \cos 2\theta_r$$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos 2\theta_r$$

$$L_{asbs} = L_{bsas} = -L_B \sin 2\theta_r$$

$$L_{fdfd} = L_{lfd} + L_{mfd}$$

$$L_{asfd} = L_{fdas} = L_{sfd} \sin \theta_r$$

$$L_{bsfd} = L_{fdbs} = -L_{sfd} \cos \theta_r$$

$$W_c = W_f = \tfrac{1}{2}(L_{asas} - L_{ls}) i_{as}^2 + \tfrac{1}{2}(L_{bsbs} - L_{ls}) i_{bs}^2 + \tfrac{1}{2}(L_{fdfd} - L_{lfd}) i_{fd}^2$$

$$+ L_{asbs}\, i_{as} i_{bs} + L_{asfd}\, i_{as} i_{fd} + L_{bsfd}\, i_{bs} i_{fd}$$

$$T_e = \frac{\partial W_c}{\partial \theta_r} = L_B \sin 2\theta_r\, i_{as}^2 - L_B \sin 2\theta_r\, i_{bs}^2$$

$$- 2L_B \cos 2\theta_r\, i_{as} i_{bs} + L_{sfd} \cos \theta_r\, i_{as} i_{fd}$$

$$+ L_{sfd} \sin \theta_r\, i_{bs} i_{fd}$$

$$= L_B \sin 2\theta_r \,(i_{as}^2 - i_{bs}^2) - 2L_B \cos 2\theta_r\, i_{as} i_{bs}$$

$$+ L_{sfd}\, i_{fd} \,(\cos \theta_r\, i_{as} + \sin \theta_r\, i_{bs})$$

1-29 $v_{as} = r_s i_{as} + \frac{\partial \lambda_{as}}{\partial t}$ $\lambda_{as} = L_{asas} i_{as} + L_{asbs} i_{bs}$

$v_{bs} = r_s i_{bs} + \frac{\partial \lambda_{bs}}{\partial t}$ $\lambda_{bs} = L_{bsas} i_{as} + L_{bsbs} i_{bs}$

where $L_{asas} = L_{ls} + L_A - L_B \cos 2\theta_r$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos 2\theta_r$$

$$L_{asbs} = L_{bsas} = -L_B \sin 2\theta_r$$

$$W_c = W_f = \frac{1}{2}(L_{asas} - L_{ls}) i_{as}^2 + \frac{1}{2}(L_{bsbs} - L_{ls}) i_{bs}^2 + L_{asbs} i_{as} i_{bs}$$

$$T_e = \frac{\partial W_c}{\partial \theta_r} = L_B \sin 2\theta_r (i_{as}^2 - i_{bs}^2) - 2 L_B \cos 2\theta_r i_{as} i_{bs}$$

1-31 $v_{as} = r_s i_{as} + \frac{\partial \lambda_{as}}{\partial t}$

$$v_{bs} = r_s i_{bs} + \frac{\partial \lambda_{bs}}{\partial t}$$

$$v_{ar} = r_r i_{ar} + \frac{\partial \lambda_{ar}}{\partial t}$$

$$v_{br} = r_r i_{br} + \frac{\partial \lambda_{br}}{\partial t}$$

where $\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{ar} \\ \lambda_{br} \end{bmatrix} = \begin{bmatrix} L_{ls}+L_{ms} & 0 & L_{sr}\cos\theta_r & -L_{sr}\sin\theta_r \\ 0 & L_{ls}+L_{ms} & L_{sr}\sin\theta_r & L_{sr}\cos\theta_r \\ L_{sr}\cos\theta_r & L_{sr}\sin\theta_r & L_{lr}+L_{mr} & 0 \\ -L_{sr}\sin\theta_r & L_{sr}\cos\theta_r & 0 & L_{lr}+L_{mr} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{ar} \\ i_{br} \end{bmatrix}$

$$W_c = W_f = \frac{1}{2} L_{ms} (i_{as}^2 + i_{bs}^2) + \frac{1}{2} L_{mr} (i_{ar}^2 + i_{br}^2)$$

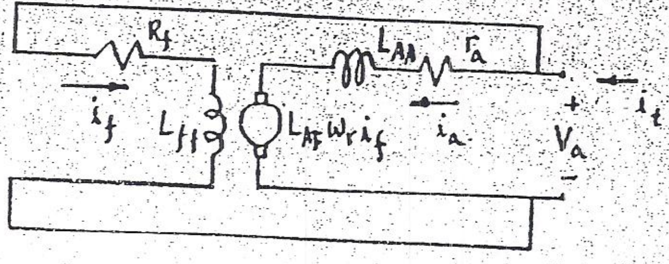
$$+ L_{sr} \cos\theta_r (i_{as} i_{ar} + i_{bs} i_{br}) + L_{sr} \sin\theta_r (i_{bs} i_{ar} - i_{as} i_{br})$$

$$T_e = \frac{\partial W_c}{\partial \theta_r} = L_{sr} \cos\theta_r (i_{bs} i_{ar} - i_{as} i_{br})$$

$$- L_{sr} \sin\theta_r (i_{as} i_{ar} + i_{bs} i_{br})$$

Chapter 2.

2-1 $I_t = \frac{100^W}{100^V} = 1\,A$



$I_f = \frac{V_a}{R_f} = 0.5\,A$

$I_a = I_t - I_f = 0.5\,A$

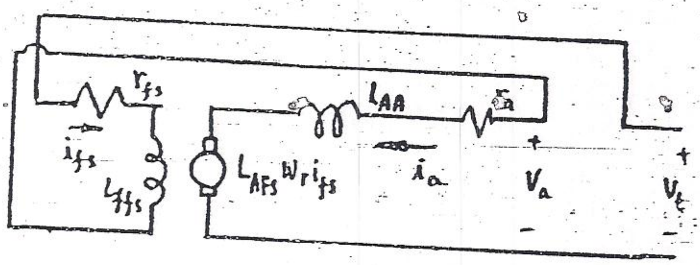
At full load, $I_a = \frac{V_a}{r_a}\left(1 - \frac{\omega_r L_{AF}}{R_f}\right) = 0.5\,A$

$$\frac{L_{AF}}{R_f} = \left(-\frac{I_a r_a}{V_a} + 1\right)/\omega_r = 0.004727$$

At no load, $I_a = 0$

$$\omega_r = \frac{R_f}{L_{Af}} \doteqdot 211.55\ \text{rad/sec} = 2020\ \text{RPM}$$

2-2 $I_a = I_{fs} = \frac{W}{V_t} = 1\,A$



$\omega_r L_{AFS} I_a = V_t - (r_a + r_{fs}) I_a$

$= 97\ (V)$

$L_{AFS} = 97/\omega_r I_a = 0.463$

At stall case, $I_a = \frac{V_t}{r_{fs} + r_a} = \frac{100}{3}$

$T_e = L_{AFS} I_a^2 = 514.6\ (N\cdot m)$

2-4. $P_{out} = T_L \omega_r$

$\because$ Steady state operation, $\frac{d\omega_r}{dt} = 0$.

$\therefore$ $T_L = T_e - B_m \omega_r = \frac{L_{AF} V_a^2}{r_a R_f} \left(1 - \frac{\omega_r L_{AF}}{R_f}\right) - B_m \omega_r$

$\frac{\partial P_{out}}{\partial \omega_r} = 0$ i.e. when

$\frac{L_{AF} V_a^2}{r_a R_f} \left(1 - \frac{2\omega_{rM} L_{AF}}{R_f}\right) - 2 B_m \omega_{rM} = 0$

$\omega_{rM} = \frac{L_{AF} V_a^2}{r_a R_f} \Big/ \left[\frac{2 L_{AF}^2 V_a^2}{r_a R_f^2} + 2 B_m \right]$,

the steady-state power output will take its maximum value

$P_{outM} = \frac{L_{AF} V_a^2}{2 r_a R_f} \omega_{rM}$

If neglect B_m $(B_m = 0)$

$\omega_{rM} = \frac{R_f}{2 L_{AF}}$ $\qquad I_{aM} = \frac{V_a}{2 r_a}$

$P_{outM} = \frac{V_a^2}{4 r_a} = \frac{1}{2} V_a I_{aM} = \frac{1}{2}\left(P_{in} - \frac{V_a^2}{R_f}\right)$

2-5. (a) $T_e = \frac{W^P}{\omega_r} = \frac{200.746}{600 \cdot 2\pi/60} = 2374.6 \text{ (N-m)}$

$$T_e = \frac{L_{AF} V_a^2}{r_a R_f} \left(1 - \frac{\omega_r L_{AF}}{R_f} \right) = 1.25 V_a^2 (1 - 0.942)$$

$$V_a = 181.7 \text{ (v)}$$

(b) $I_f = \frac{V_a}{R_f} = 15.14 \text{ (A)}$

$$I_a = \frac{V_a - \omega_r L_{AF} I_f}{r_a} = 871.1 \text{ (A)}$$

$$\text{Losses} = R_f I_f^2 + r_a I_a^2 = 11858 \text{ (w)}$$

$$\text{Input} = V_a (I_f + I_a) = 161058 \text{ (w)}$$

$$\text{EFF} = 1 - \frac{\text{losses}}{\text{Input}} = 92.6\%$$

2-7 (a) $V_a = \omega_r L_{AF} i_f + (r_a + pL_{AA}) i_a$

$$T_e = K_v i_a = J\frac{d\omega_r}{dt} + B_m \omega_r + T_L$$

$\because \Delta V_f = 0 \quad \therefore \Delta i_f = 0 \quad K_v = L_{AF} i_f = \text{constant}.$

$\because \Delta V_a = 0 \quad \therefore \quad K_v \Delta\omega_r + (r_a + SL_{AA}) \Delta i_a = 0 \qquad (1)$

$$K_v \Delta i_a = (SJ + B_m)\Delta\omega_r + \Delta T_L \qquad (2)$$

Solve $\Delta\omega_r$ from (1), and substitute $\Delta\omega_r$ into (2),

$$\left[K_v + \frac{(r_a + SL_{AA})(SJ + B_m)}{K_v}\right]\Delta i_a = \Delta T_L$$

$$\therefore \quad \frac{\Delta i_a}{\Delta T_L} = \frac{K_v}{JL_{AA}S^2 + (Jr_a + B_m L_{AA})S + B_m r_a + K_v^2}$$

(b) $\Delta V_a = K_v \Delta\omega_r + (r_a + SL_{AA})\Delta i_a \qquad (3)$

$K_v \Delta i_a = (SJ + B_m)\Delta\omega_r \qquad (4)$

Solve $\Delta\omega_r$ from (4) and substitute it into (3),

$$\Delta V_a = \left[\frac{K_v^2}{SJ + B_m} + r_a + SL_{AA}\right]\Delta i_a$$

$$\frac{\Delta i_a}{\Delta V_a} = \frac{JS + B_m}{JL_{AA}S^2 + (Jr_a + B_m L_{AA})S + B_m r_a + K_v^2}$$

2-8 $$v_a = K_v \omega_r + r_a i_a + r_a \tau_a s i_a \qquad (1)$$

$$T_e = K_v i_a = J s \omega_r \qquad (T_L = 0 \text{ and neglect } B_m)$$

$$\therefore \quad \omega_r = \frac{K_v i_a}{J s} \qquad (2)$$

Substitute (2) into (1),

$$v_a = \frac{K_v^2}{J} \cdot \frac{i_a}{s} + r_a i_a + r_a \tau_a s i_a \qquad (3)$$

$$\therefore \quad \frac{i_a}{v_a} = \frac{1}{L_{AA} s + r_a + \frac{K_v^2}{J} \frac{1}{s}}$$

$$= \frac{s}{L_{AA} s^2 + r_a s + \frac{K_v^2}{J}} \qquad (4)$$

For a RLC circuit, the transfer function is

$$\frac{i_a}{v_a} = \frac{1}{Ls + R + \frac{1}{Cs}} \qquad (5)$$

Compare (4) and (5),

$$L = L_{AA} \qquad R = r_a \qquad C = \frac{J}{K_v^2}$$

If neglect L_{AA} and R, $\quad v_a = \frac{K_v^2}{J} \frac{i_a}{s} = \frac{i_a}{Cs}$.

The unloaded dc shunt motor appears as a capacitance load to changes in the armature voltage.

(c) $v_f = (R_f + SL_{FF})\, i_f$

$$\frac{i_f}{v_f} = \frac{1}{R_f + SL_{FF}} \quad \text{or} \quad \frac{\Delta i_f}{\Delta v_f} = \frac{1}{R_f + SL_{FF}}$$

(d) $\because \; T_e = K_v i_a$ (K_v is a constant because V_f is constant)

$$\therefore \; \Delta T_e = K_v \Delta i_a$$

Using the result of (b) yields

$$\frac{\Delta T_e}{\Delta v_a} = \frac{K_v \Delta i_a}{\Delta v_a} = \frac{K_v (JS + B_m)}{JL_{AA} S^2 + (Jr_a + B_m L_{AA}) S + B_m r_a + K_v^2}$$

2-10. $T_e = \frac{L_{AF} V_a^2}{r_a R_f}\left[1 - \frac{\omega_r L_{AF}}{R_f}\right] = T_L = B_L \omega_r$

where $L_{AF} = 1.8\ H$, $R_f = 240\ \Omega$, $B_L = 0.2287\ \frac{N \cdot m \cdot s}{rad}$

and after considering all starting resistances,

$r_a = 7.8\ \Omega$

$\therefore\ \omega_r = 1 / \left[\frac{L_{AF}}{R_f} + \frac{B_L r_a R_f}{L_{AF} V_a^2}\right] = 85.99\ rad/sec = 821\ RPM$

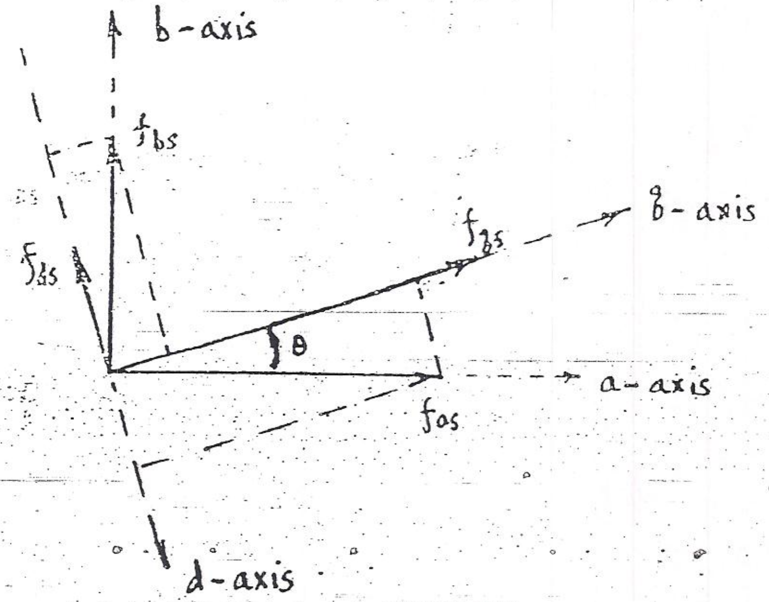
Note. In the steady state $T_L = 19.67\ N \cdot m$

Power output $= T_L \omega_r = 1691\ W = 2.27\ hp < 5\ hp$.

Chapter 3.

3-1 (a) $$(\bar{K}_{2s})^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} = \bar{K}_{2s}$$

(b)



3-2 $$\bar{V}_{abs} = p\bar{\lambda}_{abs}$$

where $\bar{V}_{abs} = [v_{as}\ v_{bs}]^T$, $\bar{\lambda}_{abs} = [\lambda_{as}\ \lambda_{bs}]^T$

$$\bar{V}_{qds} = \bar{K}_{2s}\bar{V}_{abs} = \bar{K}_{2s}\,p\bar{\lambda}_{abs} = \bar{K}_{2s}\,p\left[(\bar{K}_{2s})^{-1}\bar{\lambda}_{qds}\right]$$

$$= \bar{K}_{2s}(\bar{K}_{2s})^{-1}p\bar{\lambda}_{qds} + \bar{K}_{2s}\left[p(\bar{K}_{2s})^{-1}\right]\bar{\lambda}_{qds}$$

$$= p\bar{\lambda}_{qds} + \bar{K}_{2s}\begin{bmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}\frac{d\theta}{dt}\bar{\lambda}_{qds}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\omega\bar{\lambda}_{qds} + p\bar{\lambda}_{qds}$$

That is $v_{qs} = \omega\lambda_{ds} + p\lambda_{qs}$, $v_{ds} = -\omega\lambda_{qs} + p\lambda_{ds}$

3-3 $\quad \bar{i}_{abs} = p\bar{q}_{abs}$

where $\bar{i}_{abs} = [i_{as}\ i_{bs}]^T$, $\quad \bar{q}_{abs} = [q_{as}\ q_{bs}]^T$

$$\bar{i}_{qds} = \bar{K}_{2s}\bar{i}_{abs} = \bar{K}_{2s}\,p\,\bar{q}_{abs} = \bar{K}_{2s}\,p\left[(\bar{K}_{2s})^{-1}\bar{q}_{qds}\right]$$

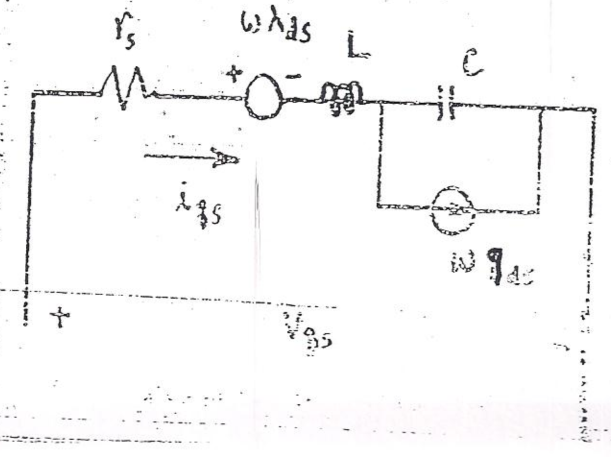
$$= \bar{K}_{2s}(\bar{K}_{2s})^{-1}p\,\bar{q}_{qds} + \bar{K}_{2s}\left[p\,(\bar{K}_{2s})^{-1}\right]\bar{q}_{qds}$$

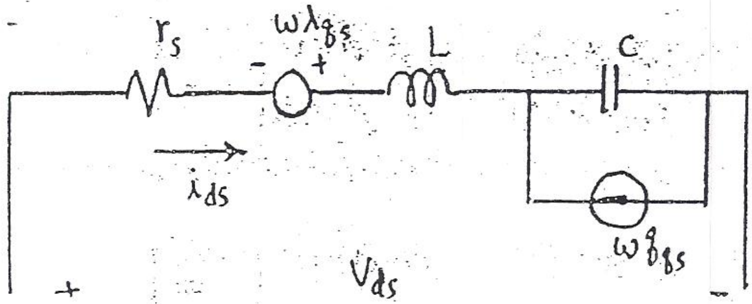
$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\omega\,\bar{q}_{qds} + p\,\bar{q}_{qds}$$

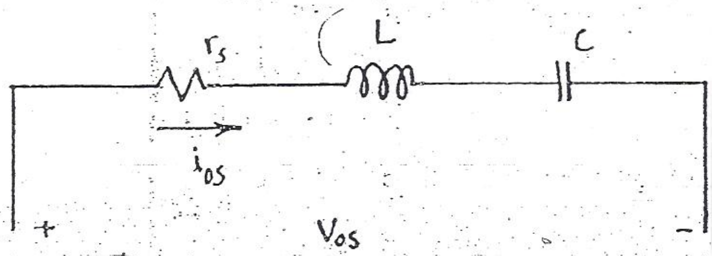
That is $\quad i_{qs} = \omega q_{ds} + p\,q_{qs}$

$$i_{ds} = -\omega q_{qs} + p\,q_{ds}$$

3-4. According to equations (3.4-2), (3.4-3), (3.4-9), (3.4-16) (3.4-22) and (3.4-28), the equivalent circuit can be drawn as follows.







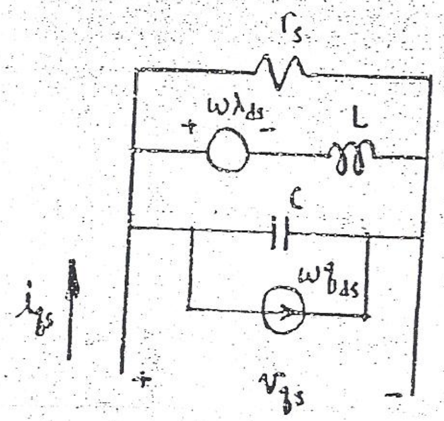
The current sources ωq_{qs} and ωq_{ds} can be replaced by equivalent voltage sources. Then the voltage equations in arbitrary reference frame are easily obtained.

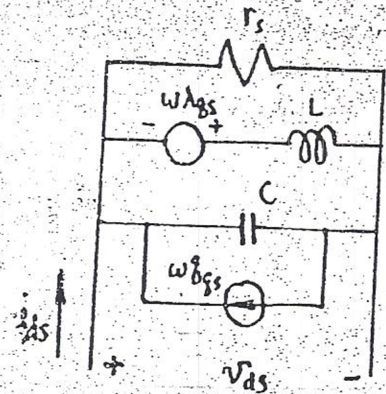
$$V_{qs} = \left(r_s + pL + \frac{1}{Cp}\right) i_{qs} + \omega \lambda_{ds} - \frac{\omega}{Cp} q_{ds}$$

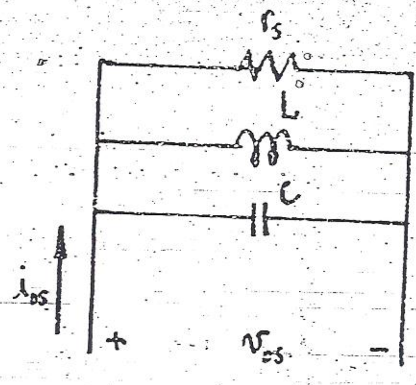
$$V_{ds} = \left(r_s + pL + \frac{1}{Cp}\right) i_{ds} - \omega \lambda_{qs} + \frac{\omega}{Cp} q_{qs}$$

$$V_{0s} = \left(r_s + pL + \frac{1}{Cp}\right) i_{0s}$$

3-5 The equivalent circuit







$$i_{qs} = \left(\frac{1}{r_s} + \frac{1}{pL} + pC\right) v_{qs} + \omega q_{ds} - \frac{\omega}{pL}\lambda_{ds}$$

$$i_{ds} = \left(\frac{1}{r_s} + \frac{1}{pL} + pC\right) v_{ds} - \omega q_{qs} + \frac{\omega}{pL}\lambda_{qs}$$

$$i_{0s} = \left(\frac{1}{r_s} + \frac{1}{pL} + pC\right) v_{0s}$$

3-6 $$f_{qs}^2 + f_{ds}^2 + f_{os}^2 = [f_{qs}\ f_{ds}\ f_{os}] \begin{bmatrix} f_{qs} \\ f_{ds} \\ f_{os} \end{bmatrix}$$

$$= (\bar{f}_{qdos})^T\ \bar{f}_{qdos} = (\bar{K}_s\ \bar{f}_{abcs})^T\ \bar{K}_s\ \bar{f}_{abcs}$$

$$= (\bar{f}_{abcs})^T\ (\bar{K}_s)^T \bar{K}_s\ \bar{f}_{abcs}$$

$$\therefore (\bar{K}_s)^T \bar{K}_s = \frac{4}{9} \begin{bmatrix} \cos\theta & \sin\theta & \frac{1}{2} \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & \frac{1}{2} \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$= \frac{4}{9} \begin{bmatrix} \frac{5}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{5}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{5}{4} \end{bmatrix}$$

3-7

$$\bar{C} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(-\frac{2\pi}{3}) & \cos(\frac{2\pi}{3}) \\ \sin\theta & \sin(-\frac{2\pi}{3}) & \sin(\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\bar{f}^{s}_{qdos} = {}^{c}\bar{K}^{s}\,\bar{f}_{d\beta o} = \begin{bmatrix} \cos[\theta(0)] & -\sin[\theta(0)] & 0 \\ \sin[\theta(0)] & \cos[\theta(0)] & 0 \\ 0 & 0 & 1 \end{bmatrix} \bar{f}_{d\beta o}$$

If $\theta(0) = 0$, $\bar{f}^{s}_{qdos} = \bar{f}_{d\beta o}$.

3-8 (a) If the transformation is $\bar{f}_{qdos} = \bar{Q}_s\,\bar{f}_{abcs}$,

$$\bar{Q}_s = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta-\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

because only d-axis changes the positive direction.

$$(\bar{Q}_s)^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta-\frac{2\pi}{3}) & -\sin(\theta-\frac{2\pi}{3}) & 1 \\ \cos(\theta+\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) & 1 \end{bmatrix}$$

(b) $\overline{V}_{abcs} = P\overline{\lambda}_{abcs}$ $\overline{V}_{qdos} = \overline{Q}_s \overline{V}_{abcs}$

$\overline{\lambda}_{qdos} = \overline{Q}_s \overline{\lambda}_{abcs}$

$$\therefore \overline{V}_{qdos} = \overline{Q}_s \overline{V}_{abcs} = \overline{Q}_s P\left[(\overline{Q}_s)^{-1}\overline{\lambda}_{qdos}\right]$$

$$= \overline{Q}_s\left[P(\overline{Q}_s)^{-1}\right]\overline{\lambda}_{qdos} + P\overline{\lambda}_{qdos}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \omega\overline{\lambda}_{qdos} + P\overline{\lambda}_{qdos}$$

That is
$$v_{qs} = -\omega\lambda_{ds} + P\lambda_{qs}$$
$$v_{ds} = \omega\lambda_{qs} + P\lambda_{ds}$$
$$v_{os} = P\lambda_{os}$$

3-9 See formulas (5.2-8), (5.2-11), (5.2-12) and (5.5-5) of Chapter 5

3-10.

$$
{}^{B-A}\bar{K} = \begin{bmatrix} \cos(\theta_A-\theta_B) & -\sin(\theta_A-\theta_B) & 0 \\ \sin(\theta_A-\theta_B) & \cos(\theta_A-\theta_B) & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} \cos(\theta_B-\theta_A) & \sin(\theta_B-\theta_A) & 0 \\ -\sin(\theta_B-\theta_A) & \cos(\theta_B-\theta_A) & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} \cos(\theta_B-\theta_A) & -\sin(\theta_B-\theta_A) & 0 \\ \sin(\theta_B-\theta_A) & \cos(\theta_B-\theta_A) & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \left({}^{A-B}K\right)^{-1}
$$

3-11 acb sequence :

$$f_{as} = \sqrt{2}\, f_s \cos\theta_{ef}$$

$$f_{bs} = \sqrt{2}\, f_s \cos\left(\theta_{ef} + \frac{2\pi}{3}\right)$$

$$f_{cs} = \sqrt{2}\, f_s \cos\left(\theta_{ef} - \frac{2\pi}{3}\right)$$

where $\theta_{ef} = \int_0^t \omega_e\, dt + \theta_{ef}(0)$

$$
\bar{f}_{qdos} = \bar{K}_s\, \bar{f}_{abcs} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta-\frac{2\pi}{3}\right) & \sin\left(\theta+\frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} \sqrt{2}\, f_s\cos\theta_{ef} \\ \sqrt{2}\, f_s\cos\left(\theta_{ef}+\frac{2\pi}{3}\right) \\ \sqrt{2}\, f_s\cos\left(\theta_{ef}-\frac{2\pi}{3}\right) \end{bmatrix}
$$

$$
= \sqrt{2}\, f_s \begin{bmatrix} \cos(\theta+\theta_{ef}) \\ \sin(\theta+\theta_{ef}) \\ 0 \end{bmatrix}
$$

Where $\theta = \int_0^t w\,dt + \theta(0)$

If $w = w_e$
$$\bar{f}^{e}_{qdos} = \sqrt{2} f_s \begin{bmatrix} \cos\left[2\int_0^t w_e dt + \theta_{ef}(0) + \theta(0)\right] \\ \sin\left[2\int_0^t w_e dt + \theta_{ef}(0) + \theta(0)\right] \\ 0 \end{bmatrix}$$

If $w = -w_e$
$$\bar{f}^{-e}_{qdos} = \sqrt{2} f_s \begin{bmatrix} \cos(\theta_{ef}(0) + \theta(0)) \\ \sin(\theta_{ef}(0) + \theta(0)) \\ 0 \end{bmatrix}$$

3-12 Let $\bar{f}^{e}_{qdos} = \bar{Q}^{e}_{s} \bar{f}_{abcs}$

where $\bar{f}_{abcs}$ is a balanced 3-phase set with a phase sequence of acb as shown in Prob. 3-11.

If
$$\bar{Q}^{e}_{s} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta + \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) \\ \sin\theta & \sin(\theta + \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix},$$

where $\theta = \int_0^t w_e dt + \theta(0)$,

then
$$\bar{f}^{e}_{qdos} = \sqrt{2} f_s \begin{bmatrix} \cos[\theta(0) - \theta_{ef}(0)] \\ \sin[\theta(0) - \theta_{ef}(0)] \\ 0 \end{bmatrix},$$

which are constants.

3-13 Let $\theta(0) = 0$, then $\tilde{F}_{as} = \tilde{F}_{qs} = -j\,\tilde{F}_{ds}$ $(\omega = \omega_e)$

$$\tilde{F}_{bs} = \tilde{F}_{as}\, e^{-j\frac{2\pi}{3}} = \tilde{F}_{qs}\left(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}\right) = -\tfrac{1}{2}\tilde{F}_{qs} - \tfrac{\sqrt{3}}{2}\tilde{F}_{ds}$$

$$\tilde{F}_{cs} = \tilde{F}_{as}\, e^{j\frac{2\pi}{3}} = \tilde{F}_{qs}\left(-\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}\right) = -\tfrac{1}{2}\tilde{F}_{qs} + \tfrac{\sqrt{3}}{2}\tilde{F}_{ds}$$

3-14 $\frac{3}{2}\left(V_{qs}I_{qs} + V_{ds}I_{ds}\right)$ چون معادل حالت دائمی است

$\theta_{ev} = \omega_e t + \theta_e(0)$

$\theta_{ev}(0) - \theta(0)$ $\theta = \omega t + \theta(0)$

$$= 3V_sI_s \cos\left[(\omega_e-\omega)t + \theta_{ev}(0) - \theta(0)\right]\cos\left[(\omega_e-\omega)t + \theta_{ei}(0) - \theta(0)\right]$$

$\theta_{ei}(0) - \theta(0)$

$$+ 3V_sI_s \sin\left[(\omega_e-\omega)t + \theta_{ev}(0) - \theta(0)\right]\sin\left[(\omega_e-\omega)t + \theta_{ei}(0) - \theta(0)\right]$$

$$= 3V_sI_s \cos\left[\theta_{ev}(0) - \theta_{ei}(0)\right] = P_e$$

$+$ $-$

$$\frac{3}{2}\left(-V_{qs}I_{ds} + V_{ds}I_{qs}\right)$$

$+$

$$= -3V_sI_s \cos\left[(\omega_e-\omega)t + \theta_{ev}(0) - \theta(0)\right]\sin\left[(\omega_e-\omega)t + \theta_{ei}(0) - \theta(0)\right]$$

$$+ 3V_sI_s \sin\left[(\omega_e-\omega)t + \theta_{ev}(0) - \theta(0)\right]\cos\left[(\omega_e-\omega)t + \theta_{ei}(0) - \theta(0)\right]$$

$$= 3V_sI_s \sin\left[\theta_{ev}(0) - \theta_{ei}(0)\right] = Q_e$$

3-15 In the stationary reference frame substituting $\omega = 0$ into (3.10-10) and (3.10-11) yields

$$i_{qs}^{s} = \frac{\sqrt{2}\, V_s}{|Z_s|}\left\{ -e^{-t/T}\cos\alpha + \cos(\omega_e t - \alpha) \right\}$$

$$i_{ds}^{s} = \frac{\sqrt{2}\, V_s}{|Z_s|}\left\{ -e^{-t/T}\sin\alpha - \sin(\omega_e t - \alpha) \right\}$$

where $\alpha = tg^{-1}\frac{\omega_e L_s}{r_s}$ $\quad T = L_s/r_s$

Likewise, in the synchronously rotating reference frame,

$\omega = \omega_e$, $\quad i_{qs}^{e} = \frac{\sqrt{2}\, V_s}{|Z_s|}\left\{ -e^{-t/T}\cos(\omega_e t + \alpha) + \cos\alpha \right\}$

$$i_{ds}^{e} = \frac{\sqrt{2}\, V_s}{|Z_s|}\left\{ -e^{-t/T}\sin(\omega_e t + \alpha) + \sin\alpha \right\}$$

$$L'_{sr} = \frac{N_s}{N_r} L_{sr} = L_{ms}$$

$$L'_{mr} = \left(\frac{N_s}{N_r}\right)^2 L_{mr} = L_{ms} \qquad L'_{\ell r} = \left(\frac{N_s}{N_r}\right)^2 L_{\ell r}$$

$$\bar{L}'_{sr} = \frac{N_s}{N_r} \bar{L}_{sr} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} L_{ms}$$

$$\bar{L}'_r = \left(\frac{N_s}{N_r}\right)^2 \bar{L}_r = \begin{bmatrix} L'_{\ell r} + L_{ms} & 0 \\ 0 & L'_{\ell r} + L_{ms} \end{bmatrix}$$

Hence $$\begin{bmatrix} \bar{\lambda}_{abs} \\ \bar{\lambda}'_{abr} \end{bmatrix} = \begin{bmatrix} \bar{L}_s & \bar{L}'_{sr} \\ (\bar{L}'_{sr})^T & \bar{L}'_r \end{bmatrix} \begin{bmatrix} \bar{i}_{abs} \\ \bar{i}'_{abr} \end{bmatrix}$$

$$\begin{bmatrix} \bar{V}_{abs} \\ \bar{V}'_{abr} \end{bmatrix} = \begin{bmatrix} \bar{r}_s + p\bar{L}_s & p\bar{L}'_{sr} \\ p(\bar{L}'_{sr})^T & \bar{r}'_r + p\bar{L}'_r \end{bmatrix} \begin{bmatrix} \bar{i}_{abs} \\ \bar{i}'_{abr} \end{bmatrix}$$

where $\bar{r}'_r = \left[\frac{N_s}{N_r}\right]^2 \bar{r}_r$

Chapter 4

4-1.
$$\bar{V}_{abs} = \bar{r}_s \, \bar{i}_{abs} + p \, \bar{\lambda}_{abs}$$

$$\bar{V}_{abr} = \bar{r}_r \, \bar{i}_{abr} + p \, \bar{\lambda}_{abr}$$

where $\bar{r}_s = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix}$, $\bar{r}_r = \begin{bmatrix} r_r & 0 \\ 0 & r_r \end{bmatrix}$

$$\begin{bmatrix} \bar{\lambda}_{abs} \\ \bar{\lambda}_{abr} \end{bmatrix} = \begin{bmatrix} \bar{L}_s & \bar{L}_{sr} \\ (\bar{L}_{sr})^T & \bar{L}_r \end{bmatrix} \begin{bmatrix} \bar{i}_{abs} \\ \bar{i}_{abr} \end{bmatrix}$$

$$\bar{L}_s = \begin{bmatrix} L_{ls} + L_{ms} & 0 \\ 0 & L_{ls} + L_{ms} \end{bmatrix}$$

$$\bar{L}_r = \begin{bmatrix} L_{lr} + L_{mr} & 0 \\ 0 & L_{lr} + L_{mr} \end{bmatrix}$$

$$\bar{L}_{sr} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} L_{sr}$$

Define $\bar{i}'_{abr} = \frac{N_r}{N_s} \bar{i}_{abs}$, $\bar{V}'_{abr} = \frac{N_s}{N_r} \bar{V}_{abr}$

$$\bar{\lambda}'_{abr} = \frac{N_s}{N_r} \bar{\lambda}_{abr}$$

Because $L_{ms} = \frac{N_s}{N_r} L_{sr}$ and $L_{ms} = \left(\frac{N_s}{N_r}\right)^2 L_{mr}$,

define

4-3

$$\lambda_{as} + \lambda_{bs} + \lambda_{cs} = [1\ 1\ 1] \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix}$$

$$= [1\ 1\ 1]\, \bar{\lambda}_{abcs} = [1\ 1\ 1] \left\{ \bar{L}_s \bar{i}_{abcs} + \bar{L}_{sr} \bar{i}_{abcr} \right\}$$

$$= [1\ 1\ 1]\, \bar{L}_s \bar{i}_{abcs} + [1\ 1\ 1]\, \bar{L}_{sr} \bar{i}_{abcr}$$

Because

$$[1\ 1\ 1]\, \bar{L}_s = [L_{ls}\ L_{ls}\ L_{ls}] = L_{ls} [1\ 1\ 1]$$

and $\quad [1\ 1\ 1]\, \bar{L}_{sr} = [0\ 0\ 0]$,

$$\lambda_{as} + \lambda_{bs} + \lambda_{cs} = L_{ls} \left(i_{as} + i_{bs} + i_{cs} \right).$$

$\because$ Stator windings are wye connected ;

$$i_{as} + i_{bs} + i_{cs} = 0$$

$$\therefore \quad \lambda_{as} + \lambda_{bs} + \lambda_{cs} = 0$$

It is obvious in this proof that it is not necessary for the rotor windings to also be connected in a 3-wire wye arrangement.

4-4

$$W_f = \frac{1}{2}(\bar{i}_{abcs})^T(\bar{L}_s - L_{ls}\bar{I})\,\bar{i}_{abcs} + (\bar{i}_{abcs})^T \bar{L}'_{sr}\,\bar{i}'_{abcr}$$

$$+ \frac{1}{2}(\bar{i}'_{abcr})^T(\bar{L}'_r - L'_{lr}\bar{I})\,\bar{i}'_{abcr}$$

(i) $$\bar{L}_s - L_{ls}\bar{I} = L_{ms}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}$$

$$(\bar{i}_{abcs})^T(\bar{L}_s - L_{ls}\bar{I})\,\bar{i}_{abcs} = L_{ms}(i_{as}^2 + i_{bs}^2 + i_{cs}^2 - i_{as}i_{bs}$$

$$- i_{as}i_{cs} - i_{bs}i_{cs})$$

$$= L_{ms}\left[(i_{as} + i_{bs} + i_{cs})^2 - 3(i_{as}i_{bs} + i_{as}i_{cs} + i_{bs}i_{cs})\right]$$

$$= 3L_{ms}\left[-i_{as}(i_{bs} + i_{cs}) - i_{bs}i_{cs}\right]$$

$$= 3L_{ms}(i_{as}^2 - i_{bs}i_{cs})$$

Let $i_{as} = \sqrt{2}\,I_s \cos\theta_{ei}$ $\qquad \theta_{ei} = \int \omega_e dt + \theta_{ei}(0)$

$i_{bs} = \sqrt{2}\,I_s \cos(\theta_{ei} - \frac{2\pi}{3})$, $\quad i_{cs} = \sqrt{2}\,I_s \cos(\theta_{ei} + \frac{2\pi}{3})$

$$i_{as}^2 - i_{bs}i_{cs} = 2I_s^2\left[\cos^2\theta_{ei} - \cos(\theta_{ei} - \tfrac{2\pi}{3})\cos(\theta_{ei} + \tfrac{2\pi}{3})\right]$$

$$= 2I_s^2\left(\frac{1}{2} - \frac{1}{2}\cos\frac{4\pi}{3}\right) = \frac{3}{2}I_s^2$$

$$\therefore \frac{1}{2}(\bar{i}_{abcs})^T(\bar{L}_s - L_{ls}\bar{I})\,\bar{i}_{abcs} = \frac{9}{4}L_{ms}I_s^2$$

(ii) Following the same derivation as (i) yields

$$\frac{1}{2}\left(\bar{i}'_{abcr}\right)^T\left(\bar{L}_r - L'_{lr}\bar{I}\right)\bar{i}'_{abcr} = \frac{9}{4} L_{ms} I_r'^2$$

(iii) $$\left(\bar{i}_{abcs}\right)^T L'_{sr} = \frac{3 I_s L_{ms}}{\sqrt{2}}\left[\cos(\theta_{ei}-\theta_r) \quad \cos\left(\theta_{ei}-\theta_r-\frac{2\pi}{3}\right) \quad \cos\left(\theta_{ei}-\theta_r+\frac{2\pi}{3}\right)\right]$$

where $\theta_r = \int \omega_r dt + \theta_r(0)$

Let $i'_{ar} = \sqrt{2} I'_r \cos\theta$, $\quad i'_{br} = \sqrt{2} I'_r \cos\left(\theta - \frac{2\pi}{3}\right)$

$i'_{cr} = \sqrt{2} I'_r \cos\left(\theta + \frac{2\pi}{3}\right)$

Here $\theta = \int (\omega_{ei} - \omega_r)\, dt + \theta(0)$

Then $$\left(\bar{i}_{abcs}\right)^T L'_{sr}\,\bar{i}'_{abcr} = \frac{9 I_s I'_r}{2} L_{ms} \cos\left[\theta_{ei}(0) - \theta_r(0) - \theta(0)\right]$$

Combining results (i), (ii) and (iii) yields

$$W_f = \frac{9}{4} L_{ms}\left\{ I_s^2 + I_r'^2 + 2 I_s I'_r \cos\left[\theta_{ei}(0) - \theta_r(0) - \theta(0)\right]\right\}$$

$= \text{Constant}$

4-5 $$W_e = W_f = \frac{1}{2} L_a i_{as}^2 + \frac{1}{2} L_b i_{bs}^2 + L_{ab} i_{as} i_{bs}$$

where $L_a = L_{ls} + L_{ms}$, $L_b = L_{ls} + L_{ms}$

$L_{ab} = L_{ms} \cos\varphi$ φ is the angle between as-axis and bs-axis. Then the torque between the as- and bs-windings

$$T_{ab} = \frac{\partial W_c}{\partial \varphi} = -i_{as} i_{bs} L_{ms} \sin\varphi = -\frac{\sqrt{3}}{2} L_{ms} i_{as} i_{bs}$$

The minus sign means the torque tries to decrease the angle φ.

4-6. $$T_e = \frac{P}{2} (\bar{i}_{abs})^T \frac{\partial}{\partial \theta_r} [\bar{L}'_{sr}] \bar{i}'_{abr}$$

$$\bar{L}'_{sr} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} L_{ms}$$ (see problem 4-1)

$$\frac{\partial}{\partial \theta_r} [\bar{L}'_{sr}] = \begin{bmatrix} -\sin\theta_r & -\cos\theta_r \\ \cos\theta_r & -\sin\theta_r \end{bmatrix} L_{ms}$$

$$\therefore T_e = -\frac{P}{2} L_{ms} \left[(i_{as} i'_{ar} + i_{bs} i'_{br}) \sin\theta_r + (i_{as} i'_{br} - i_{bs} i'_{ar}) \cos\theta_r \right]$$

$$\bar{K}_{2s}\bar{L}'_{sr}\bar{K}_{2r}^{-1} = L_{ms}\begin{bmatrix}\cos(\theta-\theta_r) & \sin(\theta-\theta_r)\\ \sin(\theta-\theta_r) & -\cos(\theta-\theta_r)\end{bmatrix}\bar{K}_{2r}^{-1} = L_{ms}\bar{I}$$

$$\bar{K}_{2r}(\bar{L}'_{sr})^T\bar{K}_{2s}^{-1} = L_{ms}\bar{K}_{2r}\begin{bmatrix}\cos(\theta-\theta_r) & \sin(\theta-\theta_r)\\ \sin(\theta-\theta_r) & -\cos(\theta-\theta_r)\end{bmatrix} = L_{ms}\bar{I}$$

$$\therefore \begin{bmatrix}\bar{\lambda}_{qds}\\ \bar{\lambda}'_{qdr}\end{bmatrix} = \begin{bmatrix}\bar{L}_s & L_{ms}\bar{I}\\ L_{ms}\bar{I} & \bar{L}'_r\end{bmatrix}\begin{bmatrix}\bar{i}_{qds}\\ \bar{i}'_{qdr}\end{bmatrix}$$

$$\begin{bmatrix}v_{qs}\\ v_{ds}\\ v'_{qr}\\ v'_{dr}\end{bmatrix} = \begin{bmatrix} r_s+\frac{p}{\omega_b}X_{ss} & \frac{\omega}{\omega_b}X_{ss} & \frac{p}{\omega_b}X_M & \frac{\omega}{\omega_b}X_M\\ -\frac{\omega}{\omega_b}X_{ss} & r_s+\frac{p}{\omega_b}X_{ss} & -\frac{\omega}{\omega_b}X_M & \frac{p}{\omega_b}X_M\\ \frac{p}{\omega_b}X_M & \left(\frac{\omega-\omega_r}{\omega_b}\right)X_M & r'_r+\frac{p}{\omega_b}X'_{rr} & \left(\frac{\omega-\omega_r}{\omega_b}\right)X'_{rr}\\ -\left(\frac{\omega-\omega_r}{\omega_b}\right)X_M & \frac{p}{\omega_b}X_M & -\left(\frac{\omega-\omega_r}{\omega_b}\right)X'_{rr} & r'_r+\frac{p}{\omega_b}X'_{rr}\end{bmatrix}\begin{bmatrix}i_{qs}\\ i_{ds}\\ i'_{qr}\\ i'_{dr}\end{bmatrix}$$

where $X_{ss} = \omega_b(L_{ls}+L_{ms})$, $X'_{rr} = \omega_b(L'_{lr}+L_{ms})$

$X_M = \omega_b L_{ms}$

4-8 (a) See Prob. 4-7.

4-7 $\quad \bar{v}_{abs} = \bar{r}_s \bar{i}_{abs} + p\bar{\lambda}_{abs}$

$$\bar{v}'_{abr} = \bar{r}'_r \bar{i}'_{abr} + p\lambda_{abr}$$

Let $\quad \bar{K}_{2s} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}, \quad \bar{K}_{2r} = \begin{bmatrix} \cos(\theta-\theta_r) & \sin(\theta-\theta_r) \\ \sin(\theta-\theta_r) & -\cos(\theta-\theta_r) \end{bmatrix}.$

By problem 3-2, in the arbitrary reference fram

$$\bar{v}_{qds} = \bar{r}_s \bar{i}_{qds} + \omega \bar{\lambda}_{dqs} + p\bar{\lambda}_{qds}$$

$$\bar{v}'_{qdr} = \bar{r}'_r \bar{i}'_{qdr} + (\omega-\omega_r)\bar{\lambda}'_{dqr} + p\bar{\lambda}'_{qdr}$$

where $\bar{\lambda}_{dqs} = \begin{bmatrix} \lambda_{ds} \\ -\lambda_{qs} \end{bmatrix}, \quad \bar{\lambda}'_{dqr} = \begin{bmatrix} \lambda'_{dr} \\ -\lambda'_{qr} \end{bmatrix}.$

The voltage equations can also be expressed in terms of the currents as follows.

From problem 4-1, $\begin{bmatrix} \bar{\lambda}_{abs} \\ \bar{\lambda}'_{abr} \end{bmatrix} = \begin{bmatrix} \bar{L}_s & \bar{L}'_{sr} \\ (\bar{L}'_{sr})^T & \bar{L}'_r \end{bmatrix} \begin{bmatrix} \bar{i}_{abs} \\ \bar{i}'_{abr} \end{bmatrix},$

after the transformation

$$\begin{bmatrix} \bar{\lambda}_{qds} \\ \bar{\lambda}'_{qdr} \end{bmatrix} = \begin{bmatrix} \bar{K}_{2s}\bar{L}_s\bar{K}_{2s}^{-1} & \bar{K}_{2s}\bar{L}'_{sr}\bar{K}_{2r}^{-1} \\ \bar{K}_{2r}(\bar{L}'_{sr})^T\bar{K}_{2s}^{-1} & \bar{K}_{2r}\bar{L}'_r\bar{K}_{2r}^{-1} \end{bmatrix} \begin{bmatrix} \bar{i}_{qds} \\ \bar{i}'_{qdr} \end{bmatrix}$$

$$\bar{K}_{2s}\bar{L}_s\bar{K}_{2s}^{-1} = \bar{L}_s, \quad \bar{K}_{2r}\bar{L}'_r\bar{K}_{2r}^{-1} = \bar{L}'_r$$

$$Z_i = 0.262 + j1.206 + \frac{(12.467 + j1.206)\,j54.02}{12.467 + j55.226}$$

$$= 11.612 + j4.948$$

$$Y_i = \frac{1}{Z_i} = 0.07289 - j0.03106$$

$$Y_{tot} = \frac{1}{Z_{tot}} = 0.07289 - j\frac{\sqrt{1-0.95^2}}{0.95}\cdot 0.07289$$

$$= 0.07289 - j0.023957$$

$\therefore \quad \omega C = 0.0071 \qquad C = 18.8 \text{ pf}$

4-28 For Fig 4.10-5, $V_B = 127.02$ V.

$I_B = 5.873$ A $\qquad T_B = 11.87$ N-m

For Fig 4.10-6, $V_B = 1327.9$

$I_B = 421.34$ A $\qquad T_B = 8904.51$

4-25 $$G = \left\{ \frac{\left(\frac{\omega_e}{\omega_b}\right)^{-2} r_s^2 + X_{ss}^2}{\left(\frac{\omega_e}{\omega_b}\right)^2 (X_M^2 - X_{ss}X'_{rr})^2 + r_s^2 X'^2_{rr}} \right\}^{1/2} = \left\{ \frac{\left(\frac{\omega_e}{\omega_b}\right)^{-2} 0.00757 + 179.08}{\left(\frac{\omega_e}{\omega_b}\right)^2 63.864 + 1.355} \right\}^{1/2}$$

(a) 120 Hz $\quad G = 0.835 \qquad$ Speed $= 2914.6$ rpm

(b) 60 Hz $\quad G = 1.657 \qquad$ Speed $= 1119.9$ rpm

(c) 30 Hz $\quad G = 3.216 \qquad$ Speed $= 240.2$ rpm

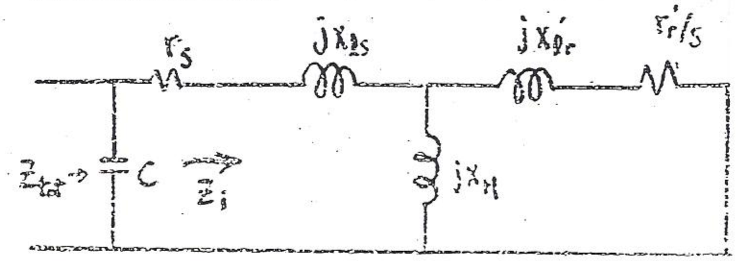
(d) 6 Hz $\quad G = 9.496 \qquad S_m = 2.1652$

Speed $= -209.7$ rpm

The negative speed means that the rotative direction of the rotor is opposite to that of the MMFs. See Problem 4-26, which shows this situation.

4-26 $\quad S = 2 \qquad T_e = 34.105$ N-m

4-27



(b) Because neglect friction and windage losses,

$$\omega_r = \omega_e, \quad s = 0 \quad \tilde{I}'_{ar} = 0$$

$$Z = r_s + jX_{ss} \approx jX_{ss} \qquad (\text{if } r_s = 0 \quad Z = jX_{ss})$$

$$|\tilde{I}_{as}| = \frac{|\tilde{V}_{as}|}{|Z|} = 5.40 \text{ A}$$

4-23 $\because$ $|\tilde{I}_{as}| \propto |\tilde{V}_{as}|$ $\qquad T_e \propto |\tilde{V}_{as}|^2$

(a) By prob. 4-22, $|\tilde{I}_{as}| = 11.70$ A

$$T_e = 0.318 \text{ NT-M}$$

(b) $|\tilde{I}_{as}| = 0.540$ A

4-24 $$G = \pm \left| \frac{r_s^2 + X_{ss}^2}{(X_M^2 - X_{ss}X'_{rr})^2 + r_s^2 X'^2_{rr}} \right|^{1/2} = \pm\, 0.4167$$

In motor action $G = 0.4167$

$$T_{e\,max} = \frac{3 \cdot 2 \cdot \frac{X_M^2}{\omega_b} G \cdot 2300^2/3}{[r_s + G(X_M^2 - X_{ss}X'_{rr})]^2 + (X_{ss} + G r_s X'_{rr})^2} = 5064.9 \text{ NT-M}$$

In generator action $G = -0.4167$

$$T_{e\,max} = -\text{[illegible]} \text{ NT-M}$$

4-18

$$Z_B = \frac{3V^2_{Base}}{P_B} = \frac{220^2}{P_B} = 6.4879$$

$$r_s = 0.2939\,\Omega, \quad X_M = 13.248\,\Omega, \quad X_{ls} = 0.5028\,\Omega$$

$$r_r' = 0.1440\,\Omega \qquad X_{lr}' = 0.2089\,\Omega$$

$$J = 2\cdot\left(\frac{P}{2}\right)^2 P_B \cdot H / \omega_b^2 = 0.472 \ \ Kg\cdot m^2$$

4-21

In per unit $\quad T_e = X_m \,Re\left[\, j\, \tilde{I}_{as}^{*}\, \tilde{I}_{ar}' \,\right]$

4-22 $\quad X_{ss} = 377 \times \left(0.0015 + \frac{3}{2}\cdot 0.035\right) = 20.36\ \Omega$

$$X_{rr} = 377 \times \left(0.0007 + \frac{3}{2}\cdot 0.035\right) = 20.06\ \Omega$$

$$X_M = 377 \times \frac{3}{2}\cdot 0.035 = 19.79$$

(a) $\quad Z = \dfrac{-16.52 + j\,9.07}{0.15 + j\,20.06} = \dfrac{18.85\ \angle 151.23^\circ}{20.06\ \angle 89.57^\circ} = 0.9396\ \angle 61.66^\circ$

$$|\tilde{I}_{as}| = \frac{|\tilde{V}_{as}|}{|Z|} = 117.04\ A$$

$$T_e = 31.83 \quad NT\text{-}M$$

Per unitized machine Parameters

hp	$V_{B(abc)}$	Z_B	S	r_s	X_{ls}	X_M	X'_{lr}	r'_r	H
3	127.02	21.63	0.05	0.0201	0.0349	1.208	0.0349	0.0377	0.7065
50	265.58	5.673	0.0528	0.0153	0.0532	2.306	0.0153	0.0402	0.7916
500	1327.9	14.18	0.015	0.0185	0.0850	3.809	0.0850	0.0132	0.5268
2250	1327.9	3.152	0.0078	0.0092	0.0717	4.138	0.0717	0.00698	0.6760

4-15

$$H = \left(\frac{1}{2}\right)\left(\frac{2}{p}\right)^2 \frac{J\omega_b^2}{P_B} = \frac{1}{2} J \left(\frac{2}{p}\omega_b\right)^2 / P_B$$

$$= \frac{1}{2} J \omega_{bm}^2 / P_B = E / P_B$$

where $E = \frac{1}{2} J \omega_{bm}^2$. ω_{bm} is the rotor mechanical speed at synchronous speed. Hence E is the stored energy of the rotor at this speed.

4-16 $\because$ $Z_B = \dfrac{3 V_{B(abc)}^2}{P_B}$

Assum z_1 is the per unit impedance of VA base P_{B1}

z_2 is the per unit impedance of VA base P_{B2}

$$\therefore z_1 \cdot \frac{3V_{B1}^2}{P_{B1}} = z_2 \cdot \frac{3V_{B2}^2}{P_{B2}}$$

$$z_2 = \left(\frac{V_{B1}}{V_{B2}}\right)^2 \frac{P_{B2}}{P_{B1}} z_1$$

If $V_{B1} = V_{B2}$, $z_2 = \dfrac{P_{B2}}{P_{B1}} z_1$.

Where $\beta = \theta - \theta_r \qquad \theta = \int_0^t \omega dt + \theta(0)$

Change the variables

$$\bar{f}_{qds} = \bar{K}_s \bar{f}_{abs} \quad , \quad \bar{f}'_{qdr_i} = \bar{K}_r \bar{f}'_{abr_i} \quad (i=1,2)$$

Then the voltage equations become

$$\bar{v}_{qds} = \bar{r}_s \bar{i}_{qds} + \frac{\omega}{\omega_b} \bar{\psi}_{dqs} + \frac{p}{\omega_b} \bar{\psi}_{qds} \qquad \bar{\psi}_{dqs} = \begin{bmatrix} \psi_{ds} & 0 \\ 0 & -\psi_{qs} \end{bmatrix}$$

$$\bar{v}'_{qdr_1} = \bar{r}'_{r_1} \bar{i}'_{qdr_1} + \frac{\omega - \omega_r}{\omega_b} \bar{\psi}'_{dqr_1} + \frac{p}{\omega_b} \bar{\psi}'_{qdr_1} \qquad \bar{\psi}'_{dqr_1} = \begin{bmatrix} \psi'_{dr_1} & 0 \\ 0 & -\psi'_{qr_1} \end{bmatrix}$$

$$\bar{v}'_{qdr_2} = \bar{r}'_{r_2} \bar{i}'_{qdr_2} + \frac{\omega - \omega_r}{\omega_b} \bar{\psi}'_{dqr_2} + \frac{p}{\omega_b} \bar{\psi}'_{qdr_2} \qquad \bar{\psi}'_{dqr_2} = \begin{bmatrix} \psi'_{dr_2} & 0 \\ 0 & -\psi'_{qr_2} \end{bmatrix}$$

and

$$\begin{bmatrix} \bar{\psi}_{qds} \\ \bar{\psi}'_{qdr_1} \\ \bar{\psi}'_{qdr_2} \end{bmatrix} = \begin{bmatrix} \bar{K}_s \bar{X}_s (\bar{K}_s)^{-1} & \bar{K}_s \bar{X}'_{sr_1} (\bar{K}_r)^{-1} & \bar{K}_s \bar{X}'_{sr_2} (\bar{K}_r)^{-1} \\ \bar{K}_r (\bar{X}'_{sr_1})^T (\bar{K}_s)^{-1} & \bar{K}_r \bar{X}'_{r_1} (\bar{K}_r)^{-1} & \bar{K}_r \bar{X}'_{r_1r_2} (\bar{K}_r)^{-1} \\ \bar{K}_r (\bar{X}'_{sr_2})^T (\bar{K}_s)^{-1} & \bar{K}_r (\bar{X}'_{r_1r_2})^T (\bar{K}_r)^{-1} & \bar{K}_r \bar{X}'_{r_2} (\bar{K}_r)^{-1} \end{bmatrix} \begin{bmatrix} \bar{i}_{qds} \\ \bar{i}'_{qdr_1} \\ \bar{i}'_{qdr_2} \end{bmatrix}$$

$$= \begin{bmatrix} \bar{X}_s & \bar{X}_{ms} & \bar{X}_{ms} \\ \bar{X}_{ms} & \bar{X}'_{r_1} & \bar{X}'_{r_1r_2} \\ \bar{X}_{ms} & \bar{X}'_{r_1r_2} & \bar{X}'_{r_2} \end{bmatrix} \begin{bmatrix} \bar{i}_{qds} \\ \bar{i}'_{qdr_1} \\ \bar{i}'_{qdr_2} \end{bmatrix}$$

where $\bar{X}_{ms} = \begin{bmatrix} X_{ms} & 0 \\ 0 & X_{ms} \end{bmatrix} = \bar{X}'_{r_1r_2}$

4-12

$$T_e = \left(\frac{P}{2}\right)\left(\bar{i}_{abs}\right)^T\left(\frac{\partial}{\partial\theta_r}\bar{L}'_{sr}\right)\bar{i}'_{abr}$$

$$= \left(\frac{P}{2}\right)\left[\left(\bar{K}_{2s}\right)^{-1}\bar{i}_{qds}\right]^T\left(\frac{\partial}{\partial\theta_r}\bar{L}'_{sr}\right)\left(\bar{K}_{2r}\right)^{-1}\bar{i}'_{qdr}$$

$$= \left(\frac{P}{2}\right)L_{ms}\,\bar{i}_{qds}^T\left(\bar{K}_{2s}\right)^{-T}\begin{bmatrix}-\sin\theta_r & -\cos\theta_r\\ \cos\theta_r & -\sin\theta_r\end{bmatrix}\left(\bar{K}_{2r}\right)^{-1}\bar{i}'_{qdr}$$

$$= \left(\frac{P}{2}\right)L_{ms}\,\bar{i}_{qds}^T\begin{bmatrix}\sin(\theta-\theta_r) & -\cos(\theta-\theta_r)\\ -\cos(\theta-\theta_r) & -\sin(\theta-\theta_r)\end{bmatrix}\left(\bar{K}_{2r}\right)^{-1}\bar{i}'_{qdr}$$

$$= \left(\frac{P}{2}\right)L_{ms}\,\bar{i}_{qds}^T\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\bar{i}'_{qdr}$$

$$= \left(\frac{P}{2}\right)L_{ms}\left(i_{qs}i'_{dr} - i_{ds}i'_{qr}\right) \qquad (a)$$

$$= \left(\frac{P}{2}\right)\left\{\left[L_{ms}i_{qs} + (L_{ms}+L'_{lr})i'_{qr}\right]i'_{dr} - \left[L_{ms}i_{ds} + (L_{ms}+L'_{lr})i'_{dr}\right]i'_{qr}\right\}$$

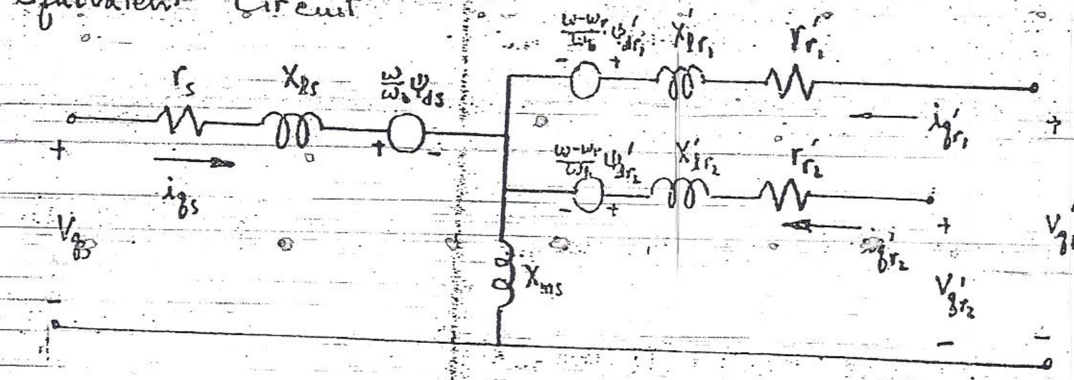
$$= \left(\frac{P}{2}\right)\left(\lambda'_{qr}i'_{dr} - \lambda'_{dr}i'_{qr}\right)$$

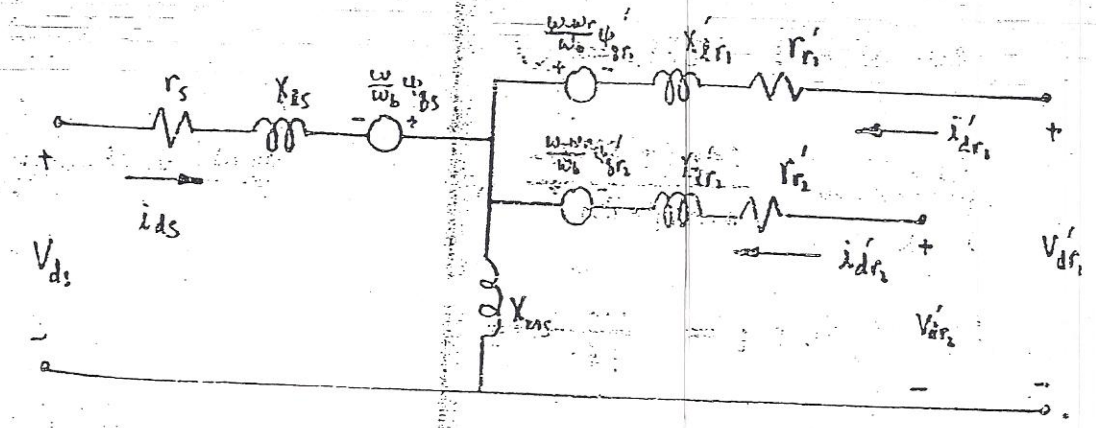
$$= \left(\frac{P}{2}\right)\frac{1}{\omega_b}\left(\psi'_{qr}i'_{dr} - \psi'_{dr}i'_{qr}\right) \qquad (b)$$

The voltage can be written as

$$
\begin{bmatrix} v_{qs} \\ v_{ds} \\ v'_{qr_1} \\ v'_{dr_1} \\ v'_{qr_2} \\ v'_{dr_2} \end{bmatrix}
=
\begin{bmatrix}
r_s+\frac{p}{\omega_b}X_{ss} & \frac{\omega}{\omega_b}X_{ss} & \frac{p}{\omega_b}X_{ms} & \frac{\omega}{\omega_b}X_{ms} & \frac{p}{\omega_b}X_{ms} & \frac{\omega}{\omega_b}X_{ms} \\
-\frac{\omega}{\omega_b}X_{ss} & r_s+\frac{p}{\omega_b}X_{ss} & -\frac{\omega}{\omega_b}X_{ms} & \frac{p}{\omega_b}X_{ms} & -\frac{\omega}{\omega_b}X_{ms} & \frac{p}{\omega_b}X_{ms} \\
\frac{p}{\omega_b}X_{ms} & \frac{\omega-\omega_r}{\omega_b}X_{ms} & r'_{r_1}+\frac{p}{\omega_b}X'_{rr_1} & \frac{\omega-\omega_r}{\omega_b}X'_{rr_1} & \frac{p}{\omega_b}X_{ms} & \frac{\omega-\omega_r}{\omega_b}X_{ms} \\
-\left(\frac{\omega-\omega_r}{\omega_b}\right)X_{ms} & \frac{p}{\omega_b}X_{ms} & -\left(\frac{\omega-\omega_r}{\omega_b}\right)X'_{rr_1} & r'_{r_1}+\frac{p}{\omega_b}X'_{rr_1} & -\left(\frac{\omega-\omega_r}{\omega_b}\right)X_{ms} & \frac{p}{\omega_b}X_{ms} \\
\frac{p}{\omega_b}X_{ms} & \frac{\omega-\omega_r}{\omega_b}X_{ms} & \frac{p}{\omega_b}X_{ms} & \frac{\omega-\omega_r}{\omega_b}X_{ms} & r'_{r_2}+\frac{p}{\omega_b}X'_{rr_2} & \frac{\omega-\omega_r}{\omega_b}X'_{rr_2} \\
-\left(\frac{\omega-\omega_r}{\omega_b}\right)X_{ms} & \frac{p}{\omega_b}X_{ms} & -\left(\frac{\omega-\omega_r}{\omega_b}\right)X_{ms} & \frac{p}{\omega_b}X_{ms} & -\left(\frac{\omega-\omega_r}{\omega_b}\right)X'_{rr_2} & r'_{r_2}+\frac{p}{\omega_b}X'_{rr_2}
\end{bmatrix}
\begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr_1} \\ i'_{dr_1} \\ i'_{qr_2} \\ i'_{dr_2} \end{bmatrix}
$$

Equivalent Circuit





4-9 $\quad \overline{v}_{abs} = \overline{r}_s \overline{i}_{abs} + p \overline{\lambda}_{abs} = \overline{r}_s \overline{i}_{abs} + \frac{p}{\omega_b} \overline{\psi}_{abs}$

$$\overline{v}_{abr_1} = \overline{r}'_{r_1} \overline{i}'_{abr_1} + p \overline{\lambda}'_{abr_1} = \overline{r}'_{r_1} \overline{i}'_{abr_1} + \frac{p}{\omega_b} \overline{\psi}'_{abr_1}$$

$$\overline{v}_{abr_2} = \overline{r}'_{r_2} \overline{i}'_{abr_2} + p \overline{\lambda}'_{abr_2} = \overline{r}'_{r_2} \overline{i}'_{abr_2} + \frac{p}{\omega_b} \overline{\psi}'_{abr_2}$$

$$\begin{bmatrix} \overline{\lambda}_{abs} \\ \overline{\lambda}'_{abr_1} \\ \overline{\lambda}'_{abr_2} \end{bmatrix} = \begin{bmatrix} \overline{L}_s & \overline{L}'_{sr_1} & \overline{L}'_{sr_2} \\ (\overline{L}'_{sr_1})^T & \overline{L}'_{r_1} & \overline{L}'_{r_1r_2} \\ (\overline{L}'_{sr_2})^T & (\overline{L}'_{r_1r_2})^T & \overline{L}'_{r_2} \end{bmatrix} \begin{bmatrix} \overline{i}_{abs} \\ \overline{i}'_{abr_1} \\ \overline{i}'_{abr_2} \end{bmatrix}$$

or
$$\begin{bmatrix} \overline{\psi}_{abs} \\ \overline{\psi}'_{abr_1} \\ \overline{\psi}'_{abr_2} \end{bmatrix} = \begin{bmatrix} \overline{X}_s & \overline{X}'_{sr_1} & \overline{X}'_{sr_2} \\ (\overline{X}'_{sr_1})^T & \overline{X}'_{r_1} & \overline{X}'_{r_1r_2} \\ (\overline{X}'_{sr_2})^T & (\overline{X}'_{r_1r_2})^T & \overline{X}'_{r_2} \end{bmatrix} \begin{bmatrix} \overline{i}_{abs} \\ \overline{i}'_{abr_1} \\ \overline{i}'_{abr_2} \end{bmatrix}$$

where $\overline{X}_s = \begin{bmatrix} X_{ls} + X_{ms} & 0 \\ 0 & X_{ls} + X_{ms} \end{bmatrix}$, $\quad \overline{X}'_{sr_1} = X_{ms} \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}$,

$$\overline{X}'_{sr_2} = X_{ms} \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}, \quad \overline{X}'_{r_1r_2} = X_{ms} \overline{I},$$

$$\overline{X}'_{r_1} = (X'_{lr_1} + X_{ms}) \overline{I}, \quad \overline{X}'_{r_2} = (X'_{lr_2} + X_{ms}) \overline{I}.$$

Let $\overline{K}_r = \begin{bmatrix} \cos\beta & \sin\beta \\ \sin\beta & -\cos\beta \end{bmatrix} \quad (\overline{K}_r)^T = (\overline{K}_r)^{-1} = K_r$.

$$\overline{K}_s = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \quad (\overline{K}_s)^T = (\overline{K}_s)^{-1} = K_s$$

$$L_{fdkd} = \left(\frac{N_{fd}}{N_{kd}}\right) L_{mkd} = \left(\frac{N_{kd}}{N_{fd}}\right) L_{mfd}$$

Let $i_j' = \left(\frac{N_j}{N_s}\right) i_j$, $v_j' = \left(\frac{N_s}{N_j}\right) v_j$

$$r_j' = \left(\frac{N_s}{N_j}\right)^2 r_j \qquad L_{\ell j}' = \left(\frac{N_s}{N_j}\right)^2 L_{\ell j}, \qquad \lambda_j' = \frac{N_s}{N_j} \lambda_j$$

Then $\begin{bmatrix} \bar{\lambda}_{abs} \\ \bar{\lambda}_{qdr}' \end{bmatrix} = \begin{bmatrix} \bar{L}_s & \bar{L}_{sr}' \\ (\bar{L}_{sr}')^T & \bar{L}_r' \end{bmatrix} \begin{bmatrix} -\bar{i}_{abs} \\ \bar{i}_{qdr}' \end{bmatrix}$

$$\bar{L}_{sr}' = \begin{bmatrix} L_{mq}\cos\theta_r & L_{md}\sin\theta_r & L_{md}\sin\theta_r \\ L_{mq}\sin\theta_r & -L_{md}\cos\theta_r & -L_{md}\cos\theta_r \end{bmatrix}$$

$$\bar{L}_r' = \begin{bmatrix} L_{\ell kq}' + L_{mq} & 0 & 0 \\ 0 & L_{\ell fd}' + L_{md} & L_{md} \\ 0 & L_{md} & L_{\ell kd}' + L_{md} \end{bmatrix}$$

$$\begin{bmatrix} \bar{V}_{abs} \\ \bar{v}_{qdr}' \end{bmatrix} = \begin{bmatrix} r_s + p\bar{L}_s & p\bar{L}_{sr}' \\ p(L_{sr}')^T & \bar{r}_r' + p\bar{L}_r' \end{bmatrix} \begin{bmatrix} -\bar{i}_{abs} \\ \bar{i}_{qdr}' \end{bmatrix}$$

Chapter 5

5-1 $$\bar{v}_{abs} = \bar{r}_s \bar{i}_{abs} + p\bar{\lambda}_{abs} \qquad \bar{f}_{abs} = [f_{as} \ f_{bs}]^T$$

$$\bar{v}_{qdr} = \bar{r}_r \bar{i}_{qdr} + p\bar{\lambda}_{qdr} \qquad \bar{f}_{qdr} = [f_{kq} \ f_{fd} \ f_{kd}]^T$$

$$\bar{r}_s = \mathrm{diag}\,[r_s \ r_s], \qquad \bar{r}_r = \mathrm{diag}\,[r_{kq} \ r_{fd} \ r_{kd}]$$

$$\begin{bmatrix} \bar{\lambda}_{abs} \\ \bar{\lambda}_{qdr} \end{bmatrix} = \begin{bmatrix} \bar{L}_s & \bar{L}_{sr} \\ (\bar{L}_{sr})^T & \bar{L}_r \end{bmatrix} \begin{bmatrix} \bar{i}_{abs} \\ \bar{i}_{qdr} \end{bmatrix}$$

$$\bar{L}_s = \begin{bmatrix} L_{ls} + L_A - L_B \cos 2\theta_r & -L_B \sin 2\theta_r \\ -L_B \sin 2\theta_r & L_{ls} + L_A + L_B \cos 2\theta_r \end{bmatrix}$$

$$\bar{L}_{sr} = \begin{bmatrix} L_{skq} \cos\theta_r & L_{sfd} \sin\theta_r & L_{skd} \sin\theta_r \\ L_{skq} \sin\theta_r & -L_{sfd} \cos\theta_r & -L_{skd} \cos\theta_r \end{bmatrix}$$

$$\bar{L}_r = \begin{bmatrix} L_{lkq} + L_{mkq} & 0 & 0 \\ 0 & L_{lfd} + L_{mfd} & L_{fdkd} \\ 0 & L_{fdkd} & L_{lkd} + L_{mkd} \end{bmatrix}$$

where $L_{skq} = \frac{N_{kq}}{N_s} L_{mq}$, $L_{sfd} = \frac{N_{fd}}{N_s} L_{md}$, $L_{skd} = \frac{N_{kd}}{N_s} L_{md}$

$$L_{mkq} = \left(\frac{N_{kq}}{N_s}\right)^2 L_{mq}, \quad L_{mfd} = \left(\frac{N_{fd}}{N_s}\right)^2 L_{md}, \quad L_{mkd} = \left(\frac{N_{kd}}{N_s}\right)^2 L_{md}$$

5-3 For a 3-phase round rotor synchronous machine,

$L_B = 0 \quad L_{mq} = L_{md}$. Then (5.2-29) is still valid.

5-4.

$$\begin{bmatrix} \bar{V}_{abcs} \\ \bar{v}'_{qdr} \end{bmatrix} = \begin{bmatrix} \bar{r}_s + p\bar{L}_s & p\bar{L}'_{sr} \\ p(\bar{L}'_{sr})^T & \bar{r}'_r + p\bar{L}'_r \end{bmatrix} \begin{bmatrix} \bar{i}_{abcs} \\ \bar{i}'_{qdr} \end{bmatrix}$$

$\bar{L}_s$ = the same as (5.2-8). $\quad \bar{f}'_{qdr} = [\, f'_{kq} \;\; f'_{kd} \,]^T$.

$$\bar{L}'_{sr} = \begin{bmatrix} L_{mq}\cos\theta_r & L_{md}\sin\theta_r \\ L_{mq}\cos(\theta_r - \frac{2\pi}{3}) & L_{md}\sin(\theta_r - \frac{2\pi}{3}) \\ L_{mq}\cos(\theta_r + \frac{2\pi}{3}) & L_{md}\sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix}$$

$$\bar{L}'_r = \begin{bmatrix} L'_{lkq} + L_{mq} & 0 \\ 0 & L'_{lkd} + L_{md} \end{bmatrix}$$

5-6

$$W_f = \frac{1}{2}(\bar{i}_{abs})^T(\bar{L}_s - L_{ls}\bar{I})\,\bar{i}_{abs} - (\bar{i}_{abs})^T \bar{L}'_{sr}\,\bar{i}'_{qdr}$$

$$+ \frac{1}{2}(\bar{i}'_{qdr})^T(\bar{L}'_r - \bar{L}'_{lr})\,\bar{i}_{qdr}$$

$$T_e = -\frac{\partial W_c}{\partial \theta_{rm}} = -\frac{\partial W_f}{\partial \theta_{rm}} = -\frac{P}{2}\frac{\partial W_f}{\partial \theta_r}$$ (Te is positive for generator action)

$$T_e = \frac{P}{2}\left\{ -\frac{1}{2}(\bar{i}_{abs})^T \frac{\partial}{\partial \theta_r}\left[\bar{L}_s - L_{ls}\bar{I}\right]\bar{i}_{abs} + (\bar{i}_{abs})^T \frac{\partial}{\partial \theta_r}(\bar{L}'_{sr})\,\bar{i}'_{qdr} \right\}$$

$$T_e = \frac{P}{2}\left\{ (\bar{i}_{abs})^T \begin{bmatrix} -L_B \sin 2\theta_r & L_B \cos 2\theta_r \\ L_B \cos 2\theta_r & L_B \sin 2\theta_r \end{bmatrix} \bar{i}_{abs} \right.$$

$$\left. + (\bar{i}_{abs})^T \begin{bmatrix} -L_{mq} \sin\theta_r & L_{md}\cos\theta_r & L_{md}\cos\theta_r \\ L_{mq}\cos\theta_r & L_{md}\sin\theta_r & L_{md}\sin\theta_r \end{bmatrix} \bar{i}'_{qdr} \right\}$$

$$= \frac{P}{2}\left\{ \frac{L_{md}-L_{mq}}{2}\left[(i_{bs}^2 - i_{as}^2)\sin 2\theta_r + 2 i_{as} i_{bs} \cos 2\theta_r \right] \right.$$

$$+ L_{mq} i'_{kq} (i_{bs}\cos\theta_r - i_{as}\sin\theta_r)$$

$$\left. + L_{md} (i'_{fd} + i'_{kd}) (i_{as}\cos\theta_r + i_{bs}\sin\theta_r) \right\}$$

5-7 $$\bar{K}_s \bar{L}_s (\bar{K}_s)^{-1} = \begin{bmatrix} L_{ls} + \frac{3}{2}L_A - \frac{3}{2}L_B \cos 2(\theta-\theta_r) & -\frac{3}{2}L_B \sin 2(\theta-\theta_r) & 0 \\ -\frac{3}{2}L_B \sin 2(\theta-\theta_r) & L_{ls} + \frac{3}{2}L_A + \frac{3}{2}L_B\cos 2(\theta-\theta_r) & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

Only if $\omega = \omega_r$, choose $\theta(0) = \theta_r(0)$; then

$$\bar{K}_s \bar{L}_s (\bar{K}_s)^{-1} = \begin{bmatrix} L_{ls} + \frac{3}{2}(L_A - L_B) & 0 & 0 \\ 0 & L_{ls} + \frac{3}{2}(L_A + L_B) & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

5-8

$$(\bar{L}'_{sr})^T (\bar{K}_s)^{-1} = \frac{3}{2}\begin{bmatrix} L_{mq}\cos(\theta-\theta_r) & L_{mq}\sin(\theta-\theta_r) & 0 \\ L_{mq}\cos(\theta-\theta_r) & L_{mq}\sin(\theta-\theta_r) & 0 \\ -L_{md}\sin(\theta-\theta_r) & L_{md}\cos(\theta-\theta_r) & 0 \\ -L_{md}\sin(\theta-\theta_r) & L_{md}\cos(\theta-\theta_r) & 0 \end{bmatrix}$$

Only if $\theta = \theta_r$

$$(\bar{L}'_{sr})^T (\bar{K}_s)^{-1} = \frac{3}{2}\begin{bmatrix} L_{mq} & 0 & 0 \\ L_{mq} & 0 & 0 \\ 0 & L_{md} & 0 \\ 0 & L_{md} & 0 \end{bmatrix}$$

5-10. See problems 5-7 and 5-8

5-11 $$\bar{v}_{abs} = \bar{r}_s \bar{i}_{abs} + p\bar{\lambda}_{abs}$$

$$\bar{v}'^{r}_{qdr} = \bar{r}'_r \bar{i}'^{r}_{qdr} + p\bar{\lambda}'^{r}_{qdr} = \bar{r}'_r \bar{i}'^{r}_{qdr} + \frac{p}{\omega_b}\bar{\psi}'^{r}_{qdr}$$

$$\bar{v}^{r}_{qds} = \bar{K}^{r}_{s}\bar{r}_s(\bar{K}^{r}_{s})^{-1}\bar{i}^{r}_{qds} + \bar{K}^{r}_{s}\, p\left[(\bar{K}^{r}_{s})^{-1}\bar{\lambda}^{r}_{qds}\right]$$

$$= \bar{r}_s \bar{i}^{r}_{qds} + \omega_r \bar{\lambda}^{r}_{dqs} + p\bar{\lambda}^{r}_{qds}$$

$$= \bar{r}_s \bar{i}^{r}_{qds} + \frac{\omega_r}{\omega_b}\bar{\psi}^{r}_{dqs} + \frac{p}{\omega_b}\bar{\psi}^{r}_{qds}$$

where $$\bar{\psi}^{r}_{dqs} = \begin{bmatrix} \psi^{r}_{ds} & 0 \\ 0 & -\psi^{r}_{qs} \end{bmatrix}$$

$$\begin{bmatrix} \bar{\Psi}^r_{qds} \\ \bar{\Psi}'^r_{qdr} \end{bmatrix} = \begin{bmatrix} \bar{K}^r_{2s}\bar{X}_s(\bar{K}^r_{2s})^{-1} & \bar{K}^r_{2s}\bar{X}'_{sr} \\ (\bar{X}'_{sr})^T(\bar{K}^r_{2s})^{-1} & \bar{X}'_r \end{bmatrix} \begin{bmatrix} -\bar{i}^r_{qds} \\ \bar{i}'^r_{qdr} \end{bmatrix}$$

$$\bar{K}^r_{2s}\bar{X}_s(\bar{K}^r_{2s})^{-1} = \omega_b \bar{K}^r_{2s}\bar{L}_s(\bar{K}^r_{2s})^{-1} = \omega_b \begin{bmatrix} L_{ls}+L_A-L_B & 0 \\ 0 & L_{ls}+L_A+L_B \end{bmatrix}$$

$$= \begin{bmatrix} X_{ls}+X_{mq} & 0 \\ 0 & X_{ls}+X_{md} \end{bmatrix}$$

$$X_{mq} = \omega_b(L_A - L_B) , \quad X_{md} = \omega_b(L_A + L_B)$$

$$\bar{K}^r_{2s}\bar{X}'_{sr} = \omega_b \bar{K}^r_{2s}\bar{L}'_{sr} = \omega_b \begin{bmatrix} L_{mq} & 0 & 0 \\ 0 & L_{md} & L_{md} \end{bmatrix}$$

$$= \begin{bmatrix} X_{mq} & 0 & 0 \\ 0 & X_{md} & X_{md} \end{bmatrix}$$

$$(\bar{X}'_{sr})^T(\bar{K}^r_{2s})^{-1} = (\bar{X}'_{sr})^T(\bar{K}^r_{2s})^T = (\bar{K}^r_{2s}\bar{X}'_{sr})^T$$

$$= \begin{bmatrix} X_{mq} & 0 \\ 0 & X_{md} \\ 0 & X_{md} \end{bmatrix}$$

The inductance matrices $\bar{L}_s$, $\bar{L}'_{sr}$, $\bar{L}'_r$ are shown in Problem 5-1.

5-14 $$T_e = \frac{P}{2}\left\{-\frac{1}{2}(\bar{i}_{abs})^T \frac{\partial}{\partial\theta_r}[\bar{L}_s - L_{ls}\bar{I}]\,\bar{i}_{abs} + (\bar{i}_{abs})^T \frac{\partial}{\partial\theta_r}(\bar{L}'_{sr})\,\bar{i}'_{qdr}\right\}$$

$$= \frac{P}{2}\left[(\bar{K}^r_{2s})^{-1}\bar{i}^r_{qds}\right]^T\left\{-\frac{1}{2}\frac{\partial}{\partial\theta_r}[\bar{L}_s - L_{ls}\bar{I}](\bar{K}^r_{2s})^{-1}\bar{i}^r_{qds} + \frac{\partial}{\partial\theta_r}(\bar{L}'_{sr})\,\bar{i}'^r_{qdr}\right\}$$

$$= \frac{P}{2}\left\{(\bar{i}^r_{qds})^T\ \bar{K}^r_{2s}\begin{bmatrix}-L_B\sin 2\theta_r & L_B\cos 2\theta_r\\ L_B\cos 2\theta_r & L_B\sin 2\theta_r\end{bmatrix}\bar{K}^r_{2s}\ \bar{i}^r_{qds}\right.$$

$$\left. + (\bar{i}^r_{qds})^T\ \bar{K}^r_{2s}\begin{bmatrix}-L_{mq}\sin\theta_r & L_{md}\cos\theta_r & L_{md}\cos\theta_r\\ L_{mq}\cos\theta_r & L_{md}\sin\theta_r & L_{md}\sin\theta_r\end{bmatrix}\bar{i}'^r_{qdr}\right\}$$

$$= \frac{P}{2}\left\{(\bar{i}^r_{qds})^T\begin{bmatrix}0 & -L_B\\ -L_B & 0\end{bmatrix}(\bar{i}^r_{qds}) + (\bar{i}^r_{qds})^T\begin{bmatrix}0 & L_{md} & L_{md}\\ -L_{mq} & 0 & 0\end{bmatrix}\bar{i}'^r_{qdr}\right\}$$

$$= \frac{P}{2}\left[L_{md}(-i^r_{ds} + i'^r_{fd} + i'^r_{kd})\,i^r_{qs} - L_{mq}(-i^r_{qs} + i'^r_{kq})\,i^r_{ds}\right]$$

$$= \frac{P}{2}(\lambda^r_{ds}i^r_{qs} - \lambda^r_{qs}i^r_{ds}) = \left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)(\psi^r_{ds}i^r_{qs} - \psi^r_{qs}i^r_{ds})$$

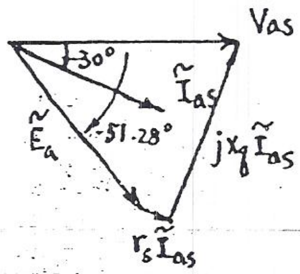
5-15 $V_B = 11.55\ \text{KV}$, $I_B = 9.38\ \text{KA}$, $Z_B = 1.23\ \Omega$

5-16 $V_B = 15.01\ \text{KV}$, $I_B = 18.54\ \text{KA}$, $Z_B = 0.810\ \Omega$

5-18 (a) $P = 3|\tilde{V}_{as}||\tilde{I}_{as}|\cos\varphi$ $\quad |\tilde{I}_a| = 60.61$ A

$$\tilde{E}_a = \tilde{V}_{as} - (r_s + jX_q)\,\tilde{I}_{as} = 216.59\,\underline{/-51.28^\circ} \text{ V}$$

$$Q = 3|V_a||I_a|\sin\varphi = 23.096 \text{ KVAR}$$

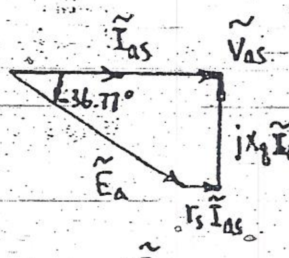


(a)

(b) $\varphi = 0$, $Q = 0$

$|\tilde{I}_{as}| = 52.49$ A

$\tilde{E}_a = 297.50\,\underline{/-36.77^\circ}$

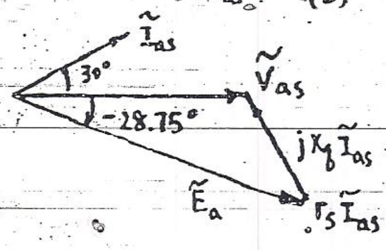


(b)

(c) $|\tilde{I}_{as}| = 60.61$ A

$\tilde{E}_a = 389.10\,\underline{/-28.75^\circ}$

$Q = -23.096$ KVAR



(C)

5-19 Assume the positive direction of current is into the stator terminals.

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_{ds}^r i_{qs}^r - \lambda_{qs}^r i_{ds}^r\right) = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(L_{md} - L_{mq}\right) i_{qs}^r i_{ds}^r$$

$$= 0$$

6-4. Under the Conditions given in this problem and if the terms involving r_s^2 are neglected,

$$i_{qs}^r(s) = \frac{\omega_b \sqrt{2} V_s}{s^2 + 2\alpha s + \omega_b^2} \cdot \frac{1}{X_q(s)}$$

$$i_{ds}^r(s) = \frac{\omega_b^2 \sqrt{2} V_s}{s(s^2 + 2\alpha s + \omega_b^2)} \cdot \frac{1}{X_d(s)} \quad , \quad \alpha = \frac{\omega_b r_s}{2}\left(\frac{1}{X_q(\infty)} - \frac{1}{X_d(\infty)}\right)$$

Then $$\psi_{qs}^r(s) = -X_q(s)\, i_{qs}^r(s) = \frac{-\sqrt{2}\,\omega_b V_s}{s^2 + 2\alpha s + \omega_b^2}$$

$$\psi_{ds}^r(s) = -X_d(s)\, i_{ds}^r(s) = \frac{-\sqrt{2}\,\omega_b^2 V_s}{s(s^2 + 2\alpha s + \omega_b^2)} \quad , \quad e_{xfd}'^r(s) = 0$$

By (6.7-21) and (6.7-22)

$$\psi_{qs}^r(t) = -\sqrt{2} V_s e^{-\alpha t} \sin \omega_b t \quad , \quad \text{if } \alpha \ll \omega_b$$

$$\psi_{ds}^r(t) = \sqrt{2} V_s \left[e^{-\alpha t} \cos \omega_b t - 1\right]$$

$$i_{qs}^r(t) = \frac{\sqrt{2} V_s}{X_q''} e^{-\alpha t} \sin \omega_b t$$

$$i_{ds}^r(t) = \sqrt{2} V_s \left[\frac{1}{X_d} + \left(\frac{\tau_{do}'}{\tau_d'}\frac{1}{X_d'} - \frac{1}{X_d}\right) e^{-t/\tau_d'} + \left(\frac{1}{X_d''} - \frac{\tau_{do}'}{\tau_d'}\frac{1}{X_d'}\right) e^{-t/\tau_d''}\right]$$
$$- \frac{\sqrt{2} V_s}{X_d''} e^{-\alpha t} \cos \omega_b t$$

Note that we must add the initial values to above solutions.

$$\psi_{qso}^r = i_{qso}^r = i_{dso}^r = 0, \quad \psi_{dso}^r = \sqrt{2} V_s$$

$\therefore$ $\psi_{ds}^r(t)$ should be $\sqrt{2} V_s e^{-\alpha t} \cos \omega_b t$

5-32 For the hydro turbine generator,

$$T_e = 98.5 \sin\delta \times 10^6 \text{ N.M}$$

Similar to problem 5-31, the following equations can be obtained.

$$\begin{cases} T_I \delta_M + 98.5 \cos\delta_M = 98.5 \\ T_I = 98.5 \sin\delta_M \end{cases}$$

Then $T_I = 71.374 \times 10^6$ N·M, $\delta_M = 133.56°$

For the steam turbine generator,

$$T_e = 6.92 \sin\delta \times 10^6 \text{ N.M}$$

$$\begin{cases} T_I \delta_M + 6.92 \cos\delta_M = 6.92 \\ T_I = 6.92 \sin\delta_M \end{cases}$$

$T_I = 5.014 \times 10^6$ N·M, $\delta_M = 133.56°$

5-32 For the hydro turbine generator,

$T_e = 98.5 \sin\delta \times 10^6$ N.M

Similar to problem 5-31, the following equations can be obtained.

$$\begin{cases} T_I \delta_M + 98.5 \cos\delta_M = 98.5 \\ T_I = 98.5 \sin\delta_M \end{cases}$$

Then $T_I = 71.374 \times 10^6$ N.M , $\delta_M = 133.56^\circ$

For the steam turbine generator,

$T_e = 6.92 \sin\delta \times 10^6$ N.M

$$\begin{cases} T_I \delta_M + 6.92 \cos\delta_M = 6.92 \\ T_I = 6.92 \sin\delta_M \end{cases}$$

$T_I = 5.014 \times 10^6$ N.M , $\delta_M = 133.56$

6-4. Under the Conditions given in this problem and if the terms involving r_s^2 are neglected,

$$i_{qs}^r(s) = \frac{\omega_b \sqrt{2} V_s}{s^2 + 2\alpha s + \omega_b^2} \cdot \frac{1}{X_q(s)}$$

$$i_{ds}^r(s) = \frac{\omega_b^2 \sqrt{2} V_s}{s(s^2 + 2\alpha s + \omega_b^2)} \cdot \frac{1}{X_d(s)} \quad , \quad \alpha = \frac{\omega_b r_s}{2}\left(\frac{1}{X_q(\infty)} - \frac{1}{X_d(\infty)}\right)$$

Then $$\psi_{qs}^r(s) = -X_q(s)\, i_{qs}^r(s) = \frac{-\sqrt{2}\,\omega_b V_s}{s^2 + 2\alpha s + \omega_b^2}$$

$$\psi_{ds}^r(s) = -X_d(s)\, i_{ds}^r(s) = \frac{-\sqrt{2}\,\omega_b^2 V_s}{s(s^2 + 2\alpha s + \omega_b^2)} \quad , \quad e'_{xfd}(s) = 0$$

By (6.7-21) and (6.7-22)

$$\psi_{qs}^r(t) = -\sqrt{2} V_s e^{-\alpha t} \sin \omega_b t \quad , \quad \text{if } \alpha \ll \omega_b$$

$$\psi_{ds}^r(t) = \sqrt{2} V_s \left[e^{-\alpha t} \cos \omega_b t - 1\right]$$

$$i_{qs}^r(t) = \frac{\sqrt{2} V_s}{X_q''} e^{-\alpha t} \sin \omega_b t$$

$$i_{ds}^r(t) = \sqrt{2} V_s \left[\frac{1}{X_d} + \left(\frac{\tau_{do}'}{\tau_d'}\frac{1}{X_d} - \frac{1}{X_d}\right)e^{-t/\tau_d'} + \left(\frac{1}{X_d''} - \frac{\tau_{do}'}{\tau_d'}\frac{1}{X_d}\right)e^{-t/\tau_d''}\right]$$

$$- \frac{\sqrt{2} V_s}{X_d''} e^{-\alpha t} \cos \omega_b t$$

Note that we must add the initial values to above solutions.

$$\psi_{qs0}^r = i_{qs0}^r = i_{ds0}^r = 0, \qquad \psi_{ds0}^r = \sqrt{2} V_s$$

$\psi_{ds}^r(t)$ should be $\sqrt{2} V_s e^{-\alpha t} \cos \omega_b t$

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)\left(\psi_{ds}^{r}\, i_{qs}^{r} - \psi_{qs}^{r}\, i_{ds}^{r}\right)$$

$$= \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\frac{V_s^2}{\omega_b}\right)\Big[\left(\frac{1}{X_q''} - \frac{1}{X_d''}\right) e^{-2\alpha t} \sin 2\omega_b t + \frac{2}{X_d} e^{-\alpha t} \sin \omega_b t$$

$$+ 2\left(\frac{\tau_{do}'}{\tau_d'}\frac{1}{X_d} - \frac{1}{X_d}\right) e^{-(\alpha + \frac{1}{\tau_d'})t} \sin \omega_b t + 2\left(\frac{1}{X_d''} - \frac{\tau_{do}'}{\tau_d'}\frac{1}{X_d}\right) e^{-(\alpha + \frac{1}{\tau_d''})t} \sin \omega_b t\Big]$$

6-5. $i_{fd}'^{r}(s) = s\,G(s)\, i_{ds}^{r}(s)$ if $e_{xfd}'^{r} = 0$ (see problem 6-7)

$$= \frac{X_{md}}{r_{fd}'} \frac{s(1+\tau_{db}s)}{1+(\tau_{d1}+\tau_{d2})s + \tau_{d1}\tau_{d3}s^2}\, i_{ds}^{r}(s)$$

$$= \frac{X_{md}}{r_{fd}'} \frac{s(1+\tau_{db}s)}{(1+\tau_{do}'s)(1+\tau_{do}''s)} \cdot \frac{(1+\tau_{do}'s)(1+\tau_{do}''s)}{X_d(1+\tau_d's)(1+\tau_d''s)} \frac{\sqrt{2}V_s\omega_b^2}{s(s^2+2\alpha s+\omega_b^2)}$$

$$= \frac{X_{md}\sqrt{2}V_s\omega_b^2}{r_{fd}' X_d} \frac{1+\tau_{db}s}{(1+\tau_d's)(1+\tau_d''s)(s^2+2\alpha s+\omega_b^2)}$$

$$\therefore \frac{1+\tau_{db}s}{(1+\tau_d's)(1+\tau_d''s)} = \frac{A}{1+\tau_d's} + \frac{B}{1+\tau_d''s}$$

where $A = \dfrac{1-\tau_{db}/\tau_d'}{1-\tau_d''/\tau_d'}$, $B = \dfrac{1-\tau_{db}/\tau_d''}{1-\tau_d'/\tau_d''}$

In general $\tau_d' \gg \tau_{db}$, $\tau_d' \gg \tau_d''$. In this case,

$$A \approx 1, \quad B \approx \frac{\tau_d''}{\tau_d'}\left(\frac{\tau_{db}}{\tau_d''} - 1\right)$$

Using (6.7-22), we can find the solution, if $\frac{1}{\tau_d'} \ll \omega_b$ and $\frac{1}{\tau_d''} \ll \omega_b$.

$$\Delta i_{fd}^{\prime\prime r}(t) = \frac{\sqrt{2}\, V_s X_{md}}{r_{fd}^{\prime} X_d \tau_d^{\prime}} \left[e^{-t/\tau_d^{\prime}} + \left(\frac{\tau_{db}}{\tau_d^{\prime\prime}} - 1 \right) e^{-t/\tau_d^{\prime\prime}} - \frac{\tau_{db}}{\tau_d^{\prime}} e^{-\alpha t} \cos \omega_b t \right]$$

Considering the initial value

$$i_{fd}^{\prime r}(t) = i_{fdo}^{\prime r} + \Delta i_{fd}^{\prime r}(t) = \frac{\sqrt{2}\, V_s}{X_{md}} + \Delta i_{fd}^{\prime r}(t)$$

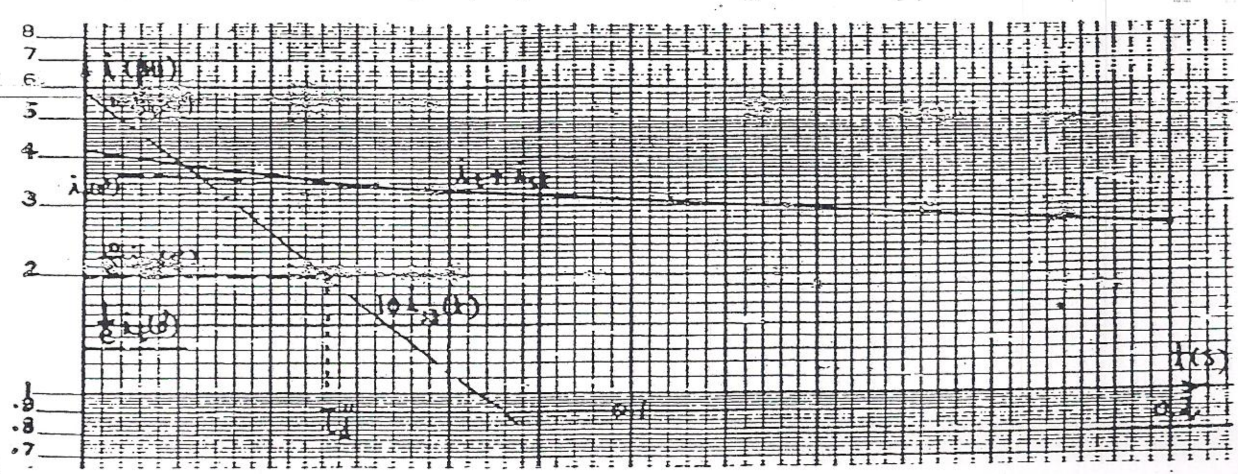
6-6 From the figure 6.10-1, the following data are obtained

t (s)	0	0.0167	0.033	0.050	0.067	0.084	0.100	0.117	0.133	0.150	0.170	0.190	0.200
i_{sc} (pu)	4.70	4.39	4.16	3.98	3.81	3.76	3.68	3.60	3.52	3.44	3.36	3.28	3.2
* $i_t + i_{st}$	4.11	3.80	3.57	3.39	3.22	3.17	3.09	3.01	2.93	2.85	2.77	2.69	2.61
* i_{st}	0.54	0.35	0.24	0.18	0.12								

$\because\ X_d = 1.7$ pu $\quad V_s = 1.0$ pu

$$i_{ss} = i_{sc}(t \to \infty) = \frac{V_s}{X_d} = 0.5882 \text{ pu}$$

$$\therefore\ i_t + i_{st} = i_{sc} - i_{ss}$$



From the figure in the previous page, we can find

$$\tau_d' = 0.470\text{ s} \qquad \text{and} \qquad i_t(0^+) = 3.57 \text{ pu.}$$

$$\because \quad i_t(0^+) = V_s\left(\frac{\tau_{do}'}{\tau_d'}\frac{1}{X_d} - \frac{1}{X_d}\right)$$

$$\therefore \quad \tau_{do}' = \left(\frac{3.57 X_d}{V_s} + 1\right)\tau_d' = 3.322\text{ s}$$

**

$$i_{st}(t) = \left[i_t(t) + i_{st}(t)\right] - 3.57e^{-t/\tau_d'}$$

From the same figure; $\tau_d'' = 0.044\text{ s}$

$$X_d'' = \frac{V_s}{i_{sc}(0^+)} = 0.212 \text{ pu.}$$

$$X_{md} = X_d - X_{ls} = 1.51 \text{ pu.}$$

(a) Using the derived time constants

From the equations for τ_d', τ_d'', τ_{do}' and X_d'', four unknowns r_{kd}', r_{fd}', X_{lfd}' and X_{lkd}' can be solved.

(b) Using the standard time constants

$$X_d' = \frac{\tau_d'}{\tau_{do}'} X_d = 0.240 \text{ pu.}$$

$$X_d' = X_{ls} + \frac{X_{md} X_{lfd}'}{X_{lfd}' + X_{md}}$$

$$\therefore \quad X_{lfd}' = \frac{X_{md}(X_d' - X_{ls})}{X_d - X_d'} = 0.052 \text{ pu}$$

$$\because \ \tau_d' = \frac{1}{\omega_b r_{fd}'}\left(X_{lfd}' + \frac{X_{md} X_{ls}}{X_d} \right)$$

$$\therefore \ r_{fd}' = \frac{1}{\omega_b \tau_d'}\left(X_{lfd}' + \frac{X_{md} X_{ls}}{X_d} \right) = 0.0012 \text{ pu.}$$

$$\because \ X_d'' = X_{ls} + \frac{X_{md} X_{lfd}' X_{lkd}'}{X_{md} X_{lfd}' + X_{md} X_{lkd}' + X_{lfd}' X_{lkd}'}$$

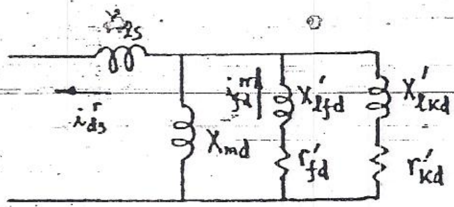
$$\therefore \ X_{lkd}' = \frac{(X_d'' - X_{ls}) X_{md} X_{lfd}'}{X_{md} X_{lfd}' - (X_d'' - X_{ls})(X_{md} + X_{lfd}')} = 0.039 \text{ pu.}$$

$$\because \ \tau_d'' = \frac{1}{\omega_b r_{kd}'}\left(X_{lkd}' + \frac{X_{md} X_{ls} X_{lfd}'}{X_{md} X_{ls} + X_{md} X_{lfd}' + X_{ls} X_{lfd}'} \right) = \frac{0.0002089}{r_{kd}'}$$

$$\therefore \ r_{kd}' = \frac{0.0002089}{\tau_d''} = 0.0047 \text{ pu.}$$

$$\tau_{do}'' = \frac{1}{\omega_b r_{kd}'}\left(X_{lkd}' + \frac{X_{md} X_{lfd}'}{X_{md} + X_{lfd}'} \right) = 0.050 \text{ s}$$

6-7



$$i_{fd}'^r = \frac{s\frac{X_{md}}{\omega_b} \parallel \left(r_{kd}' + s\frac{X_{lkd}'}{\omega_b}\right) i_{ds}^r}{s\frac{X_{md}}{\omega_b} \parallel \left(r_{kd}' + s\frac{X_{lkd}'}{\omega_b}\right) + \left(r_{fd}' + s\frac{X_{lfd}'}{\omega_b}\right)}$$

$$= \frac{s\frac{X_{md}}{\omega_b}\left(r_{kd}' + s\frac{X_{lkd}'}{\omega_b}\right) i_{ds}^r}{s\frac{X_{md}}{\omega_b}\left(r_{kd}' + s\frac{X_{lkd}'}{\omega_b}\right) + \left(r_{fd}' + s\frac{X_{lfd}'}{\omega_b}\right)\left[r_{kd}' + \frac{s}{\omega_b}(X_{md} + X_{lkd}')\right]}$$

$$= \frac{X_{md}}{r_{fd}'} \ \frac{s(1 + \tau_{d3})}{1 + (\tau_{d1} + \tau_{d2})s + \tau_{d1}\tau_{d3}s^2} \ i_{ds}^r$$

$$= sG(s) \ i_{ds}^r$$

6-8. $X_q(S) = 2.0 \dfrac{(1+0.64S)(1+0.016S)}{(1+1.59S)(1+0.05S)}$

$\tau'_{qo} = 1.59\ s$, $\tau''_{qo} = 0.05S$, $\tau'_q = 0.64S$, $\tau''_q = 0.016S$

(a) Using the derived time constants

With the selection of $X_{ls} = 0.15$ pu. there are four equations in Table 6.6-1 and four unknowns $r'_{kq1}, X'_{lkq1}, r'_{kq2}$, and X'_{lkq2}. These equations are nonlinear ; they can be solved by a computer.

(b) Let $X_q(s) = X_q \dfrac{N_X(s)}{D_X(s)}$

$$Z_{qr}(s) = \frac{SX_{mq}}{\omega_b}\left[N_X(s) - \frac{X_{ls}}{X_q} D_X(s)\right] \Big/ \left[D_X(s) - N_X(s)\right]$$

$$= \frac{1.85}{377} \; \frac{0.0042775 S^2 + 0.533S + 0.925}{0.06926S + 0.984}$$

$$= 4.61 \times 10^{-3} \; \frac{(1+0.00814S)(0.5681S+1)}{(1+0.0704S)}$$

$\therefore$ $R_{eq} = 4.61 \times 10^{-3}$ $\tau_{qa} = 0.5861$, $\tau_{qb} = 0.00814$,

$\tau_{Qa} = 0.0704$

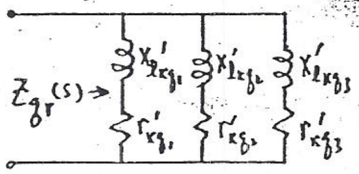
$$\begin{bmatrix} 1 & 1 \\ \tau_{qb} & \tau_{qa} \end{bmatrix} \begin{bmatrix} 1/r'_{kq1} \\ 1/r'_{kq2} \end{bmatrix} = \frac{1}{R_{eq}} \begin{bmatrix} 1 \\ \tau_{Qa} \end{bmatrix}$$

$\therefore$ $r'_{kq1} = 0.00517$ pu. , $r'_{kq2} = 0.0428$ pu ,

$X'_{lkq1} = \omega_b r'_{kq1} \tau_{qa} = 1.14$ pu, $X'_{lkq2} = \omega_b r'_{kq2} \tau_{qb} = 0.131$ pu.

6-10

$$Z_{gr}(s) = \frac{1}{\dfrac{1}{r'_{kg_1}+\frac{s}{\omega_b}X'_{lkg_1}} + \dfrac{1}{r'_{kg_2}+\frac{s}{\omega_b}X'_{lkg_2}} + \dfrac{1}{r'_{kg_3}+\frac{s}{\omega_b}X'_{lkg_3}}}$$



$$= \frac{\left(r'_{kg_1}+\frac{s}{\omega_b}X'_{lkg_1}\right)\left(r'_{kg_2}+\frac{s}{\omega_b}X'_{lkg_2}\right)\left(r'_{kg_3}+\frac{s}{\omega_b}X'_{lkg_3}\right)}{\left(r'_{kg_1}+\frac{s}{\omega_b}X'_{lkg_1}\right)\left(r'_{kg_2}+\frac{s}{\omega_b}X'_{lkg_2}\right)+\left(r'_{kg_1}+\frac{s}{\omega_b}X'_{lkg_1}\right)\left(r'_{kg_3}+\frac{s}{\omega_b}X'_{lkg_3}\right)+\left(r'_{kg_2}+\frac{s}{\omega_b}X'_{lkg_2}\right)\left(r'_{kg_3}+\frac{s}{\omega_b}X'_{lkg_3}\right)}$$

$$= R_{eq}\,\frac{(1+s\tau_{ga})(1+s\tau_{gb})(1+s\tau_{gc})}{1+sR_{eq}\left(\dfrac{\tau_{gb}+\tau_{gc}}{r'_{kg_1}}+\dfrac{\tau_{ga}+\tau_{gc}}{r'_{kg_2}}+\dfrac{\tau_{ga}+\tau_{gb}}{r'_{kg_3}}\right)+s^2R_{eq}\left(\dfrac{\tau_{ga}\tau_{gb}}{r'_{kg_3}}+\dfrac{\tau_{ga}\tau_{gc}}{r'_{kg_2}}+\dfrac{\tau_{gb}\tau_{gc}}{r'_{kg_1}}\right)}$$

$$= R_{eq}\,\frac{(1+s\tau_{ga})(1+s\tau_{gb})(1+s\tau_{gc})}{(1+s\tau_{Qa})(1+s\tau_{Qb})}$$

where $$R_{eq} = \frac{r'_{kg_1}r'_{kg_2}r'_{kg_3}}{r'_{kg_1}r'_{kg_2}+r'_{kg_1}r'_{kg_3}+r'_{kg_2}r'_{kg_3}},$$

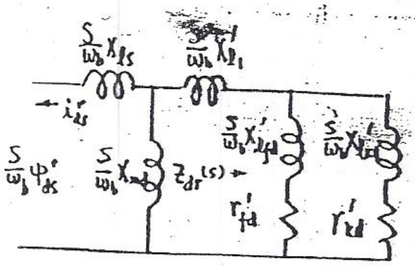
$$\tau_{ga} = \frac{X'_{lkg_1}}{\omega_b r'_{kg_1}}, \quad \tau_{gb} = \frac{X'_{lkg_2}}{\omega_b r'_{kg_2}}, \quad \tau_{gc} = \frac{X'_{lkg_3}}{\omega_b r'_{kg_3}}$$

$$\tau_{Qa} = \frac{B}{2}+\sqrt{\frac{B^2}{4}-A}, \quad \tau_{Qb} = \frac{B}{2}-\sqrt{\frac{B^2}{4}-A}$$

$$B = R_{eq}\left(\frac{\tau_{gb}+\tau_{gc}}{r'_{kg_1}}+\frac{\tau_{ga}+\tau_{gc}}{r'_{kg_2}}+\frac{\tau_{ga}+\tau_{gb}}{r'_{kg_3}}\right)$$

$$A = R_{eq}\left(\frac{\tau_{gb}\tau_{gc}}{r'_{kg_1}}+\frac{\tau_{ga}\tau_{gc}}{r'_{kg_2}}+\frac{\tau_{ga}\tau_{gb}}{r'_{kg_3}}\right)$$

6-15



$$Z_{dr}(s) = \frac{s}{\omega_b} X'_{l1} + R_{ed} \frac{(1+\tau_{da} s)(1+\tau_{db} s)}{(1+\tau_{Da} s)}$$

where R_{ed}, τ_{da}, τ_{db} and τ_{Da} are defined

by (6.3-15), (6.3-16), (6.3-17) and (6.3-18)

$$Z_{dr}(s) = \frac{\frac{s}{\omega_b} X'_{l1}(1+\tau_{Da} s) + R_{ed}(1+\tau_{da} s)(1+\tau_{db} s)}{1+\tau_{Da} s}$$

$$= R_{ed} \frac{\frac{s^2}{\omega_b^2}\left(\frac{X'_{lfd} X'_{lkd} + X'_{lfd} X'_{l1} + X'_{lkd} X'_{l1}}{r'_{fd}\, r'_{kd}}\right) + \frac{s}{\omega_b}\left(\frac{X'_{l1} + X'_{lkd}}{r'_{kd}} + \frac{X'_{l1} + X'_{lfd}}{r'_{fd}}\right) + 1}{1+\tau_{Da} s}$$

$$X_d(s) = X_{ls} + \frac{X_{md}\, Z_{dr}(s)}{Z_{dr}(s) + \frac{s X_{md}}{\omega_b}} = X_d \frac{1 + (\tau'_{d4} + \tau'_{d5}) s + \tau'_{d4} \tau'_{d6} s^2}{1 + (\tau'_{d1} + \tau'_{d2}) s + \tau'_{d1} \tau'_{d3} s^2}$$

where $\tau'_{d1} = \frac{1}{\omega_b r'_{fd}} (X'_{lfd} + X_{md} + X'_{l1})$

$$\tau'_{d2} = \frac{1}{\omega_b r'_{kd}} (X'_{lkd} + X_{md} + X'_{l1})$$

$$\tau'_{d3} = \frac{1}{\omega_b r'_{kd}} \left(X'_{lkd} + \frac{(X_{md} + X'_{l1}) X'_{lfd}}{X'_{lfd} + X_{md} + X'_{l1}}\right)$$

$$\tau'_{d4} = \frac{1}{\omega_b r'_{fd}} \left(X'_{lfd} + X'_{l1} + \frac{X_{md} X_{ls}}{X_{ls} + X_{md}}\right)$$

$$\tau'_{d5} = \frac{1}{\omega_b r'_{kd}} \left(X'_{lkd} + X'_{l1} + \frac{X_{md} X_{ls}}{X_{ls} + X_{md}}\right)$$

$$\tau'_{d6} = \frac{1}{\omega_b r'_{kd}} \left(X'_{lkd} + \frac{X_{md} X_{ls} X'_{lfd} + X'_{l1} X'_{lfd} X_{md} + X'_{l1} X'_{lfd} X_{ls}}{X'_{l1} X_{ls} + X_{md} X'_{l1} + X'_{lfd} X_{md} + X'_{lfd} X_{ls} + X_{md} X_{ls}}\right)$$

It is not difficult to find

$$G(s) = \frac{X_{md}}{r'_{fd}} \cdot \frac{1+\tau_{db} s}{1 + (\tau'_{d1} + \tau'_{d2}) s + \tau'_{d1} \tau'_{d3} s^2}, \quad \tau_{db} = \frac{X'_{lkd}}{\omega_b r'_{kd}}.$$

Clearly, $X_d(s)$ and $G(s)$ have the same denominator.

6-16. The equations are the same as (6.2-2), (6.2-4) and (6.2-5), except that $X_d(s)$ and $G(s)$ derived in problem 6-15 are used.

Chapter 7.

7-1. $\because \begin{cases} \psi_{qs}^{e} = X_{ss} i_{qs}^{e} + X_M i_{qr}^{\prime e} \\ \psi_{qr}^{\prime e} = X'_{rr} i_{qr}^{\prime e} + X_M i_{qs}^{e} \end{cases}$, $\begin{cases} \psi_{ds}^{e} = X_{ss} i_{ds}^{e} + X_M i_{dr}^{\prime e} \\ \psi_{dr}^{\prime e} = X'_{rr} i_{dr}^{\prime e} + X_M i_{ds}^{e} \end{cases}$

Let $XX = X_{ss} X'_{rr} - X_M^2$

$$\therefore \quad i_{qs}^{e} = \frac{X'_{rr}\psi_{qs}^{e} - X_M \psi_{qr}^{\prime e}}{XX}, \qquad i_{ds}^{e} = \frac{X'_{rr}\psi_{ds}^{e} - X_M \psi_{dr}^{\prime e}}{XX},$$

$$i_{qr}^{\prime e} = \frac{X_{ss}\psi_{qr}^{\prime e} - X_M \psi_{qs}^{e}}{XX}, \qquad i_{dr}^{\prime e} = \frac{X_{ss}\psi_{dr}^{\prime e} - X_M \psi_{ds}^{e}}{XX}.$$

$$v_{qs}^{e} = r_s i_{qs}^{e} + \frac{\omega_e}{\omega_b}\psi_{ds}^{e} + \frac{p}{\omega_b}\psi_{qs}^{e} = \left(\frac{X'_{rr} r_s}{XX} + \frac{p}{\omega_b}\right)\psi_{qs}^{e} - \frac{X_M r_s}{XX}\psi_{qr}^{\prime e} + \frac{\omega_e}{\omega_b}\psi_{ds}^{e}$$

$$v_{ds}^{e} = r_s i_{ds}^{e} - \frac{\omega_e}{\omega_b}\psi_{qs}^{e} + \frac{p}{\omega_b}\psi_{ds}^{e} = \left(\frac{X'_{rr} r_s}{XX} + \frac{p}{\omega_b}\right)\psi_{ds}^{e} - \frac{X_M r_s}{XX}\psi_{dr}^{\prime e} - \frac{\omega_e}{\omega_b}\psi_{qs}^{e}$$

$$v_{qr}^{\prime e} = r'_r i_{qr}^{\prime e} + \left(\frac{\omega_e-\omega_r}{\omega_b}\right)\psi_{dr}^{\prime e} + \frac{p}{\omega_b}\psi_{qr}^{\prime e} = \left(\frac{X_{ss} r'_r}{XX} + \frac{p}{\omega_b}\right)\psi_{qr}^{\prime e} - \frac{X_M r'_r}{XX}\psi_{qs}^{e} + \left(\frac{\omega_e-\omega_r}{\omega_b}\right)\psi_{dr}^{\prime e}$$

$$v_{dr}^{\prime e} = r'_r i_{dr}^{\prime e} - \left(\frac{\omega_e-\omega_r}{\omega_b}\right)\psi_{qr}^{\prime e} + \frac{p}{\omega_b}\psi_{dr}^{\prime e} = \left(\frac{X_{ss} r'_r}{XX} + \frac{p}{\omega_b}\right)\psi_{dr}^{\prime e} - \frac{X_M r'_r}{XX}\psi_{ds}^{e} - \left(\frac{\omega_e-\omega_r}{\omega_b}\right)\psi_{qr}^{\prime e}$$

$$T_e = \psi_{qr}^{\prime e} i_{dr}^{\prime e} - \psi_{dr}^{\prime e} i_{qr}^{\prime e} = \frac{X_M}{XX}\left(\psi_{qs}^{e}\psi_{dr}^{\prime e} - \psi_{qr}^{\prime e}\psi_{ds}^{e}\right)$$

The small displacement equations in the matrix form are

$$\begin{bmatrix} \Delta v_{qs}^{e} \\ \Delta v_{ds}^{e} \\ \Delta v_{qr}^{\prime e} \\ \Delta v_{dr}^{\prime e} \\ \Delta T_L \end{bmatrix} = \begin{bmatrix} \frac{X'_{rr} r_s}{XX} + \frac{p}{\omega_b} & \frac{\omega_e}{\omega_b} & -\frac{X_M r_s}{XX} & 0 & 0 \\ -\frac{\omega_e}{\omega_b} & \frac{X'_{rr} r_s}{XX} + \frac{p}{\omega_b} & 0 & -\frac{X_M r_s}{XX} & 0 \\ -\frac{X_M r'_r}{XX} & 0 & \frac{X_{ss} r'_r}{XX} + \frac{p}{\omega_b} & \frac{\omega_e - \omega_{r0}}{\omega_b} & -\psi_{dr0}^{\prime e} \\ 0 & -\frac{X_M r'_r}{XX} & -\frac{\omega_e - \omega_{r0}}{\omega_b} & \frac{X_{ss} r'_r}{XX} + \frac{p}{\omega_b} & \psi_{qr0}^{\prime e} \\ \frac{X_M}{XX}\psi_{dr0}^{\prime e} & -\frac{X_M}{XX}\psi_{qr0}^{\prime e} & -\frac{X_M}{XX}\psi_{ds0}^{e} & \frac{X_M}{XX}\psi_{qs0}^{e} & -2Hp \end{bmatrix} \begin{bmatrix} \Delta\psi_{qs}^{e} \\ \Delta\psi_{ds}^{e} \\ \Delta\psi_{qr}^{\prime e} \\ \Delta\psi_{dr}^{\prime e} \\ \frac{\Delta\omega_r}{\omega_b} \end{bmatrix}$$

In fundamental form,

$$p\begin{bmatrix}\Delta\psi_{qs}^{e}\\ \Delta\psi_{ds}^{e}\\ \Delta\psi_{qr}^{\prime e}\\ \Delta\psi_{dr}^{\prime e}\\ \dfrac{\Delta\omega_r}{\omega_b}\end{bmatrix}=\begin{bmatrix}-\omega_b\dfrac{X_{rr}'r_s}{XX} & -\omega_e & \omega_b\dfrac{X_M r_s}{XX} & 0 & 0\\ \omega_e & -\omega_b\dfrac{X_{rr}'r_s}{XX} & 0 & \omega_b\dfrac{X_M r_s}{XX} & 0\\ \omega_b\dfrac{X_M r_r'}{XX} & 0 & -\omega_b\dfrac{X_{ss}r_r'}{XX} & -(\omega_e-\omega_{r_0}) & \omega_b\psi_{dr_0}^{\prime e}\\ 0 & \omega_b\dfrac{X_M r_r'}{XX} & \omega_e-\omega_{r_0} & -\omega_b\dfrac{X_{ss}r_r'}{XX} & -\omega_b\psi_{qr_0}^{\prime e}\\ \dfrac{X_M}{2HXX}\psi_{dr_0}^{\prime e} & \dfrac{-X_M}{2HXX}\psi_{qr_0}^{\prime e} & \dfrac{-X_M}{2HXX}\psi_{ds_0}^{e} & \dfrac{X_M}{2HXX}\psi_{qs_0}^{e} & 0\end{bmatrix}\begin{bmatrix}\Delta\psi_{qs}^{e}\\ \Delta\psi_{ds}^{e}\\ \Delta\psi_{qr}^{\prime e}\\ \Delta\psi_{dr}^{\prime e}\\ \dfrac{\Delta\omega_r}{\omega_b}\end{bmatrix}+\begin{bmatrix}\omega_b\Delta v_{qs}^{e}\\ \omega_b\Delta v_{ds}^{e}\\ \omega_b\Delta v_{qr}^{\prime e}\\ \omega_b\Delta v_{dr}^{\prime e}\\ \dfrac{-1}{2H}\Delta T_L\end{bmatrix}$$

7-2. Following the same procedure as that in problem 7-1, we can find

$$\begin{bmatrix}i_{qs}^{r}\\ i_{kq_1}^{\prime r}\\ i_{kq_2}^{\prime r}\end{bmatrix}=\frac{1}{D_q}\begin{bmatrix}-(X_{kq_1}'X_{kq_2}'-X_{mq}^2) & X_{mq}X_{lkq_2}' & X_{mq}X_{lkq_1}'\\ -X_{mq}X_{lkq_2}' & X_qX_{kq_2}'-X_{mq}^2 & -X_{ls}X_{mq}\\ -X_{mq}X_{lkq_1}' & -X_{mq}X_{ls} & X_qX_{kq_1}'-X_{mq}^2\end{bmatrix}\begin{bmatrix}\psi_{qs}^{r}\\ \psi_{kq_1}^{\prime r}\\ \psi_{kq_2}^{\prime r}\end{bmatrix}$$

$$=\begin{bmatrix}-Y_{11}^{q} & Y_{12}^{q} & Y_{13}^{q}\\ -Y_{12}^{q} & Y_{22}^{q} & -Y_{23}^{q}\\ -Y_{13}^{q} & -Y_{23}^{q} & Y_{33}^{q}\end{bmatrix}\begin{bmatrix}\psi_{qs}^{r}\\ \psi_{kq_1}^{\prime r}\\ \psi_{kq_2}^{\prime r}\end{bmatrix}$$

Where $D_q = X_{mq}X_{lkq_1}'X_{lkq_2}' + X_{mq}X_{ls}X_{lkq_1}' + X_{mq}X_{ls}X_{lkq_2}' + X_{ls}X_{lkq_1}'X_{lkq_2}'$

$$\begin{bmatrix}i_{ds}^{r}\\ i_{fd}^{\prime r}\\ i_{kd}^{\prime r}\end{bmatrix}=\frac{1}{D_d}\begin{bmatrix}-(X_{fd}'X_{kd}'-X_{md}^2) & X_{md}X_{lkd}' & X_{md}X_{lfd}'\\ -X_{md}X_{lkd}' & X_dX_{kd}'-X_{md}^2 & -X_{ls}X_{md}\\ -X_{md}X_{lfd}' & -X_{ls}X_{md} & X_dX_{fd}'-X_{md}^2\end{bmatrix}\begin{bmatrix}\psi_{ds}^{r}\\ \psi_{fd}^{\prime r}\\ \psi_{kd}^{\prime r}\end{bmatrix}$$

$$=\begin{bmatrix}-Y_{11}^{d} & Y_{12}^{d} & Y_{13}^{d}\\ -Y_{12}^{d} & Y_{22}^{d} & -Y_{23}^{d}\\ -Y_{13}^{d} & -Y_{23}^{d} & Y_{33}^{d}\end{bmatrix}\begin{bmatrix}\psi_{ds}^{r}\\ \psi_{fd}^{\prime r}\\ \psi_{kd}^{\prime r}\end{bmatrix}$$

where $D_d = X_{md} X_{\ell kd} X_{\ell fd}' + X_{md} X_{\ell s} X_{\ell kd}' + X_{md} X_{\ell s} X_{\ell fd}' + X_{\ell s} X_{\ell fd}' X_{\ell kd}'$

The small displacement equations in fundamental form is

$$p \begin{pmatrix} \Delta\psi^r_{qds} \\ \Delta\psi_{rr} \end{pmatrix} = \begin{bmatrix} W_0 & Y_0' \\ Q_0 & S_0 \end{bmatrix} \begin{bmatrix} \Delta\psi^r_{qds} \\ \Delta\psi_{rr} \end{bmatrix} + \begin{bmatrix} B_{s0}' & 0 \\ 0 & B_{r0} \end{bmatrix} \begin{bmatrix} \Delta V^r_{qds} \\ \Delta V_{rr} \end{bmatrix}$$

where
$$\Delta\psi^r_{qds} = [\, \Delta\psi^r_{qs} \;\; \Delta\psi^r_{ds} \,]^T$$

$$\Delta\psi_{rr} = [\, \Delta\psi'^r_{kq1} \;\; \Delta\psi'^r_{kq2} \;\; \Delta\psi'^r_{fd} \;\; \Delta\psi'^r_{kd} \;\; \frac{\Delta\omega_r}{\omega_b} \;\; \Delta\delta \,]^T$$

$$\Delta V^r_{qds} = [\, \Delta V^r_{qs} \;\; \Delta V^r_{ds} \,]^T$$

$$\Delta V_{rr} = [\, \Delta V'^r_{kq1} \;\; \Delta V'^r_{kq2} \;\; \Delta e'^r_{xfd} \;\; \Delta V'^r_{kd} \;\; \Delta T_I \;\; 0 \,]^T$$

$$\begin{aligned}
\Delta T_e &= [\, (Y^d_{11} - Y^q_{11})\psi^r_{ds0} - Y^d_{12}\psi'^r_{fd0} - Y^d_{13}\psi'^r_{kd0} \,]\,\Delta\psi^r_{qs} + [\, (Y^d_{11} - Y^q_{11})\psi^r_{qs0} \\
&\quad + Y^q_{12}\psi'^r_{kq10} + Y^q_{13}\psi'^r_{kq20} \,]\,\Delta\psi^r_{ds} + Y^q_{12}\psi^r_{ds0}\,\Delta\psi'^r_{kq1} + Y^q_{13}\psi^r_{ds0}\,\Delta\psi'^r_{kq2} \\
&\quad - Y^d_{12}\psi^r_{qs0}\,\Delta\psi'^r_{fd} - Y^d_{13}\psi^r_{qs0}\,\Delta\psi'^r_{kd} \\
&= -i^d_1\,\Delta\psi^r_{qs} + i^q_1\,\Delta\psi^r_{ds} + i^d_2\,\Delta\psi'^r_{kq1} + i^d_3\,\Delta\psi'^r_{kq2} - i^q_2\,\Delta\psi'^r_{fd} \\
&\quad - i^q_3\,\Delta\psi'^r_{kd} \\
&= -2Hp\frac{\Delta\omega_r}{\omega_B} + \Delta T_I
\end{aligned}$$

$$W_0 = \begin{bmatrix} -r_s\omega_b Y^q_{11} & -\omega_r \\ \omega_r & -r_s\omega_b Y^d_{11} \end{bmatrix}$$

$$
Y_o' = \begin{bmatrix} r_s\omega_b Y_{12}^q & r_s\omega_b Y_{13}^q & 0 & 0 & -\psi_{ds0}^r & 0 \\ 0 & 0 & r_s\omega_b Y_{12}^d & r_s\omega_b Y_{13}^d & \psi_{qs0}^r & 0 \end{bmatrix}
$$

$$
Q_o = \begin{bmatrix} r_{kq1}'\omega_b Y_{12}^q & r_{kq2}'\omega_b Y_{13}^q & 0 & 0 & i_1^d/2H & 0 \\ 0 & 0 & r_{fd}'\omega_b Y_{12}^d & r_{kd}'\omega_b Y_{13}^d & -i_1^q/2H & 0 \end{bmatrix}^T
$$

$$
S_o = \begin{bmatrix} -r_{kq1}'\omega_b Y_{22}^q & r_{kq1}'\omega_b Y_{23}^q & 0 & 0 & 0 & 0 \\ r_{kq2}'\omega_b Y_{23}^q & -r_{kq2}'\omega_b Y_{33}^q & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_{fd}'\omega_b Y_{22}^d & r_{fd}'\omega_b Y_{23}^d & 0 & 0 \\ 0 & 0 & r_{kd}'\omega_b Y_{23}^d & -r_{kd}'\omega_b Y_{33}^d & 0 & 0 \\ -i_2^d/2H & -i_3^d/2H & i_2^q/2H & i_3^q/2H & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}
$$

$$
B_{s0}^r = \mathrm{diag}\,[\omega_b \;\; \omega_b]
$$

$$
B_{r0} = \mathrm{diag}\,\left[\omega_b \;\; \omega_b \;\; \frac{\omega_b}{\omega_e}\frac{r_{fd}'}{X_{md}} \;\; \omega_b \;\; \frac{1}{2H} \;\; 0\right]
$$

In general, it is convenient to replace ΔV_{qds}^r by ΔV_{qds}^e

$$
\begin{bmatrix} \Delta V_{qs}^r \\ \Delta V_{ds}^r \end{bmatrix} = \begin{bmatrix} \cos\delta_0 & -\sin\delta_0 \\ \sin\delta_0 & \cos\delta_0 \end{bmatrix} \begin{bmatrix} \Delta V_{qs}^e \\ \Delta V_{ds}^e \end{bmatrix} + \begin{bmatrix} -V_{ds0}^r \\ V_{qs0}^r \end{bmatrix} \Delta\delta
$$

Then
$$
p \begin{bmatrix} \Delta\psi_{qds}^r \\ \Delta\psi_{rr} \end{bmatrix} = \begin{bmatrix} W_o & Y_o \\ Q_o & S_o \end{bmatrix} \begin{bmatrix} \Delta\psi_{qds}^r \\ \Delta\psi_{rr} \end{bmatrix} + \begin{bmatrix} B_{s0} & 0 \\ 0 & B_{r0} \end{bmatrix} \begin{bmatrix} \Delta V_{qds}^e \\ \Delta V_{rr} \end{bmatrix}
$$

where W_o, Q_o, S_o, B_{r0} are the same as above

$$B_{so} = \omega_b \begin{bmatrix} \cos\delta_o & -\sin\delta_o \\ \sin\delta_o & \cos\delta_o \end{bmatrix}$$

$$Y_o = \begin{bmatrix} r_s\omega_b Y_{12}^q & r_s\omega_b Y_{13}^q & 0 & 0 & -\psi_{dso}^r & -\omega_b V_{dso}^r \\ 0 & 0 & r_s\omega_b Y_{12}^d & r_s\omega_b Y_{13}^d & \psi_{qso}^r & \omega_b V_{qso}^r \end{bmatrix}$$

7-3 If $\Delta\omega_e$ is an input variable,

$$\begin{bmatrix} \Delta v_{qs}^e \\ \Delta v_{ds}^e \\ \Delta v_{qr}^{\prime e} \\ \Delta v_{dr}^{\prime e} \\ \Delta T_L \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b}X_{ss} & \frac{\omega_{eo}}{\omega_b}X_{ss} & \frac{p}{\omega_b}X_M & \frac{\omega_{eo}}{\omega_b}X_M & 0 \\ -\frac{\omega_{eo}}{\omega_b}X_{ss} & r_s + \frac{p}{\omega_b}X_{ss} & -\frac{\omega_{eo}}{\omega_b}X_M & \frac{p}{\omega_b}X_M & 0 \\ \frac{p}{\omega_b}X_M & s_o\frac{\omega_{eo}}{\omega_b}X_M & r_r' + \frac{p}{\omega_b}X_{rr}' & s_o\frac{\omega_{eo}}{\omega_b}X_{rr}' & -X_M i_{dso}^e - X_{rr}' i_{dro}^{\prime e} \\ -s_o\frac{\omega_{eo}}{\omega_b}X_M & \frac{p}{\omega_b}X_M & -s_o\frac{\omega_{eo}}{\omega_b}X_{rr}' & r_r' + \frac{p}{\omega_b}X_{rr}' & X_M i_{qso}^e + X_{rr}' i_{qro}^{\prime e} \\ X_M i_{dro}^{\prime e} & -X_M i_{qro}^{\prime e} & -X_M i_{dso}^e & X_M i_{qso}^e & -2Hp \end{bmatrix} \begin{bmatrix} \Delta i_{qs}^e \\ \Delta i_{ds}^e \\ \Delta i_{qr}^{\prime e} \\ \Delta i_{dr}^{\prime e} \\ \frac{\Delta\omega_r}{\omega_b} \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{X_{ss}}{\omega_b} i_{dso}^e + \frac{X_M}{\omega_b} i_{dro}^{\prime e} \\ -\frac{X_{ss}}{\omega_b} i_{qso}^e - \frac{X_M}{\omega_b} i_{qro}^{\prime e} \\ \frac{X_M}{\omega_b} i_{dso}^e + \frac{X_{rr}'}{\omega_b} i_{dro}^{\prime e} \\ -\frac{X_M}{\omega_b} i_{qso}^e - \frac{X_{rr}'}{\omega_b} i_{qro}^{\prime e} \\ 0 \end{bmatrix} \Delta\omega_e$$

$$= (\bar{E}p - \bar{F})\bar{X} - \bar{B}_1 \Delta\omega_e$$

or $$\bar{E}p\bar{X} = \bar{F}\bar{X} + \bar{B}_1\Delta\omega_e + \bar{u}$$

Here, $\bar{E}$ and $\bar{F}$ are the same as (7.3-13) and (7.3-14) except ω_e is replaced by ω_{eo}.

$\bar{x}$ and $\bar{u}$ are the same as (7.3-11) and (7.3-12). If the the input voltages are balanced and the magnitude V_s is constant, and the load torque is constant, and the rotor circuit is short circuited, $\bar{u} = 0$

$$\bar{B}_1 = \frac{1}{\omega_b} \begin{bmatrix} -X_{ss}\, i^e_{dso} - X_M\, i'^e_{dro} \\ X_{ss}\, i^e_{qso} + X_M\, i'^e_{qro} \\ -X_M\, i^e_{dso} - X'_{rr}\, i'^e_{dro} \\ X_M\, i^e_{qso} + X'_{rr}\, i'^e_{qro} \\ 0 \end{bmatrix}$$

7-4. In a similar way to problem 7-3

$$\bar{E}p\bar{x} = \bar{F}x + \bar{B}_1 \omega_e + \bar{u}$$

here $\bar{x}$, $\bar{u}$ are the same as (7.3-21) and (7.3-22),

$\bar{E}$ and $\bar{F}$ are the same as (7.3-23) and (7.3-24) except ω_e is replaced by ω_{eo}.

$$\bar{B}_1 = -\frac{1}{\omega_b} \begin{bmatrix} -X_d\, i^r_{dso} + X_{md}\, i'^r_{fdo} + X_{md}\, i'^r_{kd} \\ X_q\, i^r_{qso} - X_{mq}\, i'^r_{kq1o} - X_{mq}\, i'^r_{kq2o} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \omega_b \end{bmatrix}$$

In general, it is convenient for use to replace $\begin{bmatrix} v_{qs}^{r} \\ v_{ds}^{r} \end{bmatrix}$ by $\begin{bmatrix} v_{qs}^{e} \\ v_{ds}^{e} \end{bmatrix}$. By employing the approximations that $\cos\delta\delta = 1$ $\sin\delta\delta = \delta\delta$, the transformation (7.3-25) is still valid in this case and the formulas after the transformation are similar to (7.3-31) in the text.

7-5. $\omega_e = \frac{1}{2}\omega_b = \text{constant}$, $i_{as} = i_{bs} = i_{cs} = 0$

$\therefore$ $i_{qs}^{r} = i_{ds}^{r} = 0$ $\quad V_{kq1}^{\prime r} = v_{kq2}^{\prime r} = v_{kd}^{\prime r} = 0$

Then we can form the equations

$$\begin{cases} v_{kq1}^{\prime r} = \left(r_{kq1}^{\prime} + \frac{p}{\omega_b}X_{kq1}^{\prime}\right) i_{kq1}^{\prime r} + \frac{p}{\omega_b}X_{mq}\, i_{kq2}^{\prime r} = 0 & i_{kq1}^{\prime r}(0) = 0 \\ v_{kq2}^{\prime r} = \left(r_{kq2}^{\prime} + \frac{p}{\omega_b}X_{kq2}^{\prime}\right) i_{kq2}^{\prime r} + \frac{p}{\omega_b}X_{mq}\, i_{kq1}^{\prime r} = 0 & i_{kq2}^{\prime r}(0) = 0 \end{cases}$$

The solution for these equations is $\begin{cases} i_{kq1}^{\prime r}(t) = 0 \\ i_{kq2}^{\prime r}(t) = 0 \end{cases}$.

In this case, we can select $i_{kd}^{\prime r}$ and $i_{fd}^{\prime r}$ as state variables. The state equations are

$$\begin{cases} \frac{p}{\omega_b}X_{kd}^{\prime}\, i_{kd}^{\prime r} + \frac{p}{\omega_b}X_{md}\, i_{fd}^{\prime r} + r_{kd}^{\prime}\, i_{kd}^{\prime r} = 0 & i_{kd}^{\prime r}(0) = 0 \\ \frac{p}{\omega_b}X_{md}\, i_{kd}^{\prime r} + \frac{p}{\omega_b}X_{fd}^{\prime}\, i_{fd}^{\prime r} + r_{fd}^{\prime}\, i_{fd}^{\prime r} = V_{fd}^{\prime r} & i_{fd}^{\prime r}(0) = i_{fd0}^{\prime r} \end{cases}$$

The output equations are formed in the following way.

Solving above equations for $\frac{p}{\omega_b} i_{kd}^{\prime r}$ and $\frac{p}{\omega_b} i_{fd}^{\prime r}$ yields

$$\frac{p}{\omega_b} i_{kd}^{\prime r} = \frac{r_{fd}' X_{md} i_{fd}^{\prime r} - r_{kd}' X_{fd}' i_{kd}^{\prime r} - X_{md} V_{fd}^{\prime r}}{X_{kd}' X_{fd}' - X_{md}^2}$$

$$\frac{p}{\omega_b} i_{fd}^{\prime r} = \frac{r_{kd}' X_{md} i_{kd}^{\prime r} - r_{fd}' X_{kd}' i_{fd}^{\prime r} + X_{kd}' V_{fd}^{\prime r}}{X_{kd}' X_{fd}' - X_{md}^2}$$

$$v_{ds}^r = -\frac{\omega_r}{\omega_b} X_{mq} (i_{kq1}^{\prime r} + i_{kq2}^{\prime r}) + \frac{p}{\omega_b} X_{md} (i_{kd}^{\prime r} + i_{fd}^{\prime r})$$

$$= \frac{(-r_{kd}' X_{lfd}' i_{kd}^{\prime r} - r_{fd}' X_{lkd}' i_{fd}^{\prime r} + X_{lkd}' V_{fd}^{\prime r}) X_{md}}{X_{kd}' X_{fd}' - X_{md}^2}$$

$$v_{qs}^r = \frac{\omega_r}{\omega_b} X_{md} (i_{kd}^{\prime r} + i_{fd}^{\prime r}) = \frac{X_{md}}{2} (i_{kd}^{\prime r} + i_{fd}^{\prime r})$$

Then, for this problem, $\omega_r = \omega_e =$ constant

$$v_s = \sqrt{v_{ds}^{r2} + v_{qs}^{r2}}\,.$$

7-7 (a)

$$\frac{\Delta\omega_r}{\omega_b} = \bar{C}\bar{X} \qquad \bar{C} = [0\ 0\ 0\ 0\ 1]$$

$$\bar{u} = [0\ 0\ 0\ 0\ \Delta T_L]^T = G\,\Delta T_L\ , \quad G = [0\ 0\ 0\ 0\ 1]^T$$

Then $\frac{\Delta\omega_r}{\omega_b}/\Delta T_L = \bar{C}(s\bar{I}-\bar{A})^{-1}\bar{B}\bar{G}$

where $\bar{A}$, $\bar{B}$ are given by (7.3-17) and (7.3-18)

(b) $P_e = T_e \cdot \omega_r \cdot (\frac{2}{P})$

$$\Delta P_e = (\frac{2}{P})\omega_{r0}\,\Delta T_e + (\frac{2}{P})T_{e0}\,\Delta\omega_r$$

In per unit system $\Delta P_e = \frac{\omega_{r0}}{\omega_b}\Delta T_e + T_{e0}\frac{\Delta\omega_r}{\omega_b}$

where $T_{e0} = X_M(i^e_{qs0}\,i'^e_{dr0} - i^e_{ds0}\,i'^e_{qr0})$

$$\Delta T_e = X_M[\,i'^e_{dr0}\quad -i'^e_{qr0}\quad -i^e_{ds0}\quad i^e_{qs0}\quad 0\,]\,\bar{X}$$

$$\frac{\Delta\omega_r}{\omega_b} = [0\ 0\ 0\ 0\ 1]\,\bar{X}$$

$\therefore\ \Delta P_e = \bar{C}\bar{X}$

$$\bar{C} = X_M\left[\frac{\omega_{r0}}{\omega_b}i'^e_{dr0}\quad -\frac{\omega_{r0}}{\omega_b}i'^e_{qr0}\quad -\frac{\omega_{r0}}{\omega_b}i^e_{ds0}\quad \frac{\omega_{r0}}{\omega_b}i^e_{qs0}\quad i^e_{qs0}i'^e_{dr0} - i^e_{ds0}i'^e_{qr0}\right]$$

$$\bar{u} = \bar{G}\,\Delta V_s \qquad \bar{G} = [\cos\alpha\ \ -\sin\alpha\ \ 0\ \ 0\ \ 0]^T$$

Then $\Delta P_e/\Delta V_s = \bar{C}(s\bar{I}-\bar{A})^{-1}\bar{B}\hat{G} \qquad (\alpha = \theta_{ev}(0) - \theta_e(0))$

7-8 (a)

$$\frac{\Delta\omega_r}{\omega_b} = \bar{C}\bar{X} \qquad \bar{C} = [0\ 0\ 0\ 0\ 0\ 0\ 1\ 0]$$

$$\bar{u} = \bar{G}\,\Delta T_I \qquad \bar{G} = [0\ 0\ 0\ 0\ 0\ 0\ 1\ 0]^T$$

Then $\dfrac{\Delta\omega_r}{\omega_b}\Big/\Delta T_I = \bar{C}\,(s\bar{I}-\bar{A})^{-1}\bar{B}\bar{G}$

where $\bar{A}$ and $\bar{B}$ are given by (7.3-46) and (7.3-47).

(b) In the per unit system $\Delta P_e = \dfrac{\omega_{ro}}{\omega_b}\Delta T_e + T_{eo}\dfrac{\Delta\omega_{ro}}{\omega_b}$

where $T_{eo} = (\psi^r_{dso}\, i^r_{qso} - \psi^r_{qso}\, i^r_{dso})$

$$= X_{md}(-i^r_{dso} + i'^r_{fdo})\, i^r_{qso} - X_{mq}(-i^r_{qso})\, i^r_{dso}$$

$\therefore \Delta P_e = \bar{C}\bar{X}$, $\quad \bar{X} = \left[\Delta i^r_{qs},\ \Delta i^r_{ds},\ \Delta i'^r_{kq1},\ \Delta i'^r_{kq2},\ \Delta i'^r_{fd},\ \Delta i'^r_{kd},\ \dfrac{\Delta\omega_r}{\omega_b},\ \Delta\delta\right]^T$

$$\bar{C} = \Big[X_{mq}\, i^r_{dso} \quad -X_{md}(i^r_{dso} - i'^r_{fdo}) \quad -X_{md}\, i^r_{qso} + X_{mq}\, i^r_{qso} \quad -X_{mq}\, i^r_{dso}$$

$$-X_{mq}\, i^r_{dso} \quad X_{md}\, i^r_{qso} \quad X_{md}\, i^r_{qso} \quad T_{eo} \quad 0\Big]$$

$$\bar{u} = \bar{G}\,\Delta V_s \qquad \bar{G} = [\cos\alpha\ \ -\sin\alpha\ \ 0\ \ 0\ \ 0\ 0\ 0\ 0]^T$$

Then $\Delta P_e/\Delta V_s = \bar{C}\,(s\bar{I}-\bar{A})^{-1}\bar{B}\bar{G}$.

where $\bar{A} = \bar{E}^{-1}\bar{F}$ and $\bar{B} = E^{-1}$. $\bar{E}$ and $\bar{F}$ are given by (7B-4) and (7B-5)

(c) $i_s^2 = i_{qs}^{r\,2} + i_{ds}^{r\,2}$

$$\therefore \quad \Delta i_s = \frac{i_{qso}^{r}}{i_{so}} \Delta i_{qs}^{r} + \frac{i_{dso}^{r}}{i_{so}} \Delta i_{ds}^{r} = \bar{C}\,\bar{X}$$

where $\bar{C} = \left[\frac{i_{qso}^{r}}{i_{so}} \quad \frac{i_{dso}^{r}}{i_{so}} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \right]$

$$\bar{X} = \left[\Delta i_{qs}^{r} \;\; \Delta i_{ds}^{r} \;\; \Delta i_{kq1}^{\prime r} \;\; \Delta i_{kq2}^{\prime r} \;\; \Delta i_{fd}^{\prime r} \;\; \Delta i_{kd}^{\prime r} \;\; \frac{\Delta\omega_r}{\omega_b} \;\; \Delta\delta \right]^T$$

$\bar{u} = \bar{G}\,\Delta e_{xfd}^{\prime r} \qquad \bar{G} = [\,0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0\,]^T$

$$\frac{\Delta i_s}{\Delta e_{xfd}^{\prime r}} = \bar{C}\,(s\bar{I} - \bar{A})^{-1}\,\bar{B}\,\bar{G}$$

where $\bar{A} = \bar{E}^{-1}\bar{F}$, $\bar{B} = \bar{E}^{-1}$. $\bar{E}$ and $\bar{F}$ are given by (7B-4) and (7B-5).

7-9 From Problem 7-1, the state equations using flux linkages per second as state variables can be written

as $$p\bar{X} = \bar{A}\bar{X} + \bar{B}\bar{u}$$

where $\bar{X} = [\Delta\psi_{qs}^{e} \;\; \Delta\psi_{ds}^{e} \;\; \Delta\psi_{qr}^{\prime e} \;\; \Delta\psi_{dr}^{\prime e} \;\; \frac{\Delta\omega_r}{\omega_b}]$

$$\bar{B} = \begin{bmatrix} \omega_b & 0 & 0 & 0 & 0 \\ 0 & \omega_b & 0 & 0 & 0 \\ 0 & 0 & \omega_b & 0 & 0 \\ 0 & 0 & 0 & \omega_b & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2H} \end{bmatrix}$$

$$\bar{u} = [\Delta V_{qs}^{e} \;\; \Delta V_{ds}^{e} \;\; \Delta V_{qr}^{\prime e} \;\; \Delta V_{dr}^{\prime e} \;\; \Delta T_L]^T$$

$\bar{A}$ can be found in the solution of the problem 7-1.

$$\Delta T_e = \frac{X_M}{XX}[\psi_{dro}^{\prime e} \;\; -\psi_{qro}^{\prime e} \;\; -\psi_{dso}^{e} \;\; \psi_{qso}^{e} \;\; 0]\,\bar{X} = \bar{C}\bar{X}$$

where $XX = X_{ss}X_{rr}^{\prime} - X_M^2$

$\bar{u} = \bar{G}\,\Delta V_s \qquad \bar{G} = [\cos\alpha \;\; -\sin\alpha \;\; 0 \;\; 0 \;\; 0]^T$

Then $$\Delta T_e/\Delta V_s = \bar{C}(s\bar{I} - \bar{A})^{-1}\bar{B}\,\bar{G}$$

7-10. From Problem 7-2, we have foun the state equations Using flux linkages per second as state variables as

$$p\bar{X} = \hat{A}\bar{X} + \bar{B}\bar{u}$$

where $\bar{X} = \begin{bmatrix} \Delta\psi_{qds}^{r} \\ \Delta\psi_{rr} \end{bmatrix}$, $\bar{u} = \begin{bmatrix} \Delta V_{qds}^{e} \\ \Delta V_{rr} \end{bmatrix}$,

$\bar{A} = \begin{bmatrix} W_0 & Y_0 \\ Q_0 & S_0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B_{s0} & 0 \\ 0 & B_{r0} \end{bmatrix}$.

$$\Delta T_e = \Big[(Y_{11}^{d}-Y_{11}^{q})\psi_{ds0}^{r} - Y_{12}^{d}\psi_{fd0}^{\prime r} - Y_{13}^{d}\psi_{kd0}^{\prime r} \quad (Y_{11}^{d}-Y_{11}^{q})\psi_{qs0}^{r} + Y_{12}^{q}\psi_{kq10}^{\prime r} + Y_{13}^{q}\psi_{kq20}^{\prime r}$$

$$Y_{12}^{q}\psi_{ds0}^{r} \quad Y_{13}^{q}\psi_{ds0}^{r} \quad -Y_{12}^{d}\psi_{qs0}^{r} \quad -Y_{13}^{d}\psi_{qs0}^{r} \quad 0 \quad 0 \Big]\bar{X}$$

$$= \bar{C}\bar{X}.$$

$\bar{u} = \bar{G}\,\Delta V_s \qquad G = [\cos\alpha \;\; -\sin\alpha \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T$

Then $\Delta T_e / \Delta V_s = \bar{C}(S\bar{I}-\bar{A})^{-1}\bar{B}\bar{G}$.

7-11. From Problem 7-3, The state equation can be written in fundamental form:

$$P\bar{X} = \bar{E}^{-1}\bar{F}\bar{X} + \bar{E}^{-1}\bar{B}_1\Delta W_e \qquad (\bar{u}=0 \text{ for a balanced input voltages})$$

$$= \bar{A}\bar{X} + \bar{B}\Delta W_e$$

where $\bar{A} = \bar{E}^{-1}\bar{F}$, $\bar{B} = \bar{E}^{-1}\bar{B}_1$.

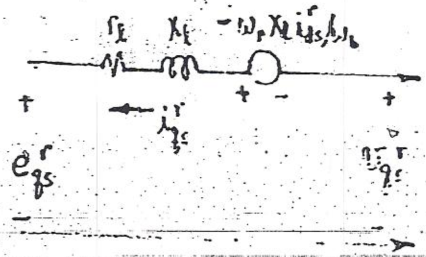
$$\frac{\Delta W_r}{\omega_b} = \bar{C}\bar{X} = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 1]\bar{X}$$

Then $\dfrac{\Delta W_r}{\Delta W_e} = W_b \dfrac{\Delta W_r}{W_b} / \Delta W_e = \bar{C}(S\bar{I}-\bar{A})^{-1}\bar{B}\cdot\omega_b$.

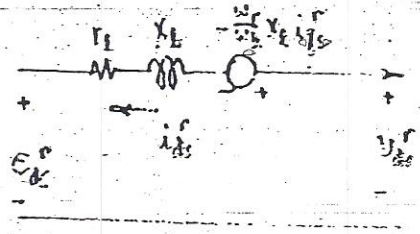
7-12. The solution formula is exactly same as problem 7-11 except using different matrices $\bar{A}$, $\bar{B}$, $\bar{E}$, $\bar{F}$, $\bar{B}_1$ and

$$\bar{C} = [\,0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\,].$$

7-13 Referring to Problem 3-4, we can find the equivalent circuits as shown in the figures,



where
$$\begin{bmatrix} e_{qs}^r \\ e_{ds}^r \\ 0 \end{bmatrix} = K_s^r \begin{bmatrix} e_{ga} \\ e_{gb} \\ E_{gc} \end{bmatrix}$$



e_{ga}, e_{gb}, and E_{gc} are bus voltages.

$$e_{qs}^r = v_{qs}^r - \frac{\omega_r}{\omega_b} X_\ell i_{ds}^r - \left(r_\ell + \frac{p}{\omega_b} X_\ell\right) i_{qs}^r$$

$$e_{ds}^r = v_{ds}^r + \frac{\omega_r}{\omega_b} X_\ell i_{qs}^r - \left(r_\ell + \frac{p}{\omega_b} X_\ell\right) i_{ds}^r$$

$$\Delta v_{qs}^r = \frac{\omega_{r0}}{\omega_b} X_\ell \Delta i_{ds}^r + \left(r_\ell + \frac{p}{\omega_b} X_\ell\right) \Delta i_{qs}^r + \frac{X_\ell}{\omega_b} i_{ds0}^r \Delta\omega_r$$

$$\Delta v_{ds}^r = -\frac{\omega_{r0}}{\omega_b} X_\ell \Delta i_{qs}^r + \left(r_\ell + \frac{p}{\omega_b} X_\ell\right) \Delta i_{ds}^r - X_\ell i_{qs0}^r \frac{\Delta\omega_r}{\omega_b}$$

$$\because \quad v_s^2 = v_{qs}^{r\,2} + v_{ds}^{r\,2}$$

$$\therefore \quad \Delta v_s = \frac{v_{qs0}^r}{v_{s0}} \Delta v_{qs}^r + \frac{v_{ds0}^r}{v_{s0}} \Delta v_{ds}^r = \left[\frac{v_{qs0}^r}{v_{s0}}\left(r_\ell + \frac{p}{\omega_b} X_\ell\right) - \frac{v_{ds0}^r}{v_{s0}} \frac{\omega_{r0}}{\omega_b} X_\ell\right] \Delta i_{qs}^r$$
$$+ \left[\frac{v_{qs0}^r}{v_{s0}} \frac{\omega_{r0}}{\omega_b} X_\ell + \frac{v_{ds0}^r}{v_{s0}}\left(r_\ell + \frac{p}{\omega_b} X_\ell\right)\right] \Delta i_{ds}^r + \left(\frac{v_{qs0}^r}{v_{s0}} X_\ell i_{ds0}^r - \frac{v_{ds0}^r}{v_{s0}} X_\ell i_{qs0}^r\right) \frac{\Delta\omega_r}{\omega_b}$$

or $\Delta v_s = \bar{C}\,\bar{X}$

$$\bar{C} = [\,C_1 \;\; C_2 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; C_7 \;\; 0\,]$$

$$C_1 = \left[\frac{v_{qso}^r}{v_{so}}\left(r_e + \frac{s}{\omega_b}x_l\right) - \frac{v_{dso}^r}{v_{so}}\frac{\omega_{ro}}{\omega_b}x_l\right], \quad C_2 = \left[\frac{v_{qso}^r}{v_{so}}\frac{\omega_{ro}}{\omega_b}x_l + \frac{v_{dso}^r}{v_{so}}\left(r_e + \frac{s}{\omega_b}x_l\right)\right]$$

$$C_7 = \frac{v_{qso}^r}{v_{so}}x_l i_{dso}^r - \frac{v_{dso}^r}{v_{so}}x_l i_{qso}^r$$

$$\bar{X} = \left[\,\Delta i_{qs}^r \;\; \Delta i_{ds}^r \;\; \Delta i'^r_{kq1} \;\; \Delta i'^r_{kq2} \;\; \Delta i'^r_{fd} \;\; \Delta i'^r_{kd} \;\; \frac{\Delta\omega_r}{\omega_b} \;\; \Delta\delta\,\right]^T$$

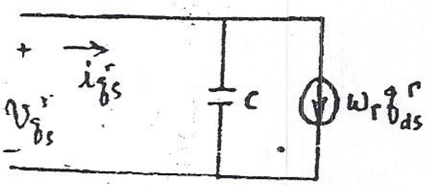
$$\bar{u} = \bar{G}\,\Delta e'^r_{xfd} \qquad \bar{G} = [\,0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0\,]$$

$$\frac{\Delta v_s}{\Delta e'^r_{xfd}} = \bar{C}\,(s\bar{I} - \bar{A})^{-1}\,\bar{B}\,\bar{G}$$

where $\bar{A} = \bar{E}^{-1}\bar{F}$, $\bar{B} = \bar{E}^{-1}$. $\bar{E}$ and $\bar{F}$ are given by (7.3-23) and (7.3-24)

7-14. The solution is the same as that of Problem 7-13 if $x_l = 0$.

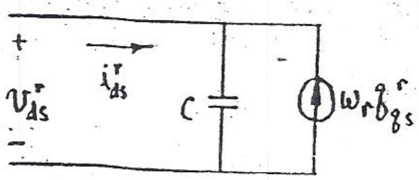
7-15. Referring to Problem 3-4, we can find the equivalent circuits as shown in the figures.



$$v_{qs}^r = \frac{1}{cp}\left(i_{qs}^r - \omega_r q_{ds}^r\right)$$

$$= \frac{1}{cp}\left(i'^r_{qs} - \frac{\omega_r}{\omega_b}B_c v_{ds}^r\right)$$

$$v_{ds}^r = \frac{1}{Cp}\left(i_{ds}^r + \omega_r q_{qs}^r\right)$$



$$= \frac{1}{Cp}\left(i_{ds}^r + \frac{\omega_r}{\omega_b} B_c v_{qs}^r\right)$$

$$\Delta v_{qs}^r = \frac{1}{Cp}\left(\Delta i_{qs}^r - \frac{\omega_{ro}}{\omega_b} B_c \Delta v_{ds}^r - \frac{v_{dso}^r}{\omega_b} B_c \Delta\omega_r\right)$$

$$\Delta v_{ds}^r = \frac{1}{Cp}\left(\Delta i_{ds}^r + \frac{\omega_{ro}}{\omega_b} B_c \Delta v_{qs}^r + B_c v_{qso}^r \frac{\Delta\omega_r}{\omega_b}\right)$$

$$= \frac{1}{Cp}\left(\Delta i_{ds}^r + \frac{\omega_{ro}}{\omega_b} B_c \frac{1}{Cp}\Delta i_{qs}^r - \frac{1}{Cp}\frac{\omega_{ro}}{\omega_b} v_{dso}^r B_c^2 \frac{\Delta\omega_r}{\omega_b}\right) - \left(\frac{1}{Cp}\right)^2 \left(\frac{\omega_{ro}}{\omega_b}\right)^2 B_c^2 \Delta v_{ds}^r$$

$$\therefore \Delta v_{ds}^r = \frac{1}{\left(\frac{p}{\omega_b}\right)^2 + \left(\frac{\omega_{ro}}{\omega_b}\right)^2}\left[X_c \frac{p}{\omega_b}\Delta i_{ds}^r + \frac{\omega_{ro}}{\omega_b}\Delta i_{qs}^r X_c - \frac{\omega_{ro}}{\omega_b} v_{dso}^r \frac{\Delta\omega_r}{\omega_b} + \frac{p}{\omega_b} v_{qso}^r \frac{\Delta\omega_r}{\omega_b}\right]$$

$$\Delta v_{qs}^r = \frac{1}{\left(\frac{p}{\omega_b}\right)^2 + \left(\frac{\omega_{ro}}{\omega_b}\right)^2}\left[X_c \frac{p}{\omega_b}\Delta i_{qs}^r - \frac{\omega_{ro}}{\omega_b} X_c \Delta i_{ds}^r - \frac{\omega_{ro}}{\omega_b} v_{qso}^r \frac{\Delta\omega_r}{\omega_b} - \frac{p}{\omega_b} v_{dso}^r \frac{\Delta\omega_r}{\omega_b}\right]$$

$$\Delta v_s = \frac{v_{qso}^r}{v_{so}}\Delta v_{qs}^r + \frac{v_{dso}^r}{v_{so}}\Delta v_{ds}^r$$

$$= \frac{1}{\left(\frac{p}{\omega_b}\right)^2 + \left(\frac{\omega_{ro}}{\omega_b}\right)^2}\Bigg[\left(\frac{v_{qso}^r}{v_{so}} X_c \frac{p}{\omega_b} + \frac{v_{dso}^r}{v_{so}} X_c \frac{\omega_{ro}}{\omega_b}\right)\Delta i_{qs}^r + \left(\frac{v_{dso}^r}{v_{so}} X_c \frac{p}{\omega_b} -\right.$$

$$\left.\frac{v_{qso}^r}{v_{so}} X_c \frac{\omega_{ro}}{\omega_b}\right)\Delta i_{ds}^r + \frac{p}{\omega_b}\left(\frac{v_{dso}^r}{v_{so}} v_{qso}^r - \frac{v_{qso}^r}{v_{so}} v_{dso}^r\right)\frac{\Delta\omega_r}{\omega_b}$$

$$- \frac{\omega_{ro}}{\omega_b}\left(v_{qso}^r \frac{v_{qso}^r}{v_{so}} + \frac{v_{dso}^r}{v_{so}} v_{dso}^r\right)\frac{\Delta\omega_r}{\omega_b}\Bigg]$$

or $\Delta v_s = \bar{C}\bar{X}$

$$\bar{C} = \left[C_1 \quad C_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad C_7 \quad 0\right]$$

$$C_1 = \frac{1}{\left(\frac{s}{\omega_b}\right)^2 + \left(\frac{\omega_{ro}}{\omega_b}\right)^2}\left(\frac{v_{qso}^r}{v_{so}} X_c \frac{s}{\omega_b} + \frac{v_{dso}^r}{v_{so}} X_c \frac{\omega_{ro}}{\omega_b}\right)$$

$$C_2 = \frac{1}{\left(\frac{s}{\omega_b}\right)^2 + \left(\frac{\omega_{r0}}{\omega_b}\right)^2} \left(\frac{v_{ds0}^r}{v_{s0}} X_E \frac{s}{\omega_b} - \frac{v_{qs0}^r}{v_{s0}} X_E \frac{\omega_{r0}}{\omega_b} \right)$$

$$C_7 = \frac{\omega_{r0}}{\omega_b} v_{s0}$$

$$\bar{X} = \left[\Delta i_{qs}^r \;\; \Delta i_{ds}^r \;\; \Delta i_{kq1}^{\prime r} \;\; \Delta i_{kq2}^{\prime r} \;\; \Delta i_{fd}^{\prime r} \;\; \Delta i_{kd}^{\prime r} \;\; \frac{\Delta \omega_r}{\omega_b} \;\; \Delta\delta \right]^T$$

$$\bar{u} = \bar{G}\, \Delta e_{xfd}^{\prime r} \qquad \bar{G} = [\,0\;0\;0\;0\;1\;0\;0\;0\,]$$

$$\frac{\Delta v_s}{\Delta e_{xfd}^{\prime r}} = \bar{C}\,(s\bar{I} - \bar{A})^{-1} \bar{B}\, \bar{G}$$

where $\bar{A} = \bar{E}^{-1}\bar{F}$, $\bar{B} = \bar{E}^{-1}$. $\bar{E}$ and $\bar{F}$ are given by (7.3-23) and (7.3-24).

Chapter 8

8-1 The rotor voltage equations in the synchronously rotating reference fram are

$$v'^e_{qr} = r'_r i'^e_{qr} + \left(\frac{\omega_e-\omega_r}{\omega_b}\right)\psi'^e_{dr} + \frac{p}{\omega_b}\psi'^e_{qr}$$

$$v'^e_{dr} = r'_r i'^e_{dr} - \left(\frac{\omega_e-\omega_r}{\omega_b}\right)\psi'^e_{qr} + \frac{p}{\omega_b}\psi'^e_{dr}$$

Neglecting the electrical transients yields

$$v'^e_{qr} = r'_r i'^e_{qr} + \left(\frac{\omega_e-\omega_r}{\omega_b}\right)\psi'^e_{dr}$$

$$v'^e_{dr} = r'_r i'^e_{dr} - \left(\frac{\omega_e-\omega_r}{\omega_b}\right)\psi'^e_{qr}$$

Or in matrix form

$$\begin{bmatrix} v'^e_{qr} \\ v'^e_{dr} \end{bmatrix} = \begin{bmatrix} r'_r & 0 \\ 0 & r'_r \end{bmatrix}\begin{bmatrix} i'^e_{qr} \\ i'^e_{dr} \end{bmatrix} + \begin{bmatrix} 0 & \frac{\omega_e-\omega_r}{\omega_b} \\ -\frac{\omega_e-\omega_r}{\omega_b} & 0 \end{bmatrix}\begin{bmatrix} \psi'^e_{qr} \\ \psi'^e_{dr} \end{bmatrix}$$

In the arbitrary reference frame

$$\begin{bmatrix} v'_{qr} \\ v'_{dr} \end{bmatrix} = ({}^{e}K)^{-1}\begin{bmatrix} r'_r & 0 \\ 0 & r'_r \end{bmatrix}{}^{e}K\begin{bmatrix} i'_{qr} \\ i'_{dr} \end{bmatrix} + ({}^{e}K)^{-1}\begin{bmatrix} 0 & \frac{\omega_e-\omega_r}{\omega_b} \\ -\frac{\omega_e-\omega_r}{\omega_b} & 0 \end{bmatrix}{}^{e}K\begin{bmatrix} \psi'_{qr} \\ \psi'_{dr} \end{bmatrix}$$

$$= \begin{bmatrix} r'_r & 0 \\ 0 & r'_r \end{bmatrix}\begin{bmatrix} i'_{qr} \\ i'_{dr} \end{bmatrix} + \begin{bmatrix} 0 & \frac{\omega_e-\omega_r}{\omega_b} \\ -\frac{\omega_e-\omega_r}{\omega_b} & 0 \end{bmatrix}\begin{bmatrix} \psi'_{qr} \\ \psi'_{dr} \end{bmatrix}.$$

From (8.2-11)

$$\tilde{V}_{qs} = r_s\tilde{I}_{qs} + \frac{\omega_e}{\omega_b}\tilde{\Psi}_{ds} = \left(r_s + j\frac{\omega_b}{\omega_b}X_{ls}\right)\tilde{I}_{qs} + j\frac{\omega_e}{\omega_b}X_M\left(\tilde{I}_{qs} + \tilde{I}'_{ds}\right)$$

here the relation $\tilde{F}_{ds} = j\tilde{F}_{qs}$ has been used

In the same way, from the rotor equation derived above,

$$\tilde{V}'_{qr} = r'_r \tilde{I}'_{qr} + \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\tilde{\psi}'_{qr} = \left(r'_r + j\frac{\omega_e-\omega_r}{\omega_b}X_{ls}\right)\tilde{I}'_{qr}$$

$$+ j\,\frac{\omega_e-\omega_r}{\omega_b}X_M\left(\tilde{I}_{qs} + \tilde{I}'_{qr}\right)$$

$$\tilde{V}'_{qr}/s = \left(r'_r/s + j\frac{\omega_e}{\omega_b}X_{ls}\right)\tilde{I}'_{qr} + j\frac{\omega_e}{\omega_b}X_M\left(\tilde{I}_{qs} + \tilde{I}'_{qr}\right)$$

By the relations $\tilde{F}_{qs} = \tilde{F}_{as}$ and $\tilde{F}'_{qr} = \tilde{F}'_{ar}$; the equivalent circuit given in Fig. 4.9-1 is obtained.

8-2 The solution is given by (5.9-1) – (5.9-19).

8-3 The flux linkages in rotor loops are conservative; hence

$\lambda'_{qr}(0^+) = 0$ and $\lambda'_{dr}(0^+) = 0$

$\because\ T_e(0^+) = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda'_{qr} i'_{dr} - \lambda'_{dr} i'_{qr}\right)$

$\therefore\ T_e(0^+) = 0.$

8-5 (a)

$$\frac{\Delta\omega_r}{\omega_b} = [0 \; 0 \; 1]\bar{X} = \bar{C}\bar{X} \qquad \bar{C} = [0 \; 0 \; 1]$$

$$\bar{u}_1 = \bar{G}\,\Delta T_L \qquad \bar{G} = [0 \; 0 \; 1]^T$$

$$\therefore \quad \frac{\Delta\omega_r}{\omega_b} / \Delta T_L = \bar{C}(S\bar{I} - \bar{A})^{-1}\bar{B}_1\bar{G}$$

where $\bar{A}$, $\bar{B}_1$ are given by (8.5-10)

(b) In the per unit system $\Delta P_e = \frac{\omega_{ro}}{\omega_b}\Delta T_e + T_{eo}\frac{\Delta\omega_{ro}}{\omega_b}$

where $T_{eo} = \frac{X_M}{D}(\psi^e_{qso}\psi'^e_{dro} - \psi'^e_{qro}\psi^e_{dso})$ (see problem 7-1)

$$D = X_{ss}X'_{rr} - X_M^2$$

$$\Delta T_e = \bar{C}_1\Delta\bar{\psi}^e_{qds} + \bar{C}_2\Delta\bar{\psi}_{rr}$$

where $\bar{C}_1 = \frac{X_M}{D}\left[\psi'^e_{dro} \;\; -\psi'^e_{qro}\right]$

$$\bar{C}_2 = \frac{X_M}{D}\left[-\psi^e_{dso} \;\; \psi^e_{qso} \;\; 0\right]$$

$$\Delta T_e = (\bar{C}_2 - \bar{C}_1\bar{W}_K^{-1}\bar{Y}_K)\Delta\bar{\psi}_{rr} + \bar{C}_1\bar{W}_K^{-1}\Delta\bar{V}^e_{qds} \qquad \text{(see (8.5-7))}$$

Then $\Delta P_e = \left\{\frac{\omega_{ro}}{\omega_b}(\bar{C}_2 - \bar{C}_1\bar{W}_K^{-1}\bar{Y}_K) + [0 \; 0 \; T_{eo}]\right\}\Delta\bar{\psi}_{rr}$

$$+ \frac{\omega_{ro}}{\omega_b}\bar{C}_1\bar{W}_K^{-1}\Delta V^e_{qds} = \bar{C}\Delta\bar{\psi}_{rr} + \bar{D}_2\Delta V^e_{qds}.$$

$$\Delta V^e_{qds} = \bar{G}\,\Delta V_s \qquad \bar{G} = [\cos\alpha \;\; -\sin\alpha]^T$$

Hence $\dfrac{\Delta P_e}{\Delta V_s} = \left\{ \bar{C}\,(s\bar{I}-\bar{A})^{-1}\bar{B}_2 + \bar{D}_2 \right\} \bar{G}$

where $\bar{A}$ and $\bar{B}_2$ are given by (8.5-10).

8-6 (a) $\dfrac{\Delta \omega_r}{\omega_b} = [\,0\;0\;0\;0\;1\;0\,]\,\bar{X} = \bar{C}\bar{X}$

$\bar{u}_1 = [\,0\;0\;0\;0\;1\;0\,]^T \Delta T_I = \bar{G}\,\Delta T_I$

Then $\dfrac{\Delta \omega_r}{\omega_b} \Big/ \Delta T_I = \bar{C}\,(s\bar{I}-\bar{A})^{-1}\bar{B}_1\bar{G}$

where $\bar{A}$ and $\bar{B}_1$ are given by (8.5-38)

(b) $\Delta P_e = \dfrac{\omega_{ro}}{\omega_b}\Delta T_e + T_{eo}\dfrac{\Delta \omega_r}{\omega_b}$ (In the per-unit system)

where $T_{eo} = (a_{11}-b_{11})\,\psi^r_{qso}\,\psi^r_{dso} + \psi^r_{dso}\,(a_{12}\psi'^r_{kq1o} + a_{13}\psi'^r_{kq2o})$

$- \psi^r_{qso}\,(b_{12}\psi'^r_{fdo} + b_{13}\psi'^r_{kdo})$ (see (5.6-5))

$\Delta T_e = \bar{C}_1\,\Delta\psi^r_{qds} + \bar{C}_2\,\Delta\psi_{rr}$

where $\bar{C}_1 = [A_{11}\;A_{12}]$, $\bar{C}_2 = [\,A_{13}\;A_{14}\;A_{15}\;A_{16}\;0\;0\,]$

A_{1i} $(i = 1, 2, \cdots 6)$ are given by (8.5-24) – (8.5-29).

$\Delta T_e = (\bar{C}_2 - \bar{C}_1\,\bar{W}_k^{-1}\bar{Y}_{lk})\,\Delta\bar{\psi}_{rr} + \bar{C}_1\,\bar{W}_k^{-1}\,\bar{T}\,\Delta\bar{V}^e_{qds}$.

(see (8.5-35))

Then $T_e = \left\{ \frac{\omega_{ro}}{\omega_b} \left(\bar{C}_2 - \bar{C}_1 \bar{W}_k^{-1} \bar{Y}_k \right) + \left[0\ 0\ 0\ 0\ T_{eo}\ 0 \right] \right\} \Delta \bar{\Psi}_{rr}$

$$+ \frac{\omega_{ro}}{\omega_b} \bar{C}_1 \bar{W}_k^{-1} \bar{T} \Delta V_{qds}^e = \bar{C} \Delta \bar{\Psi}_{rr} + \bar{D}_2 \Delta V_{qds}^e$$

$$\Delta V_{qds}^e = \bar{G} \Delta V_s \qquad \bar{G} = \left[\cos\alpha \quad -\sin\alpha \right]^T$$

Hence $\dfrac{\Delta P_e}{\Delta V_s} = \left\{ \bar{C} \left(s\bar{I} - \bar{A} \right)^{-1} \bar{B}_2 + \bar{D}_2 \right\} \bar{G}$

where $\bar{A}$ and $\bar{B}_2$ are given by (8.5-38).

9-1 $V_{as} = e_{ga} + V_{gn}$

$V_{bs} = e_{gb} + V_{gn}$

$V_{cs} = e_{gc} + V_{gn}$

Assume $\theta(0) = 0$, $\bar{K}_s^s = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\sqrt{3}/2 & \sqrt{3}/2 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$\therefore V_{qs}^s = \frac{2}{3}(V_{as} - \frac{1}{2}V_{bs} - \frac{1}{2}V_c = \frac{2}{3}(e_{ga} - \frac{1}{2}e_{gb} - \frac{1}{2}e_{gc})$

$V_{ds}^s = \frac{2}{3}(-\sqrt{3}/2\, V_{bs} + \frac{\sqrt{3}}{2}V = \frac{1}{\sqrt{3}}(-e_{gb} + e_{gc})$

9-2. Assume $v_{as} = A\cos(\omega_e t +$ $v_{bs} = B\cos(\omega_e t + \theta_2)$

$v_{ac} = C\cos(\omega_e t + \theta_3)$.

By the solution of Problem 9

$v_{qs}^s = \frac{2}{3}(v_{as} - \frac{1}{2}v_{bs})$

$v_{ds}^s = \frac{1}{\sqrt{3}}(-v_{bs} + v_{cs})$

$v_{qs}^s = \frac{2}{3}\mathrm{Re}\{(Ae^{j\theta_1} - \frac{1}{2}Ce^{j\theta_3})e^{j\omega_e t}\}$

$= \frac{2}{3}\mathrm{Re}\{[(A\cos\theta_1 - \frac{1}{2}C\cos\theta_3) + j(A\sin\theta_1$

$- \frac{1}{2}B\sin\theta_1 - \frac{1}{2}C\sin\theta_3)]$

$= V_q\cos(\omega_e t + \alpha$

Where $V_q = \frac{2}{3}\sqrt{(A\cos\theta_1 - \frac{1}{2}B\cos\theta_2 - \frac{1}{2}C\cos\theta_3)^2 + (A\sin\theta_1 - \frac{1}{2}B\sin\theta_2 - \frac{1}{2}C\sin\theta_3)^2}$

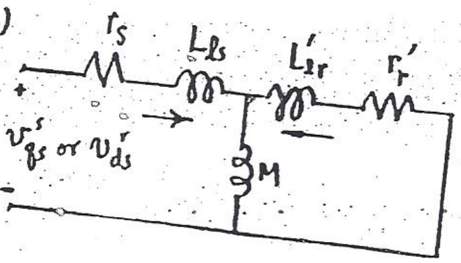
$$\alpha_q = \tan^{-1}\frac{A\sin\theta_1 - \frac{1}{2}B\sin\theta_2 - \frac{1}{2}C\sin\theta_3}{A\cos\theta - \frac{1}{2}B\cos\theta_2 - \frac{1}{2}C\cos\theta_3}$$

Like wise $v_{ds}^s = V\cos(\omega_e t + \alpha_d)$

For the case of $\omega_r =$, the equivalent circuits for the q- and d- axis are same as shown in the figure.

Let $Z_{in} = (r_s + j\omega_e L_{ls}) + \frac{j\omega_e M(r_r' + j\omega_e L_{lr}')}{r_r' + j\omega_e(M + L_{lr}')}$

$= Z_1 \angle \varphi_1$



Then $i_{qs}^s = \frac{V_q}{Z_1} + \alpha_q - \varphi_1)$

$i_{ds}^s = \frac{V_d}{Z} + \alpha_d - \varphi_1)$

Let $\frac{j\omega_e M}{r_r' + j\omega_e(M} k \angle \varphi_2$

Then $i_{qr}'^s = (\omega_e t + \alpha_q - \varphi_1 + \varphi_2)$

$i_{dr}'^s = (\omega_e t + \alpha_d - \varphi_1 + \varphi_2)$

$T_e = (\frac{3}{2})(\frac{P}{2}) i_{ds}^s i_{qr}'^s) = (\frac{3}{2})(\frac{P}{2}) M \frac{k}{} \frac{V_q V_d}{Z_1^2}$

$[\cos(\omega_e t + \alpha_d - \varphi_1 + \varphi_2) - \cos(\omega_e t + \alpha_q - \varphi_1)\cos(\omega_e t + \alpha_q - \varphi_1 + \varphi_2)]$

$= (\frac{3}{2})(\frac{P}{2}) r \alpha_q)\sin\varphi_1 = \text{Constant}$ $\therefore |T_{e(pal)}| = 0.$

9-3 Solve the Equation (9.6-29) for

$1 \le s = \frac{\omega_e - \omega_r}{\omega_e} \le 2$ using a computer, then

$\tilde{I}^s_{qs+}$, $\tilde{I}^s_{qr+}$ and $\tilde{I}'^s_{qr-}$ are obtained. Hence the $T_{e(+)}$, $T_{e(-)}$

can be calculated by (9.5-26).

For $\frac{\omega_r}{\omega_b} = -1.0$, i.e. $S = 2$,

$\tilde{I}'^s_{qr-} = 0$. Equation (9.6-29) becomes

$$\begin{bmatrix} \tilde{E} \\ \frac{\tilde{V}'^s_{qr+}}{2} \end{bmatrix} = \begin{bmatrix} r_s + j\frac{\omega_e}{\omega_b}X_{ss} & j\frac{1}{2}\frac{\omega_e}{\omega_b}X_M \\ j\frac{\omega_e}{\omega_b}X_M & \frac{r_r'}{2} + j\frac{\omega_e}{\omega_b}X'_{rr} \end{bmatrix} \begin{bmatrix} \tilde{I}^s_{qs+} \\ \tilde{I}'^s_{qr+} \end{bmatrix} \quad \text{or}$$

$$\begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 0.0453 + j2.1195 & j1.021 \\ j2.042 & 0.0111 + j2.0742 \end{bmatrix} \begin{bmatrix} \tilde{I}^s_{qs+} \\ \tilde{I}'^s_{qr+} \end{bmatrix}$$

$\tilde{I}^s_{qs+} = 0.4494 \angle 92.60°$ $\quad \tilde{I}'^s_{qr+} = 0.4424 \angle -87.09°$

$\tilde{I}^s_{qs-} = -\tilde{I}^s_{qs+} = 0.4494 \angle -87.40°$

$T_{e(+)} = X_M [Re(j \tilde{I}^{s*}_{qs+} \tilde{I}_{qr+})] = 0.00217$

$T_{e(-)} = 0$

$|T_{e(puls)}| = X_M [Re(j \tilde{I}^s_{qs-} \tilde{I}'^s_{qr+})] = 0.0390$

9-4. $\tilde{V}^s_{qs} = \frac{2}{3}\left(\tilde{E}_{ga} - \frac{1}{2}\tilde{E}_{gb} - \frac{1}{2}\tilde{E}_{gc}\right)$ (by Problem 9-1)

$$= \frac{2}{3}\left(1\angle 0 - \frac{1}{4}\angle -\frac{2\pi}{3} - \frac{1}{2}\angle\frac{\pi}{2}\right) = \frac{2}{3}\left(\frac{9}{8} - j\frac{4-\sqrt{3}}{8}\right)$$

$$= \frac{3}{4} - j\,\frac{4-\sqrt{3}}{12} = 0.7734\angle 14.144^\circ$$

$$\tilde{V}^s_{ds} = \frac{1}{\sqrt{3}}\left(-\tilde{E}_{gb} + \tilde{E}_{gc}\right) = \frac{1}{\sqrt{3}}\left(\frac{1}{4} + j\frac{4+\sqrt{3}}{4}\right)$$

$$\begin{bmatrix} \tilde{V}^s_{qs+} \\ \tilde{V}^s_{qs-} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -j\frac{1}{2} \\ \frac{1}{2} & j\frac{1}{2} \end{bmatrix} \begin{bmatrix} \tilde{V}^s_{qs} \\ \tilde{V}^s_{ds} \end{bmatrix} = \begin{bmatrix} \frac{3+\sqrt{3}}{6} - j\frac{1}{6} \\ \frac{3-2\sqrt{3}}{12} - j\frac{2-\sqrt{3}}{12} \end{bmatrix} = \begin{bmatrix} 0.8061\angle -11.93^\circ \\ 0.04466\angle 210.0^\circ \end{bmatrix}$$

By (9.5-15), we have

$$\begin{bmatrix} \tilde{V}^s_{qs+} \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + j\frac{\omega_e}{\omega_b}X_{ss} & j\frac{\omega_e}{\omega_b}X_M \\ j\frac{\omega_e}{\omega_b}X_M & \frac{r'_r}{s} + j\frac{\omega_e}{\omega_b}X'_{rr} \end{bmatrix} \begin{bmatrix} \tilde{I}^s_{qs+} \\ \tilde{I}'^s_{qr+} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{V}^s_{qs-} \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + j\frac{\omega_e}{\omega_b}X_{ss} & j\frac{\omega_e}{\omega_b}X_M \\ j\frac{\omega_e}{\omega_b}X_M & \frac{r'_r}{2-s} + j\frac{\omega_e}{\omega_b}X'_{rr} \end{bmatrix} \begin{bmatrix} \tilde{I}^s_{qs-} \\ \tilde{I}'^s_{qr-} \end{bmatrix}$$

Solving these equations for different s, $\tilde{I}^s_{qs+}$, $\tilde{I}'^s_{qr+}$, $\tilde{I}^s_{qs-}$ and $\tilde{I}'^s_{qr-}$ can be obtained; then $T_{e(+)}$, $T_{e(-)}$ and $|T_{e(puls)}|$ are easy to be found. In a special case that $\frac{\omega_r}{\omega_b} = 1$, $s = 0$.

Then $\tilde{I}'^s_{qr+} = 0$ $\quad \tilde{I}^s_{qs+} = \dfrac{\tilde{V}^s_{qs+}}{r_s + j\frac{\omega_e}{\omega_b}X_{ss}}$.

9-5. Assume the dc voltage is applied across the B and C phase terminals, and the stator circuit is a 3-wire system. Therefore with $\theta = 0$, $f_{qs}^s = f_{as}$ and

$$v_{as} = v_{qs}^s = \frac{p}{\omega_b}\varphi_{qs}^s = X_M \frac{p}{\omega_b} i_{qr}'^s \quad (\because i_{qs}^s = 0)$$

$$v_{bs} = \frac{1}{2}e_{gb} - \frac{1}{2}e_{gc} - \frac{1}{2}v_{as} \qquad (\text{by } (9.6\text{-}21))$$

$$v_{cs} = \frac{1}{2}e_{gc} - \frac{1}{2}e_{gb} - \frac{1}{2}v_{as} \qquad (\text{by } (9.6\text{-}22))$$

Then $v_{ds}^s = \frac{1}{\sqrt{3}}(v_{cs} - v_{bs})$ (by the transformation (9.5-18))

$$= \frac{1}{\sqrt{3}}(e_{gc} - e_{gb}) = -\frac{1}{\sqrt{3}}$$

Because the circuits have only dc voltage source, the steady-state currents have only dc components.

$$\therefore \quad v_{qs}^s = \frac{p}{\omega_b} X_M i_{qr}'^s = 0$$

$$0 = v_{qs}^s = (r_s + \frac{p}{\omega_b}X_{ss}) i_{qs}^s + \frac{\omega_s}{\omega_b}X_{ss} i_{ds}^s + \frac{p}{\omega_b}X_M i_{qr}'^s + \frac{\omega_s}{\omega_b}X_M i_{dr}'^s$$

$$= r_s i_{qs}^s \qquad (\because \omega_s = 0)$$

$$\therefore \quad i_{qs}^s = 0$$

$$-\frac{1}{\sqrt{3}} = v_{ds}^s = (-\frac{\omega_s}{\omega_b}X_{ss}) i_{qs}^s + (r_s + \frac{p}{\omega_b}X_{ss}) i_{ds}^s - \frac{\omega_s}{\omega_b}X_M i_{qr}'^s + \frac{p}{\omega_b}X_M i_{dr}'^s$$

$$= r_s i_{ds}^s$$

$$\therefore \quad i_{ds}^s = -\frac{1}{\sqrt{3}\, r_s}$$

$$0 = v'^{s}_{dr} = \frac{\omega_r}{\omega_b} X_M i^s_{qs} + \frac{p}{\omega_b} X_M i^s_{ds} + \frac{\omega_r}{\omega_b} X'_{rr} i'^s_{qr} + \left(r'_r + \frac{p}{\omega_b} X'_{rr}\right) i'^s_{dr}$$

$$= \frac{\omega_r}{\omega_b} X'_{rr} i'^s_{qr} + r'_r i'^s_{dr}$$

$$\therefore \quad i'^s_{dr} = -\frac{\omega_r}{\omega_b} \frac{X'_{rr}}{r'_r} i'^s_{qr}$$

$$0 = v'^s_{qr} = \frac{p}{\omega_b} X_M i^s_{qs} - \frac{\omega_r}{\omega_b} X_M i^s_{ds} + \left(r'_r + \frac{p}{\omega_b} X'_{rr}\right) i'^s_{qr} - \frac{\omega_r}{\omega_b} X'_{rr} i'^s_{dr}$$

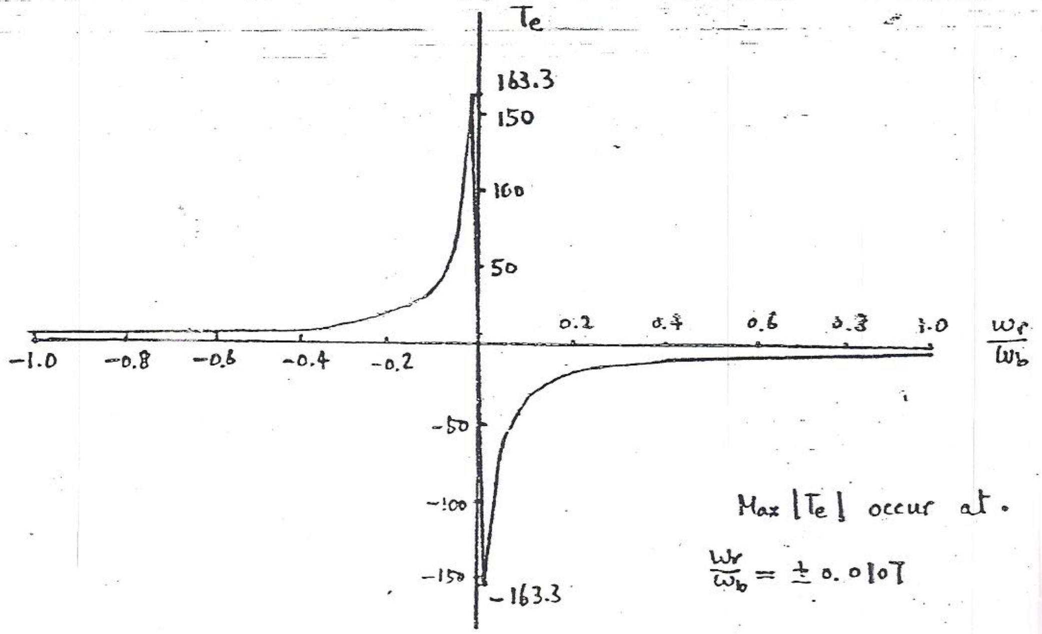
$$\left[r'_r + \left(\frac{\omega_r}{\omega_b} X'_{rr}\right)^2 \frac{1}{r'_r}\right] i'^s_{qr} = \frac{\omega_r}{\omega_b} X_M i^s_{ds} = -\frac{\omega_r}{\omega_b} \frac{X_M}{\sqrt{3}\, r_s}$$

$$\therefore \quad i'^s_{qr} = -\frac{r'_r}{\sqrt{3}\, r_s} \cdot \frac{\frac{\omega_r}{\omega_b} X_M}{r'^2_r + \left(\frac{\omega_r}{\omega_b} X'_{rr}\right)^2}$$

$$i'^s_{dr} = \frac{\frac{\omega_r}{\omega_b} X'_{rr}}{\sqrt{3}\, r_s} \; \frac{\frac{\omega_r}{\omega_b} X_M}{r'^2_r + \left(\frac{\omega_r}{\omega_b} X'_{rr}\right)^2}$$

$$T_e = X_M \left(i^s_{qs} i'^s_{dr} - i^s_{ds} i'^s_{qr}\right) = -X_M i^s_{ds} i'^s_{qr}$$

$$= \frac{r'_r}{3 r_s^2} \frac{\frac{\omega_r}{\omega_b} X_M^2}{r'^2_r + \left(\frac{\omega_r}{\omega_b} X'_{rr}\right)^2} = -15.0365 \frac{\frac{\omega_r}{\omega_b}}{0.0004928 + \left(2.0742 \frac{\omega_r}{\omega_b}\right)^2}$$



Max $|T_e|$ occur at.

$$\frac{\omega_r}{\omega_b} = \pm 0.0107$$

$$\tilde{I}_{cs} = \tilde{I}^s_{cs+} + \tilde{I}^s_{cs-} = e^{j\frac{2\pi}{3}}\tilde{I}^s_{as+} + e^{-j\frac{2\pi}{3}}\tilde{I}^s_{as-}$$

$$\begin{bmatrix} \tilde{E}_{ga} \\ \frac{\tilde{V}'^s_{qr+}}{s} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + j\frac{\omega_e}{\omega_b}X_{ss} - \frac{2}{3}Z & j\frac{\omega_e}{\omega_b}X_M & (\frac{1}{3} - j\frac{\sqrt{3}}{3})Z & 0 \\ j\frac{\omega_e}{\omega_b}X_M & \frac{r'_r}{s} + j\frac{\omega_e}{\omega_b}X'_{rr} & 0 & 0 \\ (\frac{1}{3} + j\frac{\sqrt{3}}{3})Z & 0 & r_s + j\frac{\omega_e}{\omega_b}X_{ss} - \frac{2}{3}Z & j\frac{\omega_e}{\omega_b}X_M \\ 0 & 0 & j\frac{\omega_e}{\omega_b}X_M & \frac{r'_r}{2-s} + j\frac{\omega_e}{\omega_b}X'_{rr} \end{bmatrix} \begin{bmatrix} \tilde{I}^s_{qs+} \\ \tilde{I}'^s_{qr+} \\ \tilde{I}^s_{qs-} \\ \tilde{I}'^s_{qr-} \end{bmatrix}$$

9-7 Using the results of Problem 9-1 with e_{ga} replaced by $e_{ga} - i_{as}Z$, we can find

$$\tilde{V}^s_{qs} = \frac{2}{3}(\tilde{E}_{ga} - \frac{1}{2}\tilde{E}_{gb} - \frac{1}{2}\tilde{E}_{gc}) - \frac{2}{3}\tilde{I}_{as}Z$$

$$\tilde{V}^s_{ds} = \frac{1}{\sqrt{3}}(-\tilde{E}_{gb} + \tilde{E}_{gc})$$

By (9.5-19) $\tilde{V}^s_{qs+} = \frac{1}{2}(\tilde{V}^s_{qs} - j\tilde{V}^s_{ds}) = \frac{1}{3}\tilde{E}_{ga} - \frac{1}{2}(\frac{1}{3} - \frac{j}{\sqrt{3}})\tilde{E}_{gb}$

$$- \frac{1}{2}(\frac{1}{3} + \frac{j}{\sqrt{3}})\tilde{E}_{gc} - \frac{1}{3}\tilde{I}_{as}Z$$

$$\tilde{V}^s_{qs-} = \frac{1}{2}(\tilde{V}^s_{qs} + j\tilde{V}^s_{ds}) = \frac{1}{3}\tilde{E}_{ga} - \frac{1}{2}(\frac{1}{3} + \frac{j}{\sqrt{3}})\tilde{E}_{gb}$$

$$- \frac{1}{2}(\frac{1}{3} - \frac{j}{\sqrt{3}})\tilde{E}_{gc} - \frac{1}{3}\tilde{I}_{as}Z$$

Then, the voltage equations can be found as

6. (a) Using the results of Problem 9-1 with e_{gb} replaced by $e_{gb} - i_{bs}Z$, we can find

$$\tilde{V}_{qs}^{s} = \frac{2}{3}\left(\tilde{E}_{ga} - \frac{1}{2}\tilde{E}_{gb} + \frac{1}{2}\tilde{I}_{bs}Z - \frac{1}{2}\tilde{E}_{gc}\right) = \tilde{E}_{ga} + \frac{1}{3}\tilde{I}_{bs}Z$$

$$\tilde{V}_{ds}^{s} = \frac{1}{\sqrt{3}}\left(-\tilde{E}_{gb} + \tilde{I}_{bs}Z + \tilde{E}_{gc}\right) = j\tilde{E}_{ga} + \frac{1}{\sqrt{3}}\tilde{I}_{bs}Z$$

By (9.5-19), $\tilde{V}_{qs+}^{s} = \frac{1}{2}\left(\tilde{V}_{qs}^{s} - j\tilde{V}_{ds}^{s}\right) = \tilde{E}_{ga} + \left(\frac{1}{3} - \frac{j}{\sqrt{3}}\right)\tilde{I}_{bs}Z$

$$\tilde{V}_{qs-}^{s} = \frac{1}{2}\left(\tilde{V}_{qs}^{s} + j\tilde{V}_{ds}^{s}\right) = \left(\frac{1}{3} + \frac{j}{\sqrt{3}}\right)\tilde{I}_{bs}Z$$

$$\tilde{I}_{bs} = \tilde{I}_{bs+}^{s} + \tilde{I}_{bs-}^{s} = e^{-j\frac{2\pi}{3}}\tilde{I}_{as+}^{s} + e^{j\frac{2\pi}{3}}\tilde{I}_{as-}^{s} \qquad (\tilde{I}_{bs0} = 0)$$

$$\begin{bmatrix} \tilde{E}_{ga} \\ \frac{\tilde{V}_{qr+}^{\prime s}}{s} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + j\frac{\omega_e}{\omega_b}X_{ss} - \frac{2}{3}Z & j\frac{\omega_e}{\omega_b}X_M & \left(\frac{1}{3} + j\frac{\sqrt{3}}{3}\right)Z & 0 \\ j\frac{\omega_e}{\omega_b}X_M & \frac{r_r'}{s} + j\frac{\omega_e}{\omega_b}X_{rr}' & 0 & 0 \\ \left(\frac{1}{3} - j\frac{\sqrt{3}}{3}\right)Z & 0 & r_s + j\frac{\omega_e}{\omega_b}X_{ss} - \frac{2}{3}Z & j\frac{\omega_e}{\omega_b}X_M \\ 0 & 0 & j\frac{\omega_e}{\omega_b}X_M & \frac{r_r'}{2-s} + j\frac{\omega_e}{\omega_b}X_{rr}' \end{bmatrix} \begin{bmatrix} \tilde{I}_{qs+}^{s} \\ \tilde{I}_{qr+}^{\prime s} \\ \tilde{I}_{qs-}^{s} \\ \tilde{I}_{qr-}^{\prime s} \end{bmatrix}$$

(b) Following the same procedure as (a) with e_{gc} replaced by $e_{gc} - i_{cs}Z$

$$\tilde{V}_{qs}^{s} = \tilde{E}_{ga} + \frac{1}{3}\tilde{I}_{cs}Z, \qquad \tilde{V}_{ds}^{s} = j\tilde{E}_{ga} - \frac{1}{\sqrt{3}}\tilde{I}_{cs}Z$$

$$\tilde{V}_{qs+}^{s} = \tilde{E}_{ga} + \left(\frac{1}{3} + \frac{j}{\sqrt{3}}\right)\tilde{I}_{cs}Z \qquad \tilde{V}_{qs-}^{s} = \left(\frac{1}{3} - \frac{j}{\sqrt{3}}\right)\tilde{I}_{cs}Z$$

$$\begin{bmatrix} \frac{1}{3}(\tilde{E}_{ga}+a\tilde{E}_{gb}+a^2\tilde{E}_{gc}) \\ \tilde{V}'^{s}_{qr+}/s \\ \frac{1}{3}(\tilde{E}_{ga}+a^2\tilde{E}_{gb}+a\tilde{E}_{gc}) \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3}Z+r_s+j\frac{\omega_e}{\omega_b}X_{ss} & j\frac{\omega_e}{\omega_b}X_M & \frac{1}{3}Z & 0 \\ j\frac{\omega_e}{\omega_b}X_M & \frac{r_r'}{s}+j\frac{\omega_e}{\omega_b}X_{rr}' & 0 & 0 \\ \frac{1}{3}Z & 0 & \frac{1}{3}Z+r_s+j\frac{\omega_e}{\omega_b}X_{ss} & j\frac{\omega_e}{\omega_b}X_M \\ 0 & 0 & j\frac{\omega_e}{\omega_b}X_M & \frac{r_r'}{2-s}+j\frac{\omega_e}{\omega_b}X_{rr}' \end{bmatrix} \begin{bmatrix} \tilde{I}^{s}_{qs+} \\ \tilde{I}'^{s}_{qr+} \\ \tilde{I}^{s}_{qs-} \\ \tilde{I}'^{s}_{qr-} \end{bmatrix}$$

where $a = e^{j\frac{2\pi}{3}}$.

9-8. $v_{sa} = e_{ga} - v_{as} - v_{ng} = e_{ga} - \frac{1}{2}e_{gb} - \frac{1}{2}e_{gc} - \frac{3}{2}v_{as}$

Solving the equation of (9.6-29), we can find $\tilde{I}'^{s}_{qr+}$, $\tilde{I}'^{s}_{qr-}$, hence $\tilde{I}'^{s}_{qr} = \tilde{I}'^{s}_{qr+} + \tilde{I}'^{s}_{qr-}$.

Then $\tilde{V}_{as} = \frac{j\omega_e}{\omega_b} X_M \tilde{I}'^{s}_{qr}$ is obtained.

$$\therefore \tilde{V}_{sa} = \tilde{E}_{ga} - \frac{1}{2}\tilde{E}_{gb} - \frac{1}{2}\tilde{E}_{gc} - \frac{3}{2}\tilde{V}_{as}.$$

9-9 (a) $i_{bs} = 0$, $i_{os} = 0$, $v_{os} = 0$

$$\bar{i}_{abos} = (\bar{K}_s^s)^{-1}\, \bar{i}_{qdos}^s = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & -\sqrt{3}/2 & 1 \\ -\frac{1}{2} & \sqrt{3}/2 & 1 \end{bmatrix} \begin{bmatrix} i_{qs}^s \\ i_{ds}^s \\ i_{os}^s \end{bmatrix}$$

$$\therefore \quad -\frac{1}{2} i_{qs}^s - \sqrt{3}/2\, i_{ds}^s = 0 \qquad \text{i.e.} \quad i_{qs}^s = -\sqrt{3}\, i_{ds}^s$$

$$v_{bs} = -\frac{1}{2}\left(v_{qs}^s + \sqrt{3}\, v_{ds}^s \right) = -\frac{1}{2} \frac{p}{\omega_b} \left(\psi_{qs}^s + \sqrt{3}\, \psi_{ds}^s \right)$$

$$= -\frac{X_M}{2} \frac{p}{\omega_b} \left(i_{qr}^{\prime s} + \sqrt{3}\, i_{dr}^{\prime s} \right)$$

$$v_{as} = e_{ga} - V_{ng}, \qquad v_{cs} = e_{gc} - V_{ng}$$

$$\therefore \quad V_{ng} = \frac{1}{2}(e_{ga} + e_{gc}) + \frac{1}{2} v_{bs}$$

$$v_{as} = \frac{1}{2} e_{ga} - \frac{1}{2} e_{gc} - \frac{1}{2} v_{bs}$$

$$v_{cs} = -\frac{1}{2} e_{ga} + \frac{1}{2} e_{gc} - \frac{1}{2} v_{bs}$$

By (9.5-22)

$$\tilde{V}_{qs+}^s = \tilde{E}_1 + \frac{j}{4}\frac{\omega_e}{\omega_b}\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\left(\tilde{I}_{qr}^{\prime s} + \sqrt{3}\, \tilde{I}_{dr}^{\prime s}\right) X_M$$

$$\tilde{V}_{qs-}^s = \tilde{E}_2 + \frac{j}{4}\frac{\omega_e}{\omega_b}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\left(\tilde{I}_{qr}^{\prime s} + \sqrt{3}\, \tilde{I}_{dr}^{\prime s}\right) X_M$$

where $\tilde{E}_1 = \frac{1}{2\sqrt{3}}\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)\left(\tilde{E}_{ga} - \tilde{E}_{gc}\right)$

$$\tilde{E}_2 = \frac{1}{2\sqrt{3}}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)\left(\tilde{E}_{ga} - \tilde{E}_{gc}\right)$$

$$\tilde{I}_{qr}^{\prime s} = \tilde{I}_{qr+}^{\prime s} + \tilde{I}_{qr-}^{\prime s}$$

$$\tilde{I}_{dr}^{\prime s} = \tilde{I}_{dr+}^{\prime s} + \tilde{I}_{dr-}^{\prime s} = j\,\tilde{I}_{qr+}^{\prime s} - j\,\tilde{I}_{qr-}^{\prime s}$$

$$\therefore\ \tilde{V}^{s}_{qs+} = \tilde{E}_1 + \frac{j}{2}\frac{\omega_e}{\omega_b}\left[\tilde{I}'^{s}_{qr+} + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\tilde{I}'^{s}_{qr-}\right]X_M$$

$$\begin{bmatrix} \tilde{E}_1 \\ \frac{\tilde{V}'^{s}_{qr+}}{s} \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + j\frac{\omega_e}{\omega_b}X_{ss} & j\frac{1}{2}\frac{\omega_e}{\omega_b}X_M & -\frac{1}{2}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)\frac{\omega_e}{\omega_b}X_M \\ j\frac{\omega_e}{\omega_b}X_M & \frac{r'_r}{s} + j\frac{\omega_e}{\omega_b}X'_{rr} & 0 \\ -j\frac{\omega_e}{\omega_b}X_M & 0 & \frac{r'_r}{2-s} + j\frac{\omega_e}{\omega_b}X'_{rr} \end{bmatrix} \begin{bmatrix} \tilde{I}^{s}_{qs+} \\ \tilde{I}'^{s}_{qr+} \\ \tilde{I}'^{s}_{qr-} \end{bmatrix}$$

(b) By the same procedure as (a), we can find

$$i^s_{qs} = \sqrt{3}\, i^s_{ds}, \quad v_{cs} = -\frac{1}{2}\left(v^s_{qs} - \sqrt{3}\, i'^s_{ds}\right) = -\frac{X_M}{2}\frac{p}{\omega_b}\left(i'^s_{qr} - \sqrt{3}\, i'^s_{dr}\right)$$

$$v_{as} = \frac{1}{2}e_{ga} - \frac{1}{2}e_{gb} - \frac{1}{2}v_{cs}$$

$$v_{bs} = -\frac{1}{2}e_{ga} + \frac{1}{2}e_{gb} - \frac{1}{2}v_{cs}$$

$$\tilde{V}^s_{qs+} = \tilde{E}_1 + \frac{j}{4}\frac{\omega_e}{\omega_b}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\left(\tilde{I}'^s_{qr} - \sqrt{3}\,\tilde{I}'^s_{dr}\right)$$

where $\tilde{E}_1 = \frac{1}{2\sqrt{3}}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)\left(\tilde{E}_{ga} - \tilde{E}_{gb}\right)$.

$$\therefore\ \tilde{V}^s_{qs+} = \tilde{E}_1 + \frac{j}{2}\frac{\omega_e}{\omega_b}\left[\tilde{I}'^s_{qr+} + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\tilde{I}'^s_{qr-}\right]X_M$$

$$\begin{bmatrix} \tilde{E}_1 \\ \frac{\tilde{V}'^{s}_{qr+}}{s} \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + j\frac{\omega_e}{\omega_b}X_{ss} & j\frac{1}{2}\frac{\omega_e}{\omega_b}X_M & \frac{1}{2}\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)\frac{\omega_e}{\omega_b}X_M \\ j\frac{\omega_e}{\omega_b}X_M & \frac{r'_r}{s} + j\frac{\omega_e}{\omega_b}X'_{rr} & 0 \\ -j\frac{\omega_e}{\omega_b}X_M & 0 & \frac{r'_r}{2-s} + j\frac{\omega_e}{\omega_b}X'_{rr} \end{bmatrix} \begin{bmatrix} \tilde{I}^{s}_{qs+} \\ \tilde{I}'^{s}_{qr+} \\ \tilde{I}'^{s}_{qr-} \end{bmatrix}$$

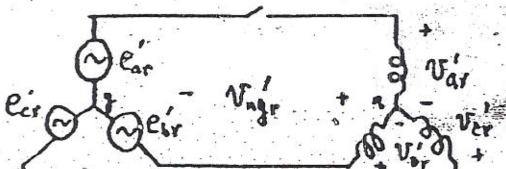
9-10. We will conduct the derivation in the rotor reference frame and choose $\theta(0)$ such that $\theta - \theta_r = 0$

Then $$\bar{K}_r^r = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & -\sqrt{3}/2 & \sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

Because $i'_{ar} = 0$, $v'_{ar} = \frac{p}{\omega_b} X_M i_{qs}^r \; (= v_{qr}'^{r})$, $\tilde{I}_{qr}'^{r} = -\tilde{I}_{qr}'^{r}$.

$$v'_{br} = e'_{br} - v'_{ngr}$$

$$v'_{cr} = e'_{cr} - v'_{ngr}$$



$$v'_{ngr} = \frac{1}{2}(e'_{br} + e'_{cr}) + \frac{1}{2}v'_{ar}$$

$$v'_{br} = \frac{1}{2}e'_{br} - \frac{1}{2}e'_{cr} - \frac{1}{2}v'_{ar}$$

$$v'_{cr} = -\frac{1}{2}e'_{br} + \frac{1}{2}e'_{cr} - \frac{1}{2}v'_{ar}$$

Folowing the similar derivation for (9.6-23), we can find

$$\tilde{V}_{qr+}'^{r} = j\frac{1}{2}\frac{\omega_e}{\omega_b} X_M \tilde{I}_{qs}^r + \tilde{E}$$

where $\tilde{E} = j\frac{1}{2\sqrt{3}}(\tilde{E}_{gb} - \tilde{E}_{gc})$. Then we have

$$\begin{bmatrix} \tilde{V}_{qs+}^r \\ 0 \\ \tilde{E}/s \end{bmatrix} = \begin{bmatrix} r_s + j\frac{\omega_e}{\omega_b}X_{ss} & j\frac{\omega_e}{\omega_b}X_M & 0 \\ 0 & -j\frac{\omega_e}{\omega_b}X_M & r_s + j\frac{\omega_e}{\omega_b}X_{ss} \\ j\frac{1}{2}\frac{\omega_e}{\omega_b}X_M & \frac{r'_r}{s} + j\frac{\omega_e}{\omega_b}X'_{rr} & -j\frac{1}{2}\frac{\omega_e}{\omega_b}X_M \end{bmatrix} \begin{bmatrix} \tilde{I}_{qs+}^r \\ \tilde{I}_{qr+}'^{r} \\ \tilde{I}_{qs-}^s \end{bmatrix}$$

Here, we assume that the voltages applied on the stator circuit are balenced.

Chapter 11

11-2. $\tilde{V}_{as} = \tilde{E}_{ga} - \tilde{I}_{as} Z_a \quad , \quad \tilde{V}_{bs} = \tilde{E}_{gb} - \tilde{I}_{bs} Z_b$

Substituting into (11.2-39) yields

$$\tilde{V}^s_{qs+} = \frac{1}{2}(\tilde{E}_{ga} + j\tilde{E}_{gb}) - \frac{1}{2} Z_a \tilde{I}_{as} - \frac{j}{2} Z_b \tilde{I}_{bs}$$

$$\tilde{V}^s_{qs-} = \frac{1}{2}(\tilde{E}_{ga} - j\tilde{E}_{gb}) - \frac{1}{2} Z_a \tilde{I}_{as} + \frac{j}{2} Z_b \tilde{I}_{bs}$$

$$\tilde{I}_{as} = \tilde{I}^s_{qs+} + \tilde{I}^s_{qs-}$$

$$\tilde{I}_{bs} = -j\tilde{I}^s_{qs+} + j\tilde{I}^s_{qs-} \qquad (\because\ \tilde{I}_{bs+} = -j\tilde{I}_{as+} \quad \tilde{I}_{bs-} = j\tilde{I}_{as-})$$

Let $\tilde{E} = \frac{1}{2}(\tilde{E}_{ga} + j\tilde{E}_{gb})$

Then $\tilde{E} = \tilde{V}^s_{qs+} + \frac{1}{2} Z_a (\tilde{I}^s_{qs+} + \tilde{I}^s_{qs-}) + \frac{1}{2} Z_b (\tilde{I}^s_{qs+} - \tilde{I}^s_{qs-})$

$$\tilde{E}^* = \tilde{V}^s_{qs-} + \frac{1}{2} Z_a (\tilde{I}^s_{qs+} + \tilde{I}^s_{qs-}) - \frac{1}{2} Z_b (\tilde{I}^s_{qs+} - \tilde{I}^s_{qs-})$$

$$\therefore \begin{bmatrix} \tilde{E} \\ \tilde{V}'^s_{qr+}/s \\ \tilde{E}^* \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(Z_a+Z_b) + r_s + j\frac{\omega_e}{\omega_b}X_{ss} & j\frac{\omega_e}{\omega_b}X_{ms} & \frac{1}{2}(Z_a - Z_b) & 0 \\ j\frac{\omega_e}{\omega_b}X_{ms} & \frac{r'_r}{s} + j\frac{\omega_e}{\omega_b}X'_{rr} & 0 & 0 \\ \frac{1}{2}(Z_a - Z_b) & 0 & \frac{1}{2}(Z_a+Z_b) + r_s + j\frac{\omega_e}{\omega_b}X_{ss} & j\frac{\omega_e}{\omega_b}X_{ms} \\ 0 & 0 & j\frac{\omega_e}{\omega_b}X_{ms} & \frac{r'_r}{2-s} + j\frac{\omega_e}{\omega_b}X'_{rr} \end{bmatrix} \begin{bmatrix} \tilde{I}^s_{qs+} \\ \tilde{I}'^s_{qr+} \\ \tilde{I}^s_{qs-} \\ \tilde{I}'^s_{qr-} \end{bmatrix}$$

11-3 $\quad v_{bs} = -v^s_{ds} = -\frac{p}{\omega_b}\psi^s_{ds} = -\frac{p}{\omega_b} X_{ms}\, i'^s_{dr}$

$$v_{as} = e_{ga}$$

By (11.2-39)

$$\tilde{V}^s_{qs+} = \frac{1}{2}\tilde{E}_{ga} + \frac{1}{2}\frac{\omega_e}{\omega_b}X_{ms}\tilde{I}'^s_{dr}$$

$$\tilde{V}^s_{qs-} = \frac{1}{2}\tilde{E}_{ga} - \frac{1}{2}\frac{\omega_e}{\omega_b}X_{ms}\tilde{I}'^s_{dr}$$

$$\tilde{I}'^s_{dr} = \tilde{I}'^s_{dr+} + \tilde{I}'^s_{dr-} = j\,\tilde{I}'^s_{qr+} - j\,\tilde{I}'^s_{qr-}$$

$$\tilde{I}^s_{ds} = \tilde{I}^s_{ds+} + \tilde{I}^s_{ds-} = 0 \qquad \therefore\ j\,\tilde{I}^s_{qs+} - j\,\tilde{I}^s_{qs-} = 0$$

$$\tilde{I}^s_{qs+} = \tilde{I}^s_{qs-}$$

$$\begin{bmatrix} \frac{1}{2}\tilde{E}_{ga} \\ \tilde{V}'^s_{qr+}/s \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + j\frac{\omega_e}{\omega_b}X_{ss} & -\frac{j}{2}\frac{\omega_e}{\omega_b}X_{ms} & \frac{j}{2}\frac{\omega_e}{\omega_b}X_{ms} \\ j\frac{\omega_e}{\omega_b}X_{ms} & \frac{r'_r}{s} + j\frac{\omega_e}{\omega_b}X'_{rr} & 0 \\ j\frac{\omega_e}{\omega_b}X_{ms} & 0 & \frac{r'_r}{2-s} + j\frac{\omega_e}{\omega_b}X'_{rr} \end{bmatrix} \begin{bmatrix} \tilde{I}^s_{qs+} \\ \tilde{I}'^s_{qr+} \\ \tilde{I}'^s_{qr-} \end{bmatrix}$$

11-5

$$\begin{cases} \psi^s_{qs} = X_{ss}\, i^s_{qs} + X_{ms}\, i'^s_{qr} \\ \psi'^s_{qr} = X_{ms}\, i^s_{qs} + X'_{rr}\, i'^s_{qr} \end{cases} \qquad i^s_{qs} = \frac{X'_{rr}\psi^s_{qs} - X_{ms}\psi'^s_{qr}}{X_{ss}X'_{rr} - X^2_{ms}}$$

$$i'^s_{qr} = \frac{X_{ss}\psi'^s_{qr} - X_{ms}\psi^s_{qs}}{X_{ss}X'_{rr} - X^2_{ms}}$$

Likewise $$i^s_{ds} = \frac{X'_{RR}\psi^s_{ds} - X_{mS}\psi'^s_{dr}}{X_{SS}X'_{RR} - X^2_{mS}}$$

$$i'^s_{dr} = \frac{X_{SS}\psi'^s_{dr} - X_{mS}\psi^s_{ds}}{X_{SS}X'_{RR} - X^2_{mS}}$$

Let $dd = X_{ss}X'_{rr} - X^2_{ms}$, $DD = X_{SS}X'_{RR} - X^2_{mS}$.

Then The voltage equations can be expressed as

$$\begin{bmatrix} v_{qs}^{s} \\ v_{ds}^{s} \\ v_{qr}^{\prime s} \\ v_{dr}^{\prime s} \end{bmatrix} = \begin{bmatrix} \frac{r_s X_{rr}'}{dd} + \frac{p}{\omega_b} & 0 & -\frac{r_s X_{ms}}{dd} & 0 \\ 0 & \frac{r_s X_{rr}'}{DD} + \frac{p}{\omega_b} & 0 & -\frac{r_s X_{ms}}{DD} \\ -\frac{r_r X_{ms}}{dd} & 0 & \frac{r_r' X_{ss}}{dd} + \frac{p}{\omega_b} & -\frac{1}{n}\frac{\omega_r}{\omega_b} \\ 0 & -\frac{r_R' X_{ms}}{DD} & n\frac{\omega_r}{\omega_b} & \frac{r_R' X_{ss}}{DD} + \frac{p}{\omega_b} \end{bmatrix} \begin{bmatrix} \psi_{qs}^{s} \\ \psi_{ds}^{s} \\ \psi_{qr}^{\prime s} \\ \psi_{dr}^{\prime s} \end{bmatrix}$$

11-6 $$T_e = \left(\frac{P}{2}\right)\left(\frac{N_S}{N_s}\right)\left(\frac{X_{ms}}{\omega_b}\right)\left(i_{qs}^{s}\, i_{dr}^{\prime s} - i_{ds}^{s}\, i_{qr}^{\prime s}\right) \qquad (\text{by } (11.4\text{-}52))$$

$$\left(\frac{P}{2}\right)\left(\frac{1}{n}\lambda_{ds}^{s}\, i_{qs}^{s} - n\lambda_{qs}^{s}\, i_{ds}^{s}\right) = \left(\frac{P}{2}\right)\Big[\frac{1}{n} i_{qs}^{s} i_{ds}^{s}(L_{lS} + L_{mS}) + \frac{1}{n}L_{mS}\, i_{dr}^{\prime s}\, i_{qs}^{s} - n\, i_{qs}^{s}\, i_{ds}^{s}(L_{ls} + L_{ms}) - n L_{ms}\, i_{qr}^{\prime s}\, i_{ds}^{s}\Big]$$

$$= \left(\frac{P}{2}\right)\left(\frac{N_S}{N_s}\right)\left(\frac{X_{ms}}{\omega_b}\right)\left(i_{qs}^{s}\, i_{dr}^{\prime s} - i_{ds}^{s}\, i_{qr}^{\prime s}\right) + \left(\frac{P}{2}\right) i_{qs}^{s}\, i_{ds}^{s}\left(\frac{1}{n}L_{lS} - n L_{ls}\right)$$

$$(\text{by } (11.4\text{-}51))$$

Only if $\left(\frac{P}{2}\right) i_{qs}^{s}\, i_{ds}^{s}\left(\frac{1}{n}L_{lS} - n L_{ls}\right) = 0$,

$$\left(\frac{P}{2}\right)\left(\frac{1}{n}\lambda_{ds}^{s}\, i_{qs}^{s} - n\lambda_{qs}^{s}\, i_{ds}^{s}\right) = \left(\frac{P}{2}\right)\left(\frac{N_S}{N_s}\right)\left(\frac{X_{ms}}{\omega_b}\right)\left(i_{qs}^{s}\, i_{dr}^{\prime s} - i_{ds}^{s}\, i_{qr}^{\prime s}\right)$$

$$= T_e,$$

i.e. $\frac{1}{n}L_{lS} - nL_{ls} = 0$ or $L_{lS} = n^2 L_{ls}$.

11-7 $\because$ $v_r = 0$, $\therefore$ $Z_B = 0$,

$$Z_A = (r_r' + j r_r' f_e)$$

By (11.5-16) $\dfrac{\tilde{V}_{qs}^s}{Z_s - j f_e X_{ms} Z_A} = \dfrac{\tilde{V}_{as}}{Z_s - j f_e X_{ms} Z_A}$

$$\tilde{I}_{ds}^s \overline{X_{ms} Z_A} = \frac{-\tilde{V}_{bs}}{Z_S - j n^2 f_e X_{ms} Z_A}$$

By (11-5- $-Z_A \tilde{I}_{qs}^s$

$-Z_A \tilde{I}_{ds}^s$

Then $= \tilde{I}_{qs}^s \tilde{I}_{ds}^s (-Z_A + Z_A) = 0$

By (11- lear that the pulsating torque is zero.

11-9. f Problem 11-2

$$\begin{bmatrix} \tilde{E} \\ \tilde{E}^* \end{bmatrix} \begin{bmatrix} 2 f_e X_{ms} Z_B - j 2 f_e X_{ms} Z_A & Z_s + Z_a - Z_b - \frac{Z_s}{n^2} \\ & Z_s + Z_a + Z_b + \frac{Z_s}{n^2} + 2 f_e X_{ms} Z_B - j 2 f_e X_{ms} Z_A \end{bmatrix} \begin{bmatrix} \tilde{I}_{qs+}^s \\ \tilde{I}_{qs-}^s \end{bmatrix}$$

11-10. $= 1$, By (11.5-37),

$$2.02 = j \frac{Z_s}{1.18} + \frac{1.18 \cdot 66.8^2}{68.92 - j4.12}$$

$\therefore$ $r_B = -7.759$ $X_B = -13.225$

(b) $S = 0.05$, $f_e = 1$. By (11.5-37)

$$2.02 + j69.59 - \frac{j\,66.8^2}{68.92 - j82.4} = j\frac{Z_s}{1.18} + \frac{1.18 \cdot 66.8^2}{68.92 - j82.4}$$

$$j\frac{Z_s}{1.18} = 2.435 + j5.342 \qquad \therefore\ r_B = -0.837 \ ,\ X_B = -98.723$$

(c) $S = 0$, $f_e = 1$. $Z_B + jZ_A = 0$

By (11.5-36) $\qquad 2.02 + j69.59 = j\dfrac{Z_s}{1.18}$

$$\therefore\ r_B = 74.976 \ ,\ X_B = -98.23 .$$

11-12. $f_r = 0$, $f_e = 1$. $Z_B = 0$

$$\tilde{I}_{qs}^{s} = \frac{\tilde{V}_{qs}^{s}}{Z_s - jX_{ms}Z_A} = \frac{110\angle 0}{2.50 + j63.0 - j60.1^2\,(j2.4 - 61.8)/(2.4 + j61.8)^2} = 16.10\angle -44.24^\circ$$

$$\tilde{I}_{ds}^{s} = \frac{\tilde{V}_{ds}^{s}}{Z_S - jn^2X_{ms}Z_A} = \frac{110\angle 0}{10.7 + j51.181 + 0.81 \cdot 60.1^2/(2.4 + j61.8)} = 8.377\angle -17.32^\circ$$

$$\tilde{I}_{qr}^{\prime s} = -Z_A\tilde{I}_{qs}^{s} = \frac{-jX_{ms}\tilde{I}_{qs}^{s}}{r_r' + jX_{rr}'} = 15.646\angle 137.99^\circ$$

$$\tilde{I}_{dr}^{\prime s} = -Z_A\tilde{I}_{ds}^{s} = 8.140\angle 164.90^\circ .$$

$$T_e = 2 \cdot 0.9 \cdot \frac{60.1}{377} \cdot \mathrm{Re}\left(\tilde{I}_{qs}^{s*}\tilde{I}_{dr}^{\prime s} - \tilde{I}_{ds}^{s*}\tilde{I}_{qr}^{\prime s}\right)$$

$$= 1.323 \ \text{N·m} .$$

rated torque: $\dfrac{0.25 \cdot 746 \cdot (\frac{P}{2})}{377} = 0.9894$ N·M

Chapter 12

12-1 Referring to Problem 7-1, we can find

$$\overline{W}_x = -\begin{bmatrix} \omega_b \frac{X'_{rr} r_s}{XX} & 0 & 0 \\ 0 & \omega_b \frac{X'_{rr} r_s}{XX} & 0 \\ 0 & 0 & \omega_b \frac{r_s}{X_{\ell s}} \end{bmatrix}, \quad \overline{W}_\omega = \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\overline{W}_r = \omega_b \frac{X_M r_s}{XX} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \overline{W}_s = \omega_b \frac{X_M r'_r}{XX} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\overline{W}_q = -\begin{bmatrix} \omega_b \frac{X_{ss} r'_r}{XX} & 0 & 0 \\ 0 & \omega_b \frac{X_{ss} r'_r}{XX} & 0 \\ 0 & 0 & \omega_b \frac{r'_r}{X'_{\ell r}} \end{bmatrix}, \quad \overline{W}_{\omega r} = \begin{bmatrix} 0 & -(\omega-\omega_r) & 0 \\ \omega-\omega_r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\overline{B}_r = \overline{B}_s = \omega_b \overline{I},$$

where $XX = X_{ss} X'_{rr} - X_M^2$

In block 2: $\quad T_e = \frac{X_M}{XX}(\psi_{qs}\psi'_{dr} - \psi'_{qr}\psi_{ds}) \quad$ (in per unit)

In block 3:

$$\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i_{qr} \\ i_{dr} \\ i_{0r} \end{bmatrix} = \begin{bmatrix} \frac{X'_{rr}}{XX} & 0 & 0 & -\frac{X_M}{XX} & 0 & 0 \\ 0 & \frac{X'_{rr}}{XX} & 0 & 0 & -\frac{X_M}{XX} & 0 \\ 0 & 0 & \frac{1}{X_{\ell s}} & 0 & 0 & 0 \\ -\frac{X_M}{XX} & 0 & 0 & \frac{X_{ss}}{XX} & 0 & 0 \\ 0 & -\frac{X_M}{XX} & 0 & 0 & \frac{X_{ss}}{XX} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{X'_{\ell r}} \end{bmatrix} \begin{bmatrix} \psi_{qs} \\ \psi_{ds} \\ \psi_{0s} \\ \psi'_{qr} \\ \psi'_{dr} \\ \psi'_{0r} \end{bmatrix}.$$

In block 4: $\quad \frac{\omega_r}{\omega_b} = \frac{1}{2Hp}(T_e - T_L)$

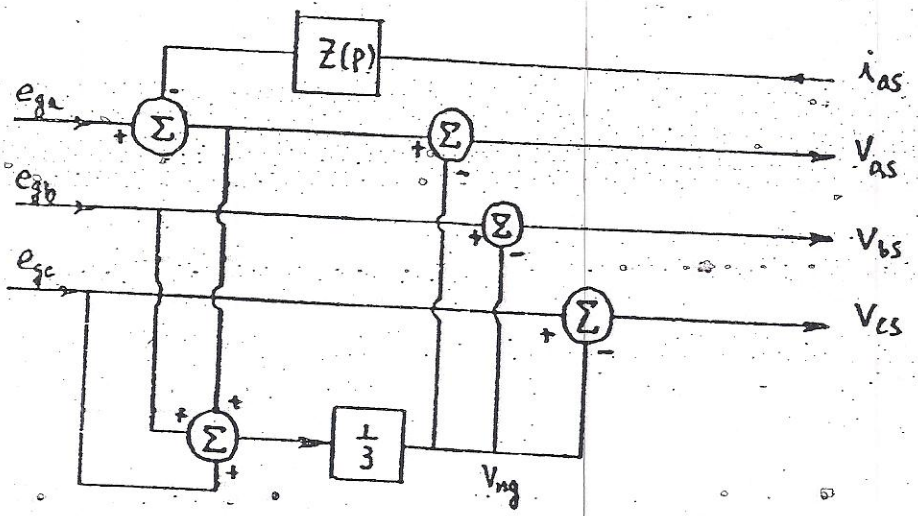
12-3 $\quad V_{as} = e_{ga} - Z(p)\, i_{as} - V_{ng}$

$$V_{bs} = e_{gb} - V_{ng}$$

$$V_{cs} = e_{gc} - V_{ng}$$

$$V_{ng} = \frac{1}{3}\left[(e_{ga} - Z(p)\, i_{as}) + e_{gb} + e_{gc} \right]$$

The block diagram for this simulation is show in the figure below.



This Simulation is to be used in Conjunction with Fig. 12.2-1 with $\omega = 0$.

12-4. (a) $i_{os} = V_{os} = 0$. $\quad i_{bs} = 0 \quad \therefore \quad i_{qs}^{s} = -\sqrt{3}\, i_{ds}^{s}$

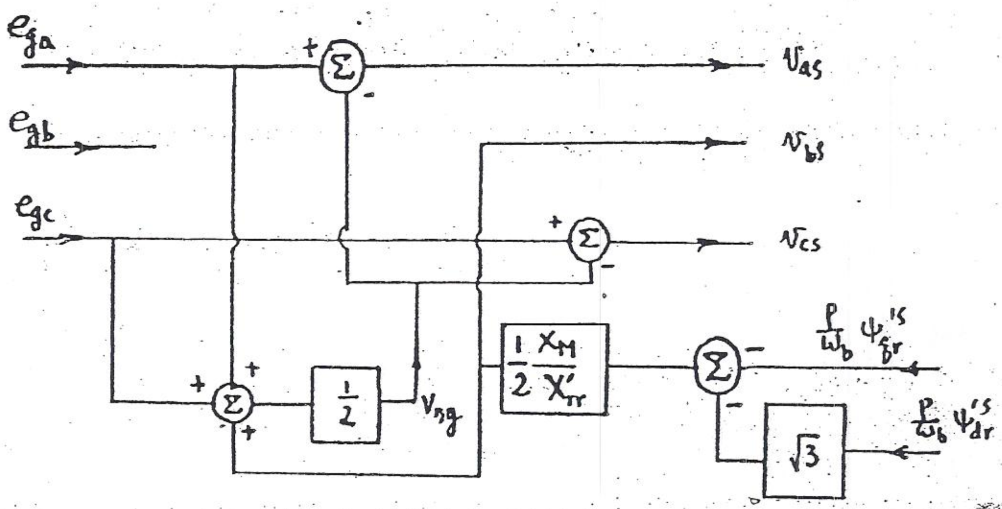
$$v_{bs} = -\frac{1}{2}\left(v_{qs} + \sqrt{3}\, v_{ds} \right) = -\frac{1}{2}\frac{p}{\omega_b}\left(\psi_{qs}^{s} + \sqrt{3}\, \psi_{ds}^{s} \right)$$

$$= -\frac{1}{2}\frac{X_M}{X'_{rr}}\frac{p}{\omega_b}\left(\psi_{qr}^{\prime s} + \sqrt{3}\, \psi_{dr}^{\prime s} \right)$$

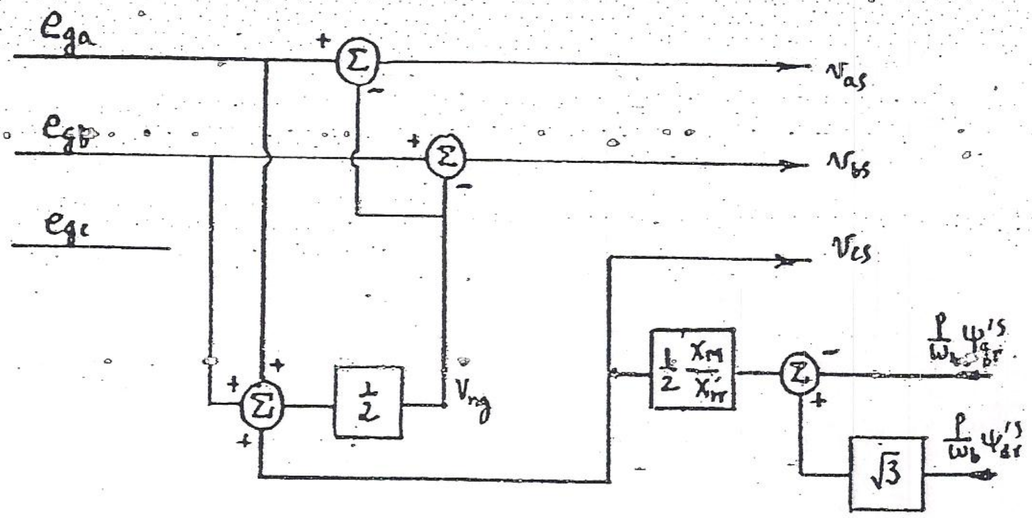
$$v_{as} = e_{ga} - V_{ng}$$

$$v_{cs} = e_{gc} - V_{ng}$$

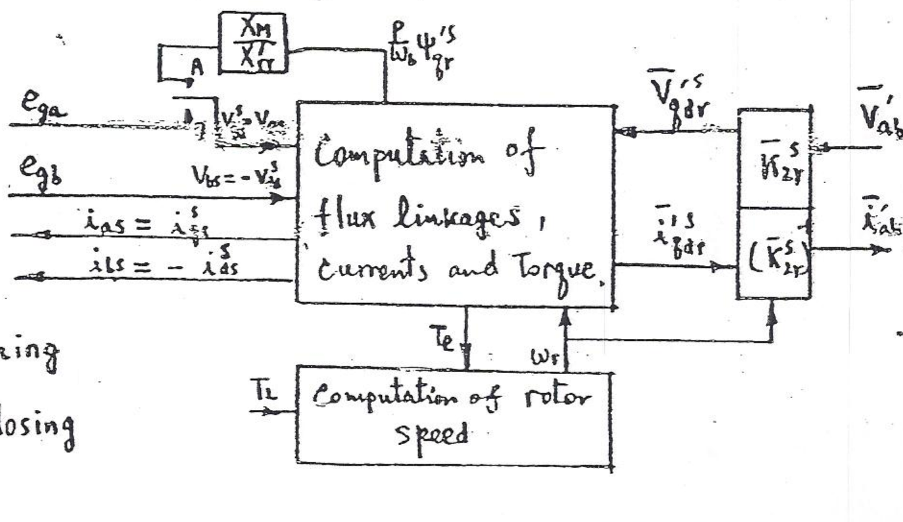
$$\therefore \quad v_{ng} = \frac{1}{2}(e_{ga} + e_{gc}) + \frac{1}{2} V_{bs}$$



(b)



12-5. For 2-phase symmetrical induction machine



A: opening

B: reclosing

For a 2-phase unsymmetrical induction machine, the block diagram is the same as above, but the formulas for computation of flux linkages, currents and torque are different from those for a symmetrical machine and in general, $\bar{V}'_{abr} = 0$.

12-6. We will conduct the analysis in the rotor reference frame and choose $\theta(0)$ such that

$$\theta - \theta_r = 0.$$

Then
$$\bar{K}_r^r = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\sqrt{3}/2 & \sqrt{3}/2 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

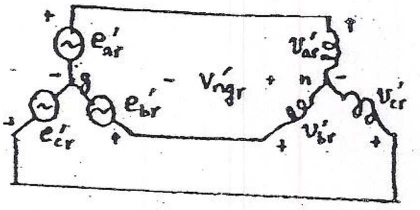
When rotor phase a is opening,

$$v'_{ar} = \frac{p}{\omega_b}\psi'^r_{qr} = \frac{p}{\omega_b}\psi'^r_{mq} = \frac{p}{\omega_b}\left(\frac{X_M}{X_{ss}}\right)\psi^r_{qs}$$

$$v'_{br} = e'_{br} - V'_{ngr}$$

$$v'_{cr} = e'_{cr} - V'_{ngr}$$

$$V'_{ngr} = \frac{1}{2}(e'_{br} + e'_{cr}) + \frac{1}{2}v'_{ar}$$



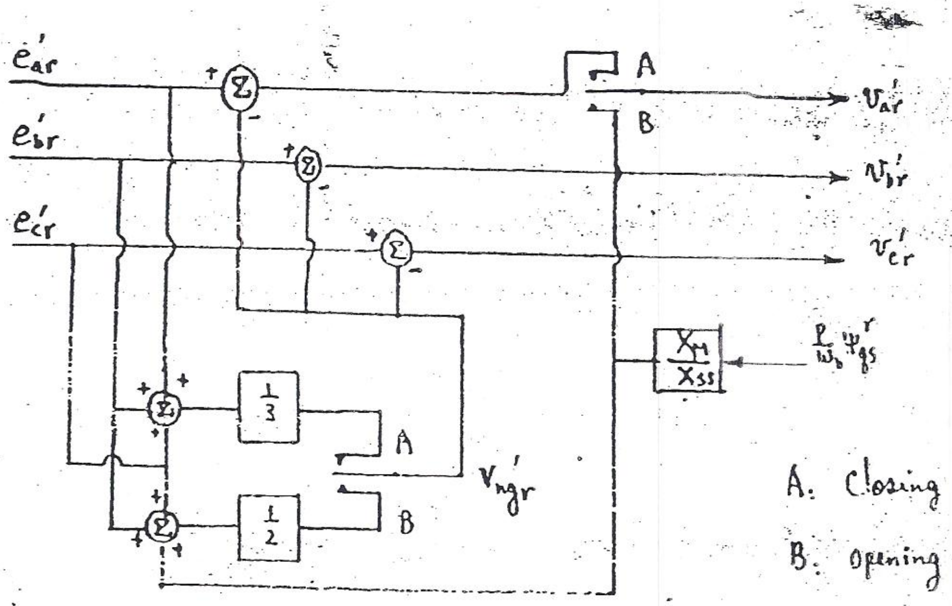
when rotor phase a is closing

$$v'_{ar} = e'_{ar} - V'_{ngr}$$

$$v'_{br} = e'_{br} - V'_{ngr}$$

$$v'_{cr} = e'_{cr} - V'_{ngr}$$

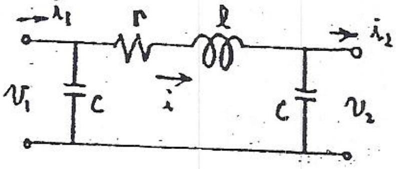
$$V'_{ngr} = \frac{1}{3}(e'_{ar} + e'_{br} + e'_{cr})$$



Above simulation is to be used in conjunction with Fig 12.2-1 with $\omega = \omega_r$.

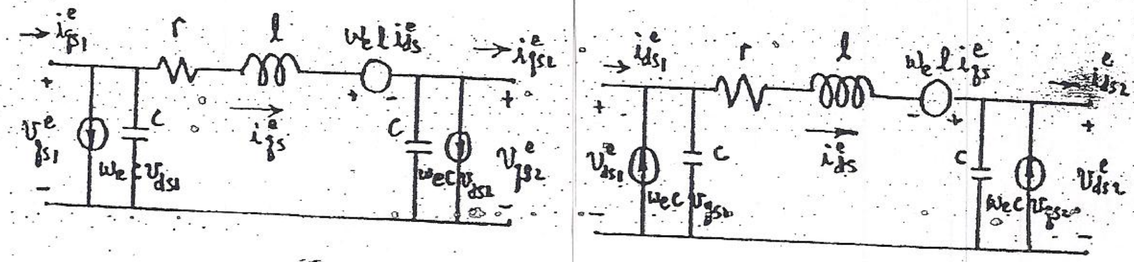
Chapter 13

13-1 Suppose the π equivalent circuit is in the form shown in the figure (a).



(a)

By the problem 3-4, we can find the q-axis and d-axis equivalent circuits shown in Figures (b) and (c).



(b) (c)

$$i^e_{qs1} = \omega_e c\, v_{ds1} + pc\, v_{qs1} + i^e_{qs} \quad , \quad i^e_{qs} = \omega_e c\, v_{ds2} + pc\, v_{qs2} + i^e_{qs2}$$

$$v^e_{qs1} = v^e_{qs2} + (r + pl)\, i^e_{qs} + \omega_e l\, i^e_{ds}$$

$$i^e_{ds} = -\omega_e c\, v^e_{qs2} + pc\, v^e_{ds2} + i^e_{ds2}$$

$$\therefore \quad v^e_{qs1} = v^e_{qs2} + (r+pl)(\omega_e c\, v_{ds2} + pc\, v^e_{qs2} + i^e_{qs2}) + \omega_e l\,(-\omega_e c\, v^e_{qs2} + pc\, v^e_{ds2} + i^e_{ds2})$$

$$= (1 + (r+pl)pc - \omega_e^2 lc)\, v^e_{qs2} + [(r+pl)\omega_e c + p\omega_e c l]\, v^e_{ds2} + (r+pl)\, i^e_{qs2} + \omega_e l\, i^e_{ds2}$$

Likewise, $v^e_{ds1} = [1 + (r+pl)pc - \omega_e^2 lc]\, v^e_{ds2} - [(r+pl)\omega_e c + p\omega_e c l]\, v^e_{qs2} + (r+pl)\, i^e_{ds2} - \omega_e l\, i^e_{qs2}$

13-2. If the electrical transients are neglected, the voltage equations obtained in Problem 13-1 will become

$$v_{qs1}^e = (1-\omega_e^2 \ell C)\, v_{qs2}^e + r\omega_e C\, v_{ds2}^e + r\, i_{qs2}^e + \omega_e \ell\, i_{ds2}^e$$

$$v_{ds1}^e = (1-\omega_e^2 \ell C)\, v_{ds2}^e - r\omega_e C\, v_{qs2}^e + r\, i_{ds2}^e - \omega_e \ell\, i_{qs2}^e$$

In the case of stead state condition of the positive sequence,

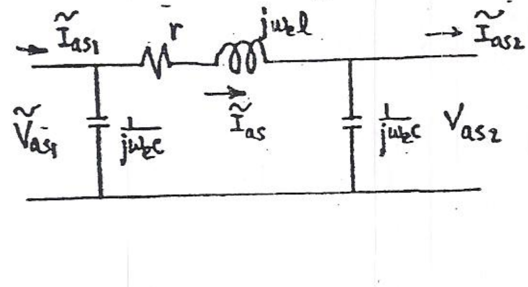
$$\sqrt{2}\,\tilde{V}_{as1} = V_{qs1}^e - jV_{ds1}^e = (1-\omega_e^2 \ell C)(V_{qs2}^e - jV_{ds2}^e) + (r + j\omega_e \ell)\cdot$$

$$(I_{qs2}^e - jI_{ds2}^e) + r\omega_e C\, j\,(V_{qs2}^e - jV_{ds2}^e)$$

$$= \sqrt{2}\left[(1-\omega_e^2 \ell C)\tilde{V}_{as2} + j r\omega_e C\,\tilde{V}_{as2} + (r + j\omega_e \ell)\tilde{I}_{as2}\right]$$

or $\tilde{V}_{as1} = (1-\omega_e^2 \ell C)\tilde{V}_{as2} + j r\omega_e C\,\tilde{V}_{as2} + (r + j\omega_e \ell)\tilde{I}_{as2}$

This equation is the same as that for the π equivalent circuit of the positive sequence of the transmission line shown in the Figure.

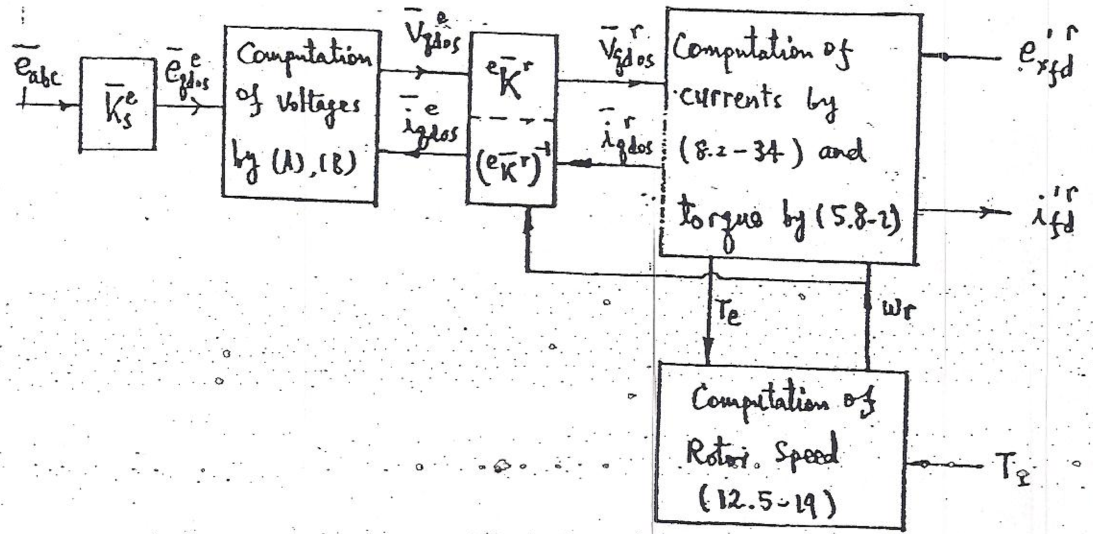


13-3 Neglecting electrical transients, we can find the voltage equations for the transmission line by Problem 13-2 where we change the direction of the currents i_{qs2}^e and i_{ds2}^e, and use E_q^e, E_d^e instead of v_{qs1}^e and v_{ds1}^e, v_{qs}^e and v_{ds}^e instead

v_{ds2}^e by v_{qs}^e and v_{ds}^e, i_{qs2}^e and i_{ds2}^e by i_{qs}^e and i_{ds}^e.

$$e_{qs}^e = (1 - w_e^2 lc)\, v_{qs}^e + r w_e c\, v_{ds}^e - r\, i_{qs}^e - w_e l\, i_{ds}^e \qquad \text{(A)}$$

$$e_{ds}^e = (1 - w_e^2 lc)\, v_{ds}^e - r w_e c\, v_{qs}^e - r\, i_{ds}^e + w_e l\, i_{qs}^e \qquad \text{(B)}$$



$$V_{qs}^e = \frac{2V_i}{\pi}\left(1 + \frac{2}{35}\cos 6w_et - \frac{2}{143}\cos 12w_et + \cdots\right)$$

$$= \frac{2V_i}{\pi}\left(1 + \frac{7}{35}\cos 6w_et - \frac{5}{35}\cos 6w_et - \frac{1}{11}\cos 12w_et + \frac{1}{13}\cos 12w_et + \cdots\right)$$

$$= \frac{2V_i}{\pi}\left(1 + \frac{1}{5}\cos 6w_et - \frac{1}{7}\cos 6w_et - \frac{1}{11}\cos 12w_et + \frac{1}{13}\cos 12w_et + \cdots\right)$$

Likewise $V_{ds}^e = \frac{2V_i}{\pi}\left(1 + \frac{1}{5}\sin 6w_et + \frac{1}{7}\sin 6w_et - \frac{1}{11}\sin 12w_et - \frac{1}{13}\sin 12w_et + \cdots\right)$

Chapter 14

14-1

$$i_{qs}^r = \lambda_{qs}^r / L_q, \quad i_{ds}^r = (\lambda_{ds}^r - \lambda_m'^r)/L_d$$

$$i_{os} = \lambda_{os} / L_{ls}$$

$$\therefore \quad V_{qs}^r = (r_s/L_q + p)\,\lambda_{qs}^r + \omega_r \lambda_{ds}^r$$

$$V_{ds}^r = (r_s/L_d + p)\,\lambda_{ds}^r - \omega_r \lambda_{qs}^r - (r_s/L_d)\,\lambda_m'^r$$

$$V_{os} = (r_s/L_{ls} + p)\,\lambda_{os}$$

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left[\left(\frac{1}{L_q} - \frac{1}{L_d}\right)\lambda_{qs}^r \lambda_{ds}^r + \frac{1}{L_d}\lambda_{qs}^r \lambda_m'^r\right]$$

14-2

$$V_{qs}^r = 0, \qquad r_s I_{qs}^r + \omega_r L_d I_{ds}^r = -\omega_r \lambda_m'^r$$

$$V_{ds}^r = \sqrt{2}\,V_s \qquad -\omega_r L_q I_{qs}^r + r_s I_{ds}^r = \sqrt{2}\,V_s$$

$$\therefore \quad I_{qs}^r = -\frac{\omega_r (r_s \lambda_m'^r + \sqrt{2}\,V_s L_d)}{\omega_r^2 L_q L_d + r_s^2}$$

$$I_{ds}^r = \frac{\sqrt{2}\,V_s r_s - \omega_r^2 \lambda_m'^r L_q}{\omega_r^2 L_q L_d + r_s^2}$$

$$T_e = -\left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\frac{(\sqrt{2}\,V_s L_d + r_s \lambda_m'^r)\left[\sqrt{2}\,V_s r_s (L_d - L_q) + \lambda_m'^r (r_s^2 + \omega_r^2 L_q^2)\right]}{(r_s^2 + \omega_r^2 L_q L_d)^2}$$

14-3. If $V_{qs}^r = \sqrt{2} V_s$ and $V_{ds}^r = 0$

$$T_{e1} = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right) \frac{r_s \lambda_m'^r}{r_s^2 + \omega_r^2 L_s^2} \left(\sqrt{2} V_s - \omega_r \lambda_m'^r \right)$$

If $V_{qs}^r = 0$ and $V_{ds}^r = -\sqrt{2} V_s$

$$T_{e2} = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right) \frac{r_s \lambda_m'^r}{r_s^2 + \omega_r^2 L_s^2} \sqrt{2} V_s \omega_r \left(\frac{L_s}{r_s} - \frac{\lambda_m'^r}{\sqrt{2} V_s} \right)$$

$T_{e1} = T_{e2}$ $\quad \omega_r = \frac{r_s}{L_s} = \frac{3.4}{12.1} \times 10^3 = 281$ rad/s

At this speed, the intersection occurs.

14-4. During free acceleration, $T_L = 0$; if B_m is neglected,

$$\omega_r = \frac{1}{K_e \tau_s \tau_m} \frac{V_{qs}^r(s)}{s^2 + \frac{1}{\tau_s} s + \frac{1}{\tau_s}\frac{1}{\tau_m}}$$

where $\tau_s = 3.56 \times 10^{-3}$ s, $\lambda_m'^r = 0.0827$ (V-s) $= K_e$

$r_s = 3.4\ \Omega$, $J = 10^{-4}$ kg·m²

$V_{qs}^r(s) = \sqrt{2} V_s / s = 15.91 / s$

$\tau_m = J r_s / \left[\frac{3}{2}\left(\frac{P}{2}\right)^2 \lambda_m'^{r2}\right] = 8.285 \times 10^{-3}$ s

$$\therefore \omega_r(s) = \frac{6.523 \times 10^6}{s\left(s^2 + 2.809 \times 10^2 s + 3.390 \times 10^4\right)}$$

$$= \frac{192.4}{s} + \frac{-(s+140.45) - 140.45}{(s+140.45)^2 + 119.07^2} \cdot 192.4$$

$$\therefore\ \omega_r(t) = 192.4\left(1 - e^{-140.45t}\cos 119.07t - 1.180\,e^{-140.45t}\sin 119.07t\right)$$

During step changes in load, find the steady-state solution at first.

(i) when $T_l = 0.1$ N.m $\qquad T_e = T_l$ (B_m is neglected)

$$T_e = \left(\tfrac{3}{2}\right)\cdot\left(\tfrac{P}{2}\right)\lambda_m'^r I_{qs}^r \qquad \therefore\ I_{qs}^r = 0.40306\ \text{A}.$$

By (14.4-5), $\quad V_{qs}^r = r_s I_{qs}^r + \frac{L_s^2}{r_s} I_{qs}^r \omega_r^2 + \lambda_m'^r \omega_r$

$$\therefore\ \omega_r = \frac{-\lambda_m'^r + \sqrt{(\lambda_m'^r)^2 - 4\frac{L_s^2}{r_s} I_{qs}^r\left(r_s I_{qs}^r - V_{qs}^r\right)}}{2\frac{L_s^2}{r_s} I_{qs}^r} = 169.76\ \text{rad/s}$$

By (14.4-4) $\quad I_{ds}^r = \frac{\omega_r L_s}{r_s} I_{qs}^r = 0.24351\ \text{A}$

(ii) when $T_e = 0.4$ N.m ; $\quad I_{qs}^r = 1.61224$ A

$\omega_r = 114.996$ rad/s

$I_{ds}^r = 0.6598$

By (14.5-9) $\because\ \Delta V_{qr}^r(s) = 0$, $B_m = 0 \qquad \Delta T_L(s) = \pm 0.3/s$

$$s\,\omega_r = -\frac{\left(\frac{P}{2}\right)\left(\frac{1}{J}\right)\left(s + \frac{1}{\tau_s}\right)\cdot(\pm 0.3)}{s\left(s^2 + \frac{1}{\tau_s}s + \frac{1}{\tau_s}\frac{1}{\tau_m}\right)}$$

where $\tau_s = 3.56\times 10^{-3}$ s , $J = 2\times 10^{-4}$ kg.m^2

$$\tau_m = J r_s / \left[\tfrac{3}{2}\left(\tfrac{P}{2}\right)^2 \lambda_m'^{r2}\right] = 1.6571\times 10^{-2}\ \text{s}$$

14-5 From the Solution of Problem 14-1,

if $v_{qs}^r = 0$, $\lambda_{ds}^r = \dfrac{(p + r_s/L_q)}{\omega_r}\lambda_{qs}^r$.

$$v_{ds}^r = \left[\frac{(p + r_s/L_d)(p + r_s/L_q)}{\omega_r} - \omega_r\right]\lambda_{qs}^r - (r_s/L_d)\lambda_m'^r$$

This equation is not linear, The transfer function can be found only for the equations of small displacements.

$$\Delta v_{ds}^r = -\left[\frac{(p + r_s/L_d)(p + r_s/L_q)}{\omega_{r_0}^2} + 1\right]\lambda_{qs_0}^r \Delta\omega_r$$

$$+ \left[\frac{(p + r_s/L_d)(p + r_s/L_q)}{\omega_{r_0}} - \omega_{r_0}\right]\Delta\lambda_{qs}^r$$

$$\Delta\omega_r = \frac{P/2}{Jp + B_m}(\Delta T_e - \Delta T_L) \qquad (\text{In our case, } \Delta T_L = 0)$$

Assume $L_d = L_q = L_s$; then

$$\Delta T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\frac{\lambda_m'^r}{L_s}\Delta\lambda_{qs}^r .$$

$$= \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\frac{\lambda_m'^r}{L_s}\left\{\Delta v_{ds}^r + \frac{(p + r_s/L_s)^2 + \omega_{r_0}^2}{\omega_{r_0}^2}\lambda_{qs_0}^r \frac{P/2 \cdot \Delta T_e}{Jp + B_m}\right\}$$

$$\frac{(p + r_s/L_s)^2 - \omega_{r_0}^2}{\omega_{r_0}}$$

$$\therefore \quad \frac{\Delta T_e}{\Delta v_{ds}^r} = \frac{6P\,\omega_{r_0}^2\,(JS + B_m)\left[(S + r_s/L_s)^2 - \omega_{r_0}^2\right]\lambda_m'^r}{-3P^2\lambda_m'^r\,\lambda_{qs_0}^r\left[(S + r_s/L_s)^4 - \omega_{r_0}^4\right] + 8\,\omega_{r_0}^3 L_s (SJ + B_m)} .$$

$$\therefore \Delta W_r = \mp \frac{3000\,(S + 280.992)}{S\,(S^2 + 280.992S + 1.6957\times10^4)}$$

$$= \mp \left[\frac{49.71}{S} - \frac{62.63}{S+87.75} + \frac{12.914}{S+193.24} \right]$$

$$\Delta W_r(t) = \mp \left[49.71 - 62.63e^{-87.75t} + 12.914e^{-193.24t} \right]$$

$\therefore$ When the load torque is suddenly stepped from 0.1 N.m to 0.4 N.m (At this moment, t is assumed to be 0),

$$W_r = 169.76 - \Delta W_r(t) = 120.05 + 62.63e^{-87.75t} - 12.914e^{-193.24t}$$

Note: From this solution, the steady-state operation will be established with $W_r = 120.05$ rad/s, which is not equal to 114.996 rad/s solved in step (ii). This error is because the simplified transfer function (14.5-9) is obtained with neglecting the term $W_r^2 L_q L_d$.

When the load torque is stepped back to 0.1 N.m ($t = 0$)

$$W_r = 120.05 + \Delta W_r = 169.76 - 62.63e^{-87.75t} + 12.914e^{-193.24t}$$