Solutions Manual

Electric Machines

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CHAPTER 2: MAGNETIC CIRCUITS AND INDUCTION

Note Unless otherwise specified, leakage and fringing are neglected.

2.1 *A square loop of side 2 d is placed with two of its sides parallel to an infinitely long conductor carrying current I. The centre line of the square is at distance b from the conductor. Determine the expression for the total flux passing through the loop. What would be the loop flux if the loop is placed such that the conductor is normal to the plane of the loop? Does the loop flux in this case depend upon the relative location of the loop with respect to the conductor?*

Solution

At distance r from conductor

$$H = \frac{I}{2\pi r} \text{ A/m}$$

$$B = \mu_0 H = \frac{\mu_0 I}{2\pi r} \text{ T}$$

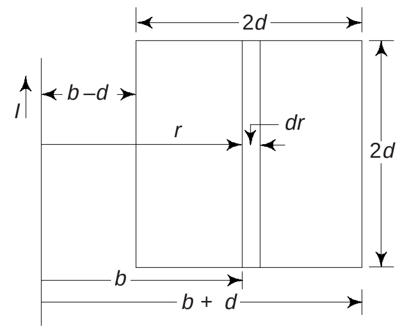


Fig. P2.1

Flux passing through elemental strip

$$d\phi = B\, dA = \frac{\mu_0 I}{2\pi r} \times 2d\, dr$$

$$= \left(\frac{\mu_0 I d}{\pi}\right) \frac{dr}{r}$$

Hence $$\phi = \frac{\mu_0 I d}{\pi} \int_{b-d}^{b+d} \frac{dr}{r}$$

$$= \frac{\mu_0 I d}{\pi} \ln \left(\frac{b+d}{b-d}\right) \text{ Wb}$$

If conductor is normal to the plane of the loop, flux through loop is zero, independent of its relative location.

2.2 *For the magnetic circuit of Fig. P2.2, find the flux density and flux in each of the outer limbs and the central limbs. Assume μ_r for iron of the core to be (a) ∞ (b) 4500.*

Solution

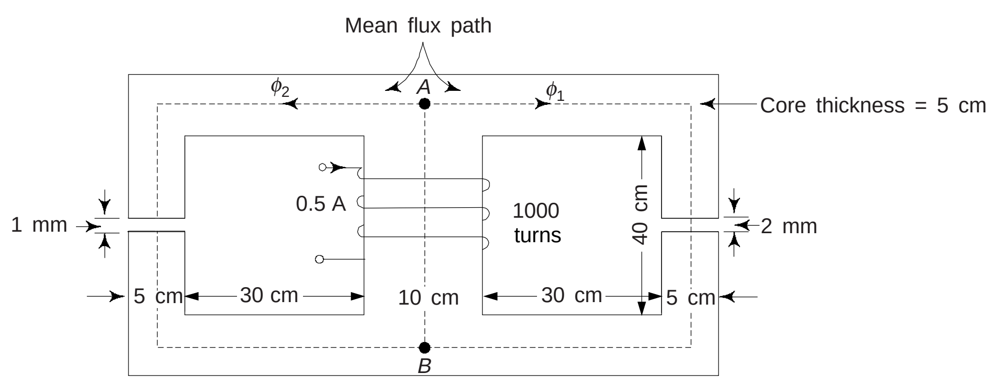


Fig. P2.2

(a) $\mu_r = \infty$. The corresponding electrical analog of magnetic circuit is shown in Fig. P2.2(a).

$$Ni = 1000 \times 0.5 = 500 \text{ AT}$$

$$R_{g1} = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 25 \times 10^{-4}} = 0.6366 \times 10^6 \text{ AT/Wb}$$

$$R_{g2} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 25 \times 10^{-4}} = 0.3183 \times 10^6 \text{ AT/Wb}$$

$$\phi_1 = \frac{500}{0.6366 \times 10^6} = 0.785 \text{ mWb}$$

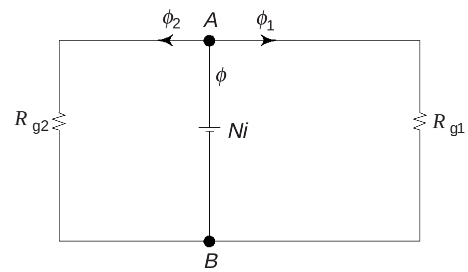


Fig. P2.2(a)

$$B_1 = \frac{0.785 \times 10^{-3}}{25 \times 10^{-4}} = 0.314 \text{ T}$$

$$\phi_2 = \frac{500}{0.3183 \times 10^6} = 1571 \text{ mWb}$$

$$B_2 = \frac{157 \times 10^{-3}}{25 \times 10^{-4}} = 0.628 \text{ T}$$

$$\phi = \phi_1 + \phi_2 = 2.356 \text{ mWb}$$

$$B = \frac{2.356 \times 10^{-3}}{50 \times 10^{-4}} = 0.471 \text{ T}$$

(b) $\mu_r = 4{,}500$. The corresponding analogous electrical circuit is given in Fig. P2.2(b). Effect of air-gaps on iron path length is negligible.

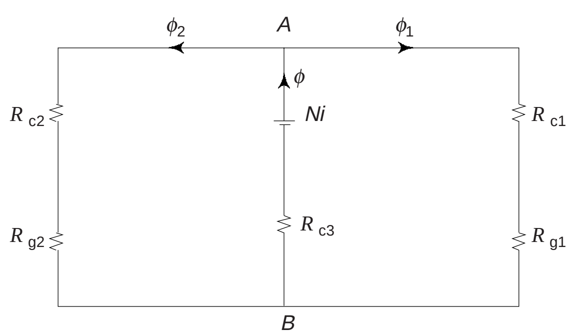


Fig. P2.2(b)

$$l_{c1} = l_{c2} = (40 + 5) + 2 \times (30 + 5 + 2.5) = 120 \text{ cm}$$

$$R_{c1} = R_{c2} = \frac{120 \times 10^{-2}}{4\pi \times 10^{-7} \times 4{,}500 \times 25 \times 10^{-4}} = 0.085 \times 10^6 \text{ AT/Wb}$$

$$l_{c3} = 40 + 5 = 45 \text{ cm}$$

$$R_{c3} = \frac{45 \times 10^{-2}}{4\pi \times 10^{-7} \times 4{,}500 \times 50 \times 10^{-4}} = 0.016 \times 10^6 \text{ AT/Wb}$$

$$R_{eq} = [(R_{c1} + R_{g1}) \parallel (R_{c2} + R_{g2})] + R_{c3}$$

$$R_{c1} + R_{g1} = 0.6366 + 0.084 = 0.7206 \times 10^6$$

$$R_{c2} + R_{g2} = 0.3183 + 0.085 = 0.4033 \times 10^6$$

$$R_{eq} = \left[\frac{0.7206 \times 0.4033}{(0.7206 + 0.4033)} + 0.016\right] \times 10^6 = 0.2746 \times 10^6 \text{ AT/Wb}$$

$$\phi = \frac{500}{0.2742 \times 10^6} = 1823 \text{ mWb}$$

$$B = \frac{1.823 \times 10^{-3}}{50 \times 10^{-4}} = 0.365 \text{ T}$$

$$\phi_1 = 1.823 \times \frac{0.4033}{1.1239} = 0.654 \text{ mWb}$$

$$B_1 = \frac{0.653 \times 10^{-3}}{25 \times 10^{-4}} = 0.261 \text{ T}$$

$$\phi_2 = 1.823 \times \frac{0.7206}{1.1239} = 1.17 \text{ mWb}$$

$$B_2 = \frac{1.17 \times 10^{-3}}{25 \times 10^{-4}} = 0.468 \text{ T}$$

2.3 *For the magnetic circuit shown in Fig. P2.3, calculate the exciting current required to establish a flux of 2 mWb in the air-gap. Take fringing into account empirically. Use the B–H curve of Fig. 2.15.*

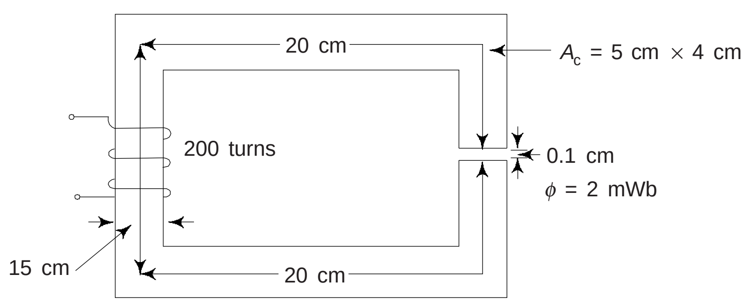


Fig. P2.3

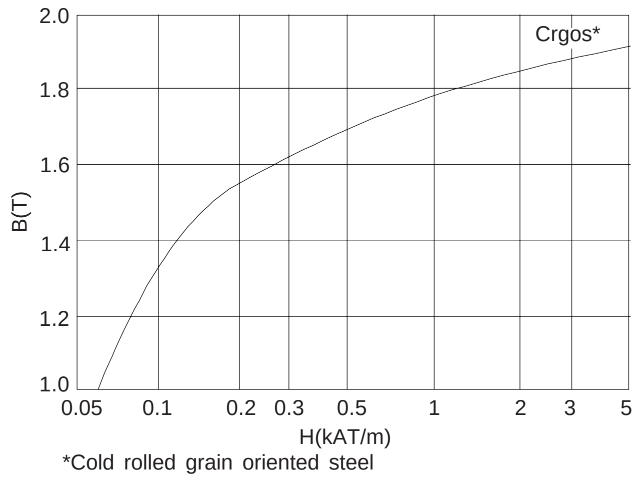


Fig. P2.3(a)

Solution Taking fringing into account empirically

$$A_g = (5 + 0.1)\ (4 + 0.1) = 20.91 \times 10^{-4}\ \text{m}^2$$

$$B_g = \frac{2 \times 10^{-3}}{20.91 \times 10^{-4}} = 0.957\ \text{T}$$

$$H_g = \frac{0.957}{4\pi \times 10^{-7}} = 7.616 \times 10^5\ \text{AT}$$

$$\text{AT}_g = 7.616 \times 10^5 \times 0.1 \times 10^{-2} = 761.6$$

$$B_c = \frac{2 \times 10^{-3}}{20 \times 10^{-4}} = 1\ \text{T}$$

Corresponding H_c is obtained from B–H curve of Fig. 2.15

$$H_c = 0.06\ \text{kAT/m} = 60\ \text{AT/m}$$

$$AT_c = 60 \times 55 \times 10^{-2} = 33$$

$$i = \frac{1}{200}\ (33 + 761.6) = 3.973\ \text{A}$$

2.4 *A steel ring has a mean diameter of 20 cm, a cross-section of 25 cm*^{2} *and a radial air-gap of 0.8 mm cut across it. When excited by a current of 1A through a coil of 1000 turns wound on the ring core, it produces an air-gap flux of 1 mWb. Neglecting leakage and fringing, calculate (a) relative permeability of steel, and (b) total reluctance of the magnetic circuit.*

Solution

$$l_c = \pi \times 20 - 0.08 = 62.75\ \text{cm};\quad l_g = 0.08\ \text{cm};\quad A_g = 25\ \text{cm}^2$$

$$R_g = \frac{0.8 \times 10^{-3}}{4\pi \times 10^{-7} \times 25 \times 10^{-4}} = 0.255 \times 10^6$$

$$F = 1000 \times 1 = 1000\ \text{AT}$$

$$\phi = 1\ \text{mWb}$$

(a) $$R_{\text{(core)}} = (1 - 0.255) \times 10^6 = 0.745 \times 10^6 \quad \text{(i)}$$

(b) $$R_{\text{(total)}} = \frac{1000}{1} \times 10^3 = 1 \times 10^6 \quad \text{(ii)}$$

From (i)

$$0.745 \times 10^6 = \frac{62.75 \times 10^{-2}}{4\pi \times 10^{-7} \times \mu_{rc} \times 25 \times 10^{-4}}$$

$$\therefore \quad \mu_{rc} = 268$$

2.5 *The core made of cold-rolled silicon steel (B–H curve of Fig. 2.15) is shown in Fig. P2.5. It has a uniform cross-section (not iron) of 5.9 cm² and a mean length of 30 cm. Coils A, B and C carry 0.4, 0.8 and 1 A respectively in the directions shown. Coils A and B have 250 and 500 turns respectively. How many turns must coil C have to establish a flux of 1 mWb in the core?*

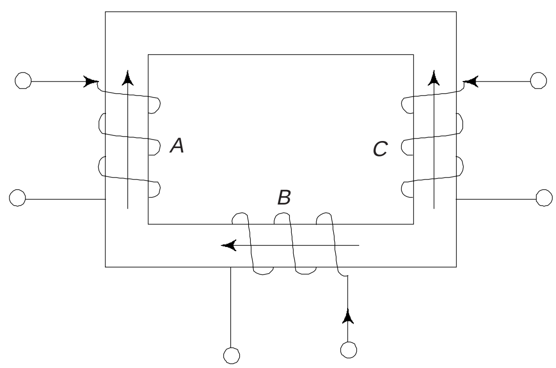


Fig. P2.5

Solution $N_A i_A = 250 \times 0.4 = 100$ AT

$N_B i_B = 500 \times 0.8 = 400$ AT

$$B = \frac{1 \times 10^{-3}}{5.9 \times 10^{-4}} = 1.695 \text{ T}$$

Corresponding H from B–H curve of Fig. 2.15 is

$$H = 0.5 \text{ kAT/m} = 500 \text{ AT/m}$$

$$(AT)_{net} = 500 \times 30 \times 10^{-2} = 150$$

$$= (AT)_A + (AT)_B - (AT)_C$$

or $$(AT)_C = 100 + 400 - 150 = 350$$

$$N_C i_C = 350 \quad \therefore \quad N_C = 350$$

2.6 *In the magnetic circuit shown in Fig. P2.6, the coil F_1 is supplying 4000 AT in the direction indicated. Find the AT of coil F_2 and current direction to produce air-gap flux of 4 mWb from top to bottom. The relative permeability of iron may be taken as 2500.*

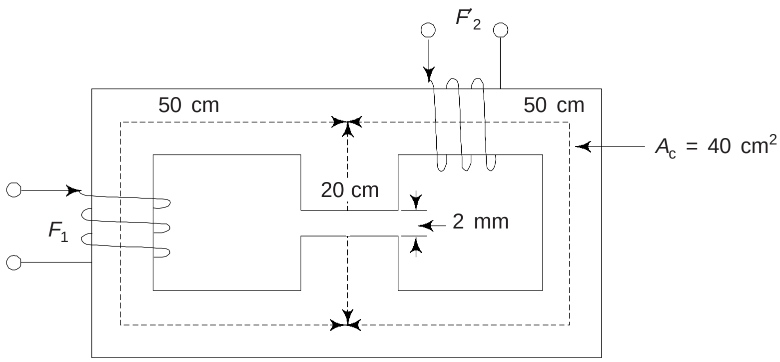


Fig. P2.6

Solution The equivalent electric circuit is shown in Fig. 2.6(a).

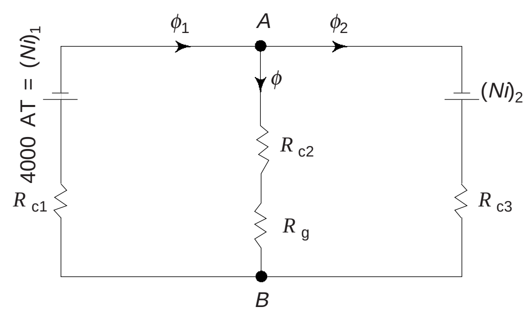


Fig. P2.6(a)

$$l_{c1} = l_{c3} = 0.5 \text{ m}$$

$$R_{c1} = R_{c3} = \frac{0.5}{4\pi \times 10^{-7} \times 2{,}500 \times 40 \times 10^{-4}}$$

$$= 3.98 \times 10^4 \text{ AT/Wb}$$

$$R_{c2} = \frac{20 \times 10^{-2}}{4\pi \times 10^{-7} \times 2{,}500 \times 40 \times 10^{-4}}$$

$$= 1.59 \times 10^4 \text{ AT/Wb}$$

$$R_g = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 40 \times 10^{-4}} = 39.79 \times 10^4 \text{ AT/Wb}$$

$$\phi = 4 \text{ mWb}$$

$$(\text{AT})_{AB} = \phi\,(R_{c2} + R_g)$$

$$= 4 \times 10^{-3}\,(159 + 39.79) \times 10^4 = 1{,}655$$

$$\phi_1 = \frac{4{,}000 - 1{,}655}{3.98 \times 10^4} = 58.9 \text{ mWb}$$

$$\phi_2 = \phi_1 - \phi = 58.9 - 4 = 54.9 \text{ mWb}$$

$$\frac{1{,}655 + (Ni)_2}{3.98 \times 10^4} = 54.9 \times 10^{-3}$$

$$1{,}655 + (Ni)_2 = 2{,}185 \quad \text{or} \quad (Ni)_2 = 530 \text{ AT}$$

2.7 *For the magnetic circuit shown in Fig. P2.7, the air-gap flux is 0.24 mWb and the number of turns of the coil wound on the central limb is 1000.*

Calculate (a) the flux in the central limb and (b) the current required. The magnetization curve of the core is as follows:

H (AT/m)	*200*	*400*	*500*	*600*	*800*	*1060*	*1400*
B (T)	*0.4*	*0.8*	*1.0*	*1.1*	*1.2*	*1.3*	*1.4*

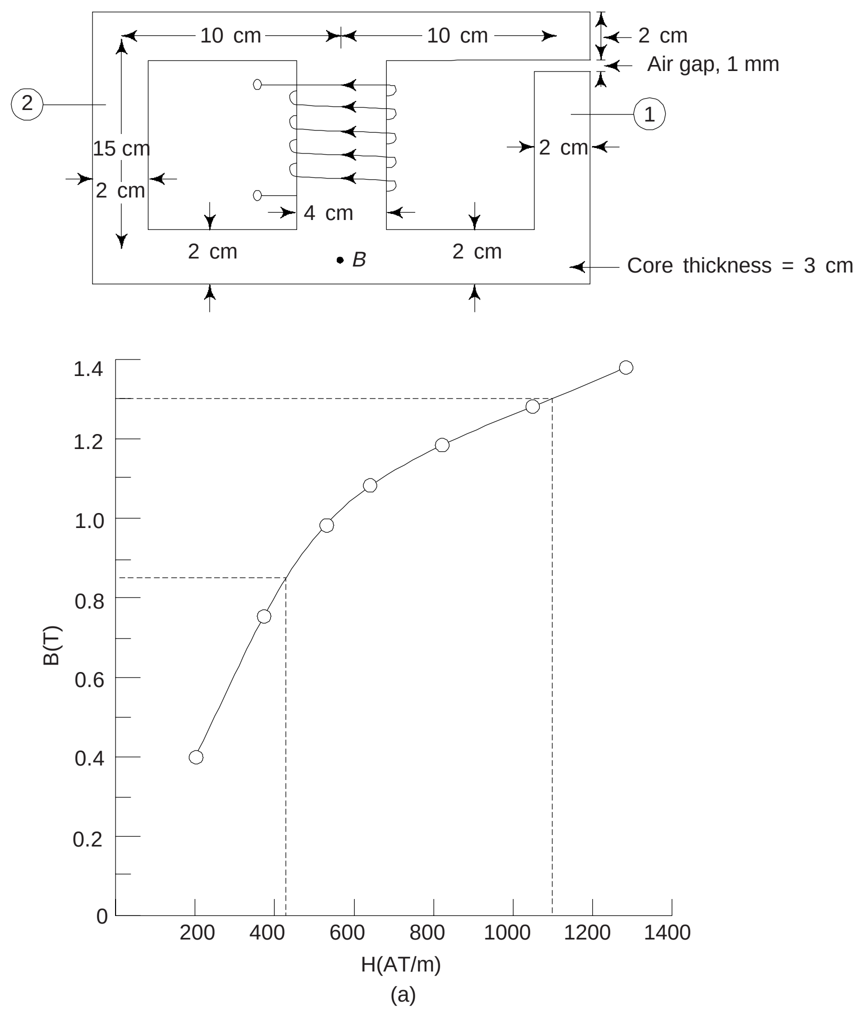


Fig. P2.7

Solution

$$B_g = \frac{0.24 \times 10^{-3}}{(2 \times 3) \times 10^{-4}} = 0.4 \text{ T}$$

$$H_g = \frac{0.4}{4\pi \times 10^{-7}} = 31.83 \times 10^4 \text{ AT/m}$$

$$B_c = B_g = 0.4 \text{ T (no fringing)}$$

From the B–H data given, H_1 = 200 AT/m

$$(\text{AT})_{AB} = H_1 l_1 + H_g l_g$$
$$= 200 \times (2 \times 10 + 15) \times 10^{-2} + 31.83 \times 10^4 \times 1 \times 10^{-3} = 388.3$$

For the left limbs

$$H_2 = \frac{(\text{AT})_{\text{AB}}}{l_2} = \frac{388.3}{35\times10^{-2}} = 1{,}109 \text{ AT/m}$$

From the B–H curve given

$$B_2 = 1.31 \text{ T}$$
$$\phi_2 = 1.31 \times (2 \times 3 \times 10^{-4}) = 0.786 \text{ mWb}$$
$$\phi_c \text{ (central limb)} = 0.24 + 0.786 = 1.026 \text{ mWb}$$
$$B_c \text{ (central limb)} = \frac{1.026\times10^{-3}}{4\times3\times10^{-4}} = 0.855 \text{ T}$$
$$H_c \text{ (central limb)} = 430 \text{ AT/m}$$
$$\text{AT}_C \text{ (central limb)} = 430 \times 15 \times 10^{-2} = 64.5$$
$$(\text{AT})_{\text{total}} = (\text{AT})_{\text{AB}} + 64.5$$
$$= 388.3 + 64.5 = 452.8$$
$$i = \frac{452.8}{1{,}000} = 0.453 \text{ A}$$

2.8 *The magnetic circuit shown in Fig. P2.8 has a coil of 500 turns wound on the central limb which has an air-gap of 1 mm. The magnetic path from A to B via each outer limb is 100 cm and via the central limb 25 cm (air-gap length excluded). The cross-sectional area of the central limb is 5 cm × 3 cm and each outer limb is 2.5 cm × 3 cm. A current of 0.5 A in the coil produces an air-gap flux of 0.35 mWb. Find the relative permeability of the medium.*

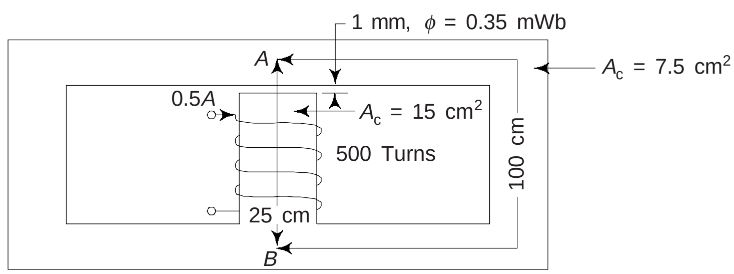


Fig. P2.8

Solution Figure P2.8(a) gives the electric equivalent of the magnetic circuit of Fig. P2.8.

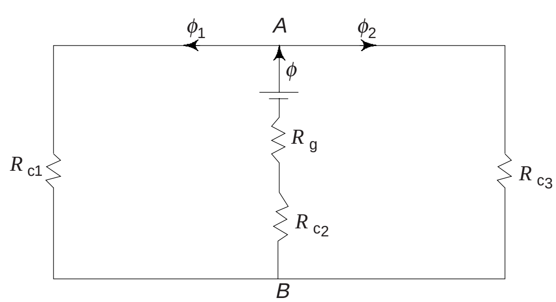


Fig. P2.8(a) *Equivalent Electric Circuit of Fig. P2.18*

$$R_{c1} = R_{c3} = \frac{100 \times 10^{-2}}{4\pi \times 10^{-7} \times \mu_r \times 7.5 \times 10^{-4}} = \frac{1061 \times 10^{9}}{\mu_r} \text{ AT/Wb}$$

$$R_{c2} = \frac{25 \times 10^{-2}}{4\pi \times 10^{-7} \times \mu_r \times 15 \times 10^{-4}} = \frac{0.133}{\mu_r} \times 10^9 \text{ AT/Wb}$$

$$R_g = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 15 \times 10^{-4}} = 0.5305 \times 10^6 \text{ AT/Wb}$$

$$Ni = \phi(R_{c1} \parallel R_{c3} + R_g + R_{c2})$$

$$500 \times 0.5 = 0.35 \times 10^{-3} \left(\frac{0.531 \times 10^9}{\mu_r} + \frac{0.133 \times 10^9}{\mu_r} + 0.5305 \times 10^6 \right)$$

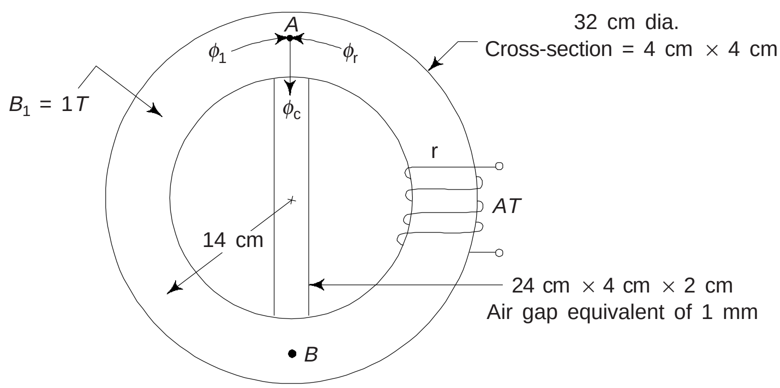
$$\mu_r = 3{,}612$$

2.9 *A cast steel ring has an external diameter of 32 cm and a square cross-section of 4 cm side. Inside and across the ring, a cast steel bar 24 × 4 × 2 cm is fitted, the butt-joints being equivalent to a total air-gap of 1 mm. Calculate the ampere-turns required on half of the ring to produce a flux density of 1 T in the other half. Given:*

H (AT/m)	*0*	*200*	*400*	*600*	*800*	*1000*	*1200*	*1400*	*1600*
B (T)	*0*	*0.11*	*0.32*	*0.6*	*0.8*	*1.0*	*1.18*	*1.27*	*1.32*

Solution Figure P2.9(a) shows the sketch of the cast steel ring.

B_1 = 1T; H_1 = 1,000 AT/m (From graph of Fig. P2.9(b))



(a)

Fig. P2.9(a)

$$(AT)_{AB} = AT_1 = 1{,}000 \times \pi \times 14 \times 10^{-2} = 439.82$$

$$\phi_1 = 1 \times 16 \times 10^{-4} = 1.6 \text{ mWb}$$

$$AT_C = (AT)_{AB} = 439.82$$

$$439.82 = \frac{B_c}{4\pi \times 10^{-7}} \times 1 \times 10^{-3} + H_c \times 0.28$$

$$439.82 = 795.77\ B_c + 2.28\ H_c$$

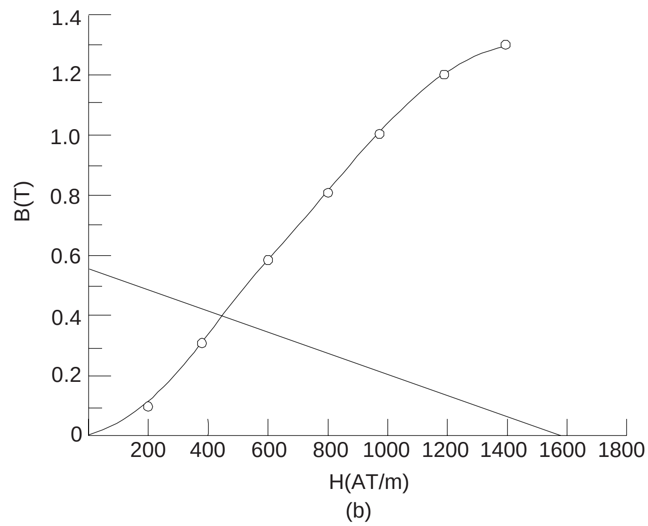


Fig. P2.9(b)

Intersection of the above straight line with B–H curve gives

$$B_c = 0.39 \text{ T};\ \phi_c = 0.39 \times 8 \times 10^{-4} = 0.312 \text{ mWb}$$

$$\phi_r = \phi_1 + \phi_c = (1.6 + 0.312) \text{ mWb} = 1.912 \text{ mWb}$$

$$B_r = \frac{1.912 \times 10^{-3}}{16 \times 10^{-4}} = 1195 \text{ T}$$

From graph, $H_r = 1230$ AT/m

$\therefore \quad AT_r = 1230 \times \pi \times 14 \times 10^{-2} = 540.98$

Total $\quad AT = 439.82 + 540.98 = 980.8$

2.10 *In Prob. 2.2 the B-H curve of the core material is characterized by the data given below. Find the flux and flux densities in the three limbs of the core.*

H (AT/m)	*50*	*100*	*150*	*200*	*250*	*300*	*350*
B (T)	*0.14*	*0.36*	*0.66*	*1.00*	*1.22*	*1.32*	*1.39*

Hint This problem can be solved by the graphical-cum-iterative technique.

Solution The B–H curve as per the data is drawn in Fig. P2.10.

Using the solution of 2.1(a) as a starting point:

$B_1 = 0.34 \rightarrow H_1 = 90;\quad H_1 l_1 = 90 \times 1.2 = 108$

$B_2 = 0.628 \rightarrow H_2 = 145;\quad H_2 l_2 = 145 \times 1.2 = 174$

$B = 0.471 \rightarrow H = 120;\quad Hl = 120 \times 0.45 = 54$

$$AT_{g1} = 500 - 108 - 54 = 338 \qquad B_1 \text{ (new)} = \frac{440 \times 4\pi \times 10^{-7}}{2 \times 10^{-3}} = 0.212$$

$$AT_{g2} = 500 - 174 - 54 = 272 \qquad B_2 \text{ (new)} = \frac{272 \times 4\pi \times 10^{-7}}{1 \times 10^{-3}} = 0.342$$

$$B\text{(new)} = \frac{0.212 \times 0.342}{2} = 0.277$$

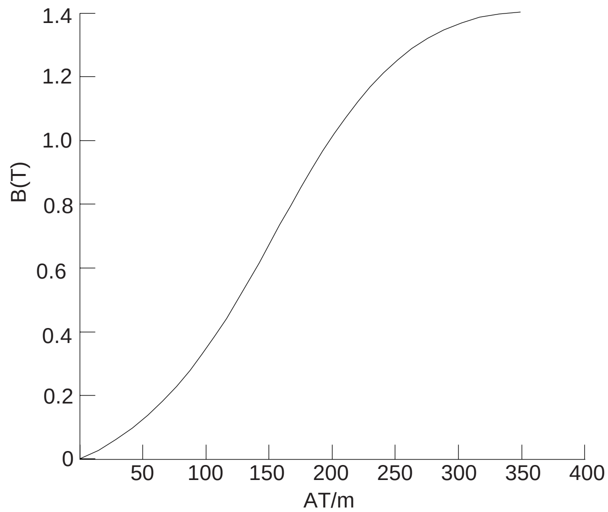


Fig. P2.10

$$B_1 = 0.212 \qquad H_1 = 70 \qquad H_1 l_1 = 70 \times 1.2 = 84$$
$$B_2 = 0.342 \qquad H_2 = 95 \qquad H_2 l_2 = 95 \times 1.2 = 114$$
$$B_1 = 0.277 \qquad H = 83 \qquad Hl = 83 \times 0.45 = 38$$

$$AT_{g1} = 500 - 84 - 38 = 378 \qquad B_1 = \frac{378 \times 4\pi \times 10^{-7}}{2 \times 10^{-3}} = 0.237$$

$$AT_{g2} = 500 - 114 - 38 = 348 \qquad B_2 = \frac{348 \times 4\pi \times 10^{-7}}{1 \times 10^{-3}} = 0.437$$

$$B = (0.237 + 0.437)/2 = 0.337 \text{ T}$$

$$B_1 = 0.237 \qquad H_1 = 70 \qquad H_1 l_1 = 70 \times 1.2 = 84$$
$$B_2 = 0.437 \qquad H_2 = 120 \qquad H_2 l_2 = 120 \times 1.2 = 144$$
$$B_4 = 0.337 \qquad H = 95 \qquad Hl = 95 \times 0.43 = 40.85$$

$$AT_{g1} = 500 - 84 - 43 = 373 \qquad B_1 = \frac{373 \times 4\pi \times 10^{-7}}{2 \times 10^{-3}} = 0.234$$

$$AT_{g2} = 500 - 144 - 43 = 313 \qquad B_2 = 313 \times 4\pi \times 10^{-4} = 0.4$$

$$B = \frac{0.634}{2} = 0.317 \text{ T}$$

(Almost converged)

2.11 *A ring of magnetic material has a rectangular cross-section. The inner diameter of the ring is 20 cm and the outer diameter is 25 cm, its thickness being 2 cm. An air-gap of 1 mm length is cut across the ring. The ring is wound with 500 turns and when carrying a current of 3 A produces a flux density of 1.2 T in the air-gap. Find (a) magnetic field intensity in the magnetic material and in the air-gap (b) relative permeability of the magnetic material, and (c) total reluctance of the magnetic circuit and component values.*

Solution Figure P2.11 gives the sketch of the magnetic ring.

$$Ni = 500 \times 3 = 1{,}500 \text{ AT}$$

$$B_c = B_g = 1.2 \text{ T (no fringing)}$$

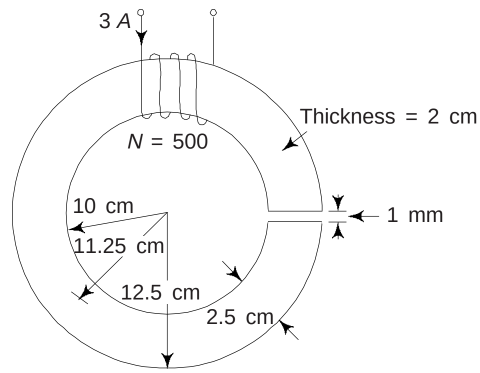


Fig. P2.11

(a) $$H_g = \frac{1.2}{4\pi \times 10^{-7}} = 9.549 \times 10^5 \text{ AT/m}$$

$$500 \times 3 = 9.549 \times 10^5 \times 1 \times 10^{-3} + H_c \times 2\pi \times 11.25 \times 10^{-2}$$

$\therefore$ $$H_c = 771.16 \text{ AT/m}$$

(b) $$H_c = \frac{B_c}{\mu_0 \mu_r}$$

$\therefore$ $$\mu_r = \frac{1.2}{4\pi \times 10^{-7} \times 771.16} = 1{,}238.3$$

(c) $$R_{\text{total}} = R_g + R_c$$

$$= \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 2 \times 2.5 \times 10^{-4}} + \frac{2\pi \times 11.25 \times 10^{-2}}{4\pi \times 10^{-7} \times 1{,}238.3 \times 2 \times 2.5 \times 10^{-4}}$$

$$= 1.592 \times 10^6 + 0.909 \times 10^6 = 2.5 \times 10^6 \text{ AT/Wb}$$

2.12 *For the magnetic ring of Prob. 2.11, the exciting current is again 3 A. Find the following:*

(a) Inductance of the coil

(b) energy stored in the magnetic material and in the air-gap

(c) rms emf induced in the coil when it carries alternating current of 3 sin 314 t.

Solution Refer to Fig. P2.11.

$$\phi = BA = 1.2 \times 2.5 \times 2 \times 10^{-4} = 0.6 \text{ mWb}$$

(a) $$\lambda = N\phi = 500 \times 0.6 \times 10^{-3} = 0.3 \text{ WbT}$$

$$L = \frac{\lambda}{i} = \frac{0.3}{3} = 0.1\text{H}$$

(b) $$W_{fc} = A_c l_c \int_0^{B_c} H_c \, dB_c = \frac{A_c l_c}{\mu_0 \mu_r} \int_0^{B_c} B_c \, dB_c = \frac{1}{2}\left(\frac{A_c l_c}{\mu_0 \mu_r}\right)\mathbf{B}_c^2$$

μ_r (as determined in Prob. 2.9) = 1,238.3

$$W_{fc} = \frac{1}{2} \times \frac{5 \times 10^{-4} \times 22.5 \times \pi \times 10^{-2}}{4\pi \times 10^{-7} \times 1{,}238.3} \times (1.2)^2$$

$$= 0.1635 \text{ J}$$

$$B_g = B_c = 1.2 \text{ T (no fringing)}$$

$$W_{fg} = \frac{1}{2} \times \frac{5 \times 10^{-4} \times 10^{-3}}{4\pi \times 10^{-7}} (1.2)^2$$

$$= 0.2865 \text{ J}$$

(c) $$e = \frac{d\lambda}{dt} = \frac{d}{dt}\, 0.3 \sin 314\, t$$
$$= 94.2 \cos 314\, t \text{ V}$$

2.13 *Assume that the core of the magnetic circuit of Fig. P2.3 has μ_r = 2500.*

(a) Calculate the energy stored in the core and in the air-gap for an excitation current of 5 A. What will be these values if $\mu_r = \infty$?

(b) What will be the excitation current to produce a sinusoidally varying flux of 0.5 sin 314 t mWb in the air-gap?

(c) Calculate the inductance of the coil. What will be the inductance if $\mu_r = \infty$?

Solution

(a) $$R_c = \frac{70 \times 10^{-2}}{4\pi \times 10^{-7} \times 2{,}500 \times 20 \times 10^{-4}} = 0.111 \times 10^6 \text{ AT/Wb}$$

$$R_g = \frac{0.1 \times 10^{-2}}{4\pi \times 10^{-7} \times 20.91 \times 10^{-4}} = 0.381 \times 10^6 \text{ AT/Wb}$$

$$R_{\text{total}} = 0.492 \times 10^6 \text{ AT/Wb}$$

$$200 \times 5 = \phi\, R_{\text{total}}$$

or $$\phi = \frac{200 \times 5}{0.492 \times 10^6} = 2.033 \text{ mWb}$$

$$W_f = (1/2)\ \phi^2 R$$

$$W_f \text{ (core)} = (1/2) \times (2.033 \times 10^{-3})^2 \times 0.111 \times 10^6 = 0.23 \text{ J}$$

$$W_f \text{ (air-gap)} = (1/2) \times (2.033 \times 10^{-3})^2 \times 0.381 \times 10^6 = 0.787 \text{ J}$$

Let $$\mu_r = \infty \quad \Rightarrow \quad R_c = 0$$

$$R_{\text{total}} = R_g = 0.381 \times 10^6 \text{ AT/Wb}$$

$$\phi = \frac{200 \times 5}{0.381 \times 10^6} = 2.625 \text{ mWb}$$

$$W_f \text{ (air-gap)} = (1/2) \times (2.625 \times 10^{-3}) \times 0.38 \times 10^6 = 1.312 \text{ J}$$

$$W_f \text{ (core)} = 0 \text{ (as } R_c = 0)$$

(b) $$\frac{N}{R_{\text{total}}} = 0.5 \times 10^{-3} \sin 314\, t$$

$$i = \left(\frac{0.492 \times 10^6 \times 0.5 \times 10^{-3}}{200} \right) \sin 314\, t$$

$$= 1.23 \sin 314\, t \text{ A}$$

(c) $$L = N^2\, P = \frac{(200)^2}{0.492 \times 10^6} = 0.0813 \text{ H}$$

If $$\mu_r = \infty$$

$$R_{\text{total}} = R_g = 0.381 \times 10^6 \text{ AT/Wb}$$

∴ $$L = \frac{(200)^2}{0.381 \times 10^6} = 0.105 \text{ H}$$

2.14 *The magnetic circuit of Fig. P2.14 has a magnetic core of relative permeability 1600 and is wound with a coil of 1500 turns excited with sinusoidal ac voltage, as shown. Calculate the maximum flux density of the core and the peak value of the exciting current. What is the peak value of the energy stored in the magnetic system and what percentage of it resides in the air-gap?*

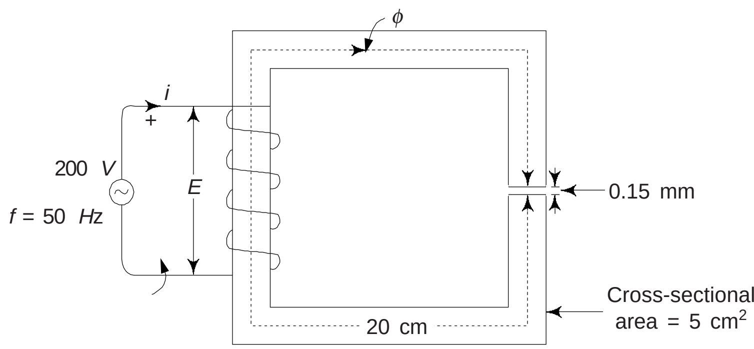


Fig. P2.14

Solution

$$E \approx V = 200 = 4.44 \times 50 \times 1500 \times \phi_{max}$$

or

$$\phi_{max} = \frac{200}{4.44 \times 50 \times 1500} = 0.6 \text{ mWb}$$

$$B_{max} = \frac{0.6}{1000 \times 5 \times 10^{-4}} = \frac{6}{5} = 1.2 \text{ T}$$

$$R_c = \frac{20 \times 10^{-2}}{4\pi \times 10^{-7} \times 1600 \times 5 \times 10^{-4}} = 0.2 \times 10^6$$

$$R_g = \frac{0.15 \times 10^{-3}}{4\pi \times 10^{-7} \times 5 \times 10^{-4}} = 0.239 \times 10^6$$

$$R_{(total)} = 0.439 \times 10^6$$

$$F_{max} = i_{max} \times 1500 = 0.439 \times 10^6 \times 0.6 \times 10^{-3}$$

$$i_{max} = 0.176 \text{ A}$$

$$W(\text{peak}) = \left(\frac{1}{2}\right)\phi_{max}^2 \; R_{(total)} = \frac{1}{2}\,(0.6 \times 10^{-3})^2 \times 0.439 \times 10^6$$

$$= 0.079 \text{ J}$$

$$\% \text{ of energy in air-gap} = \frac{0.239}{0.439} \times 100 = 54.4$$

2.15 *The material of the core of Fig. P2.15 wound with two coils as shown, is sheet steel (B-H curve of Fig. 2.15. Coil 2 carries a current of 2 A in the direction shown. What current (with direction) should coil 1 carry to establish a flux density of 1.4 T in the core in the indicated direction?*

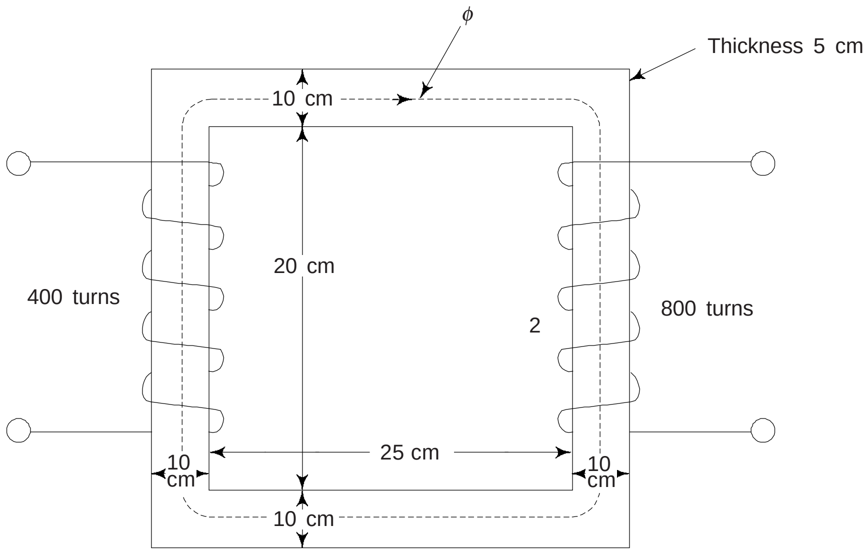


Fig. P2.15

Solution

$$B_c = 1.4\text{ T},\; H_c \text{ (from Fig. 2.15)} = 450\text{ AT/m.}$$
$$l_c = 2 \times (20 + 10) + 2\,(25 + 10) = 130\text{ cm}$$
$$AT_{net} = 450 \times 1.3 = 585$$
$$AT_2 = 800 \times 2 = 1600$$
$$AT_{net} = AT_1 \text{ (opposition to } AT_2) - AT_2$$
$$585 = AT_1 - 1600$$
$$AT_1 = 2185$$
$$I_1 = \frac{2185}{400} = 5.463\text{ A}$$

2.16 *The flux in a magnetic core is alternating sinusoidal at a frequency of 600 Hz. The maximum flux density is 2 T and the eddy-current loss is 15 W. Find the eddy-current loss in the core if the frequency is raised to 800 Hz and the maximum flux density is reduced to 1.5 T.*

$$P_e = k_e f^2\, B_m^2$$

$$\therefore \quad \frac{P_e}{15} = \left(\frac{800}{600}\right)^2 \left(\frac{15}{2}\right)^2$$

$$\therefore \quad P_e = 15\text{ W}$$

2.17 *The core-loss (hysteresis + eddy-current loss) for a given specimen of magnetic material is found to be 2000 W at 50 Hz. Keeping the flux density constant, the frequency of the supply is raised to 75 Hz resulting in a core loss of 3200 W. Compute separately hysteresis and eddy current losses at both the frequencies.*

Hint: (see page 46–47)

Solution

$$P_L = P_e + P_h = k_e f^2 B_m^2 V + k_h f B_m^n V$$

$$= k_e' f^2 + k_h' f \text{ (since } B_m \text{ constant)}$$

$$\frac{P_L}{f} = k_e' f + k_h'$$

For $P_L = 2{,}000$, $f = 50$ Hz

or $$\frac{2000}{50} = 50 k_e' + k_h'$$

$$50 k_e' + k_h' = 40 \quad (1)$$

$P_L = 3{,}200$, $f = 75$ Hz

$$\frac{3200}{75} = 75 k_e' + k_h'$$

$$75 k_e' + k_h' = \frac{128}{3} \quad (2)$$

Solving Eqs (1) and (2)

$$k_e' = \frac{8}{75} \; 0.1067; \qquad k_h' = \frac{104}{3} = 34.67$$

At $f = 50$ Hz

$\therefore$ $$P_e = k_e' f^2 = 266.7 \text{ W}; \qquad P_h = k_h' f = 1{,}733 \text{ W}$$

At $f = 75$ Hz

$$P_e = k_e' f^2 = 600 \text{ W}; \qquad P_h = k_h' f = 2{,}600 \text{ W}$$

CHAPTER 3: TRANSFORMERS

3.1 *The emf per turn of a single-phase 2200/220 V, 50 Hz transformer is approximately 12 V. Calculate (a) the number of primary and secondary turns, and (b) the net cross-sectional area of core for a maximum flux density of 1.5 T.*

Solution

(a) emf per turn = 12 V

or $$\frac{V_1}{N_1} = \frac{V_2}{N_2} = 12 \text{ V} \qquad \text{(i)}$$

Also $V_1 = 2200$ V; $V_2 = 220$ V

Therefore, from (i) we have

$$N_1 = \frac{V_1}{12} = \frac{2200}{12} = 183.33 = 183 \text{ (turns cannot be fractional)}$$

Similarly, $$N_2 = \frac{V}{12} = \frac{220}{12} = 18.33 = 18$$

(b) $V_1 \approx E_1 = 4.44\ fN_1\ \phi_{max}$

or $$\phi_{max} = \frac{V_1}{4.44\, f N_1} = \frac{12}{4.44 \times 50} = 0.054 \text{ Wb}$$

$$A_c = \frac{\phi_{max}}{B_{max}} = \frac{0.054}{15} = 0.036 \text{ m}^2$$

3.7 *A 100 kVA, 1100/230 V, 50 Hz transformer has an HV winding resistance of 0.1 Ω and a leakage reactance of 0.4 Ω. The LV winding has a resistance of 0.006 Ω and a leakage reactance of 0.01 Ω. Find the equivalent winding resistance, reactance and impedance referred to the HV and LV sides. Convert these to pu values.*

Solution HV suffix 1, LV suffix 2

$$\bar{z}_1 = r_1 + j\,x_{l1} = (0.1 + j\,0.4)\ \Omega$$
$$\bar{z}_2 = r_2 + j\,x_{l2} = (0.006 + j\,0.01)\ \Omega$$

$$\bar{z}_2' = a^2(r_2 + j\,x_{l2}) = \left(\frac{1100}{230}\right)^2 (0.006 + j\,0.01) = (0.137 + j\,0.229)\ \Omega$$

$$\bar{Z}(\text{HV}) = \bar{z}_1 + \bar{z}_2' = (0.237 + j\,0.629)\ \Omega$$

Similarly $$\bar{Z}(\text{LV}) = \bar{z}_1' + \bar{z}_2' \left(\frac{230}{1100}\right)^2 (0.1 + j\,0.4) + (0.006 + j\,0.01)$$

$$= (0.0104 + j\,0.0275)\ \Omega$$

Since pu value is the same whether referred to HV or LV side, referring it only to HV winding, we get:

Z_B (base impedance referred to HV winding)

$$Z_B(\text{HV}) = \frac{(\text{kV})_B^2}{(\text{MVA})_B} = \frac{(11)^2}{100 \times 10^{-3}} = 12.1\ \Omega$$

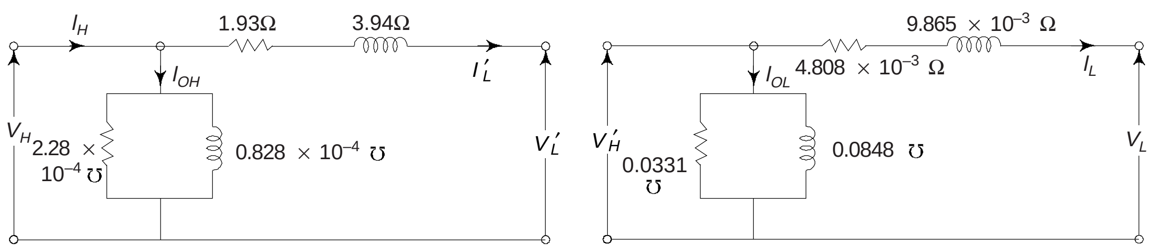
$$\bar{z}_{pu}\ (\text{HV}) = \frac{0.237 + j\,0.629}{12.1} = (0.019 + j\,0.052) \text{ pu}$$

Similarly $$Z_B = (\text{LV}) = \frac{(0.23)^2}{100 \times 10^{-3}} = 0.529\ \Omega$$

$$Z_{pu}\ (\text{LV}) = \frac{0.0104 + j\,0.0275}{0.529} = (0.0196 + j\ 0.052)\ \text{pu}$$

$$= Z_{pu}(\text{HV})\ (\text{notice})$$

3.8 *A 50 kVA, 2200/110 V transformer when tested gave the following results:*
OC test, measurements on the LV side: 400 W, 10 A, 110 V
SC test, measurements on the HV side: 808 W, 20.5 A, 90 V
Compute all the parameters of the equivalent circuit referred to the HV and LV sides of the transformer.



(a) Equivalent Circuit Referred to HV Side (b) Equivalent Circuit Referred to LV Side

Fig. P3.8 *Equivalent Circuit Referred to HV Side, Equivalent Circuit Referred to LV Side*

Solution OC Test (LV side)

$$y_0 = \frac{10}{110} = 0.091\ \mho$$

$$G_i = \frac{400}{(110)^2} = 0.0331\ \mho$$

$$B_m = \left(y_0^2 - G_1^2\right)^{0.5} = 0.0848\ \mho$$

SC Test (HV side)

$$Z = \frac{90}{20.5} = 4.39\ \Omega$$

$$R = \frac{808}{(20.5)^2} = 1.923\ \Omega$$

$$X = (Z^2 - R^2)^{0.5} = 3.946\ \Omega$$

Transformation ratio, $$\frac{N_H}{N_L} = \frac{2200}{110} = 20$$

Equivalent circuit referred to HV side

$$G_i(\text{HV}) = 0.091 \times \left(\frac{1}{20}\right)^2 = 2.28 \times 10^{-4}\ \mho$$

$$B_m(\text{HV}) = 0.0331 \times \left(\frac{1}{20}\right)^2 = 0.828 \times 10^{-4}\ \mho$$

R(HV) and X(HV) have already been calculated above.

For equivalent circuit referred to LV side

$$R(\text{LV}) = 1.923 \times \left(\frac{1}{20}\right)^2 = 4.808 \times 10^{-3}\ \Omega$$

$$X(\text{LV}) = 3.946 \times \left(\frac{1}{20}\right)^2 = 9.865 \times 10^{-3}\ \Omega$$

3.10 *A 20 kVA, 2000/200 V, 50 Hz transformer is operated at no-load on rated voltage, the input being 150 W at 0.12 power factor. When it is operating at rated load, the voltage drops in the total leakage reactance and the total resistance are, respectively, 2 and 1% of the rated voltage. Determine the input power and power factor when the transformer delivers 10 kW at 200 V at 0.8 pf lagging to a load on the LV side.*

Solution $\quad \bar{Z}_{\text{pu}} = (0.01 + j\ 0.02)$

$$Z_B(\text{HV}) = \frac{(\text{kV})_B^2}{(\text{MVA})_B} = \frac{2^2}{20 \times 10^{-3}} = 200\ \Omega$$

$\therefore \bar{Z}\,(\text{HV}) = (0.01 + j\ 0.02) \times 200 = (2 + j\ 4)\ \Omega$

Load on transformer = 10 kW at 200 V and 0.8 pf, lagging current drawn by load.

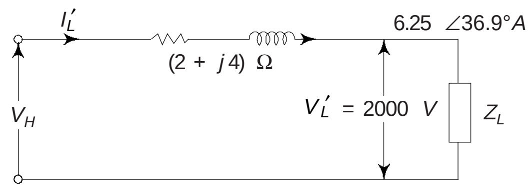


Fig. P3.10(a)

$$I_L = \frac{10 \times 10^3}{200 \times 0.8} = 62.5\ \text{A}$$

$$I_L(\text{HV}) = \frac{62.5}{10} = 6.25\ \text{A}$$

$$P_0 = 10\ \text{kW}; \quad Q_0 = 10 \tan \cos^{-1} 0.8 = 75\ \text{kVAR}$$

$$V_H = 2000 + 6.25\,(2 \times 0.8 + 4 \times 0.6) = 2025\ \text{V}$$

Parameters of the magnetizing branch (Fig. P3.10(b))

$$G_i = \frac{150}{(2000)^2} = 37.5 \times 10^{-6}\ \mho$$

$$\cos \theta_0 = 0.12; \ \theta_0 = 83.1°$$

$$B_m = G_i \tan \theta_0 = 37.5 \times 10^{-6} \times 8.273$$

$$= 310 \times 10^{-6}\ \mho$$

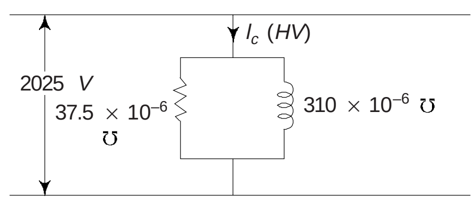


Fig. P3.10(b)

Active power loss in series resistance

$$= (6.25)^2 \times 2 = 0.078\ \text{kW}$$

Reactive power loss in series reactance

$$= (6.25)^2 \times 4 = 0.156\ \text{kVAR}$$

Active power loss in shunt conductance $= (2025)^2 \times 37.5 \times 10^{-6}$

$$= 0.154\ \text{kW}$$

Reactive power loss in shunt susceptance $= (2025)^2 \times 310 \times 10^{-6}$

$$= 1.271\ \text{kVAR}$$

Input power $= (10 + 0.078 + 0.154) + j\,(7.5 + 0.156 + 1.271)$

$$= 10.23 + j\ 8.926$$

Real power input = 10.23 kW

$$\text{pf} = \cos \tan^{-1} \frac{8.926}{10.23} = 0.753 \text{ lagging.}$$

3.11 *A single-phase load is fed through a 66 kV feeder whose impedance is 120 + j 400 Ω and a 66/6.6 kV transformer of equivalent impedance (referred to LV) 0.4 + j 1.5 Ω. The load is 250 kW at 0.8 leading power factor and 6 kV.*

(a) Compute the voltage at the sending end of the feeder.

(b) Compute the voltage at the primary terminals of the transformer.

(c) Compute the complex power input at the sending end of the feeder.

Solution Impedance of 66 kV feeder = (120 + j 400) Ω

Equivalent impedance of transformer

referred to LV side = (0.4 + j 1.5) Ω

referred to HV side = (0.4 + j 1.5) × 10^2 Ω

= (40 + j 150) Ω

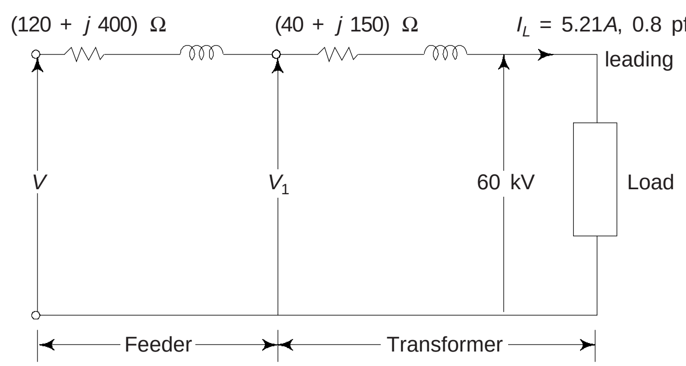


Fig. P3.11

Load of 250 kW at 0.8 pf leading at 6 kV

$$I_L = \frac{250}{6 \times 0.8} = 52.08 \text{ A}$$

$$I'_L \text{ (HV side)} = \frac{52.08}{10} = 5.21 \text{ A}$$

(a) Sending end voltage,

$$V = 60 \times 10^3 + 5.21\,(160 \times 0.8 - 550 \times 0.6) = 58.95 \text{ kV}$$

(b) Primary voltage of transformer

$$V_1 = 60 \times 10^3 + 5.21\,(40 \times 0.8 - 150 \times 0.6) = 59.7 \text{ kV}$$

Active power loss = $(5.21)^2 \times 160 \times 10^{-3}$ = 4.34 kW

Reactive power loss = $(5.21)^2 \times 550 \times 10^{-3}$ = 14.92 kVAR

Power received = (250 – j 250 tan $\cos^{-1}$ 0.8) = 250 – j 187.5

Complex power input (sending end) = (250 + 4.34) + j(–187.5 + 14.92)

= (254.34 – j 172.6) kVA

$$\text{pf} = \cos \tan^{-1} \frac{172.6}{254.34} = 0.827 \text{ leading}$$

3.12 *An audio-frequency ideal transformer is employed to couple a 60 Ω resistance load to an electric source which is represented by a constant voltage of 6 V in series with an internal resistance of 2400 Ω.*

(a) Determine the turn-ratio required to ensure maximum power transfer by matching the load and source impedances (i.e., by increasing the 60 Ω secondary impedance to 2400 Ω when referred to the primary).

(b) Find the load current, voltage and power under the conditions of maximum power transfer.

Solution

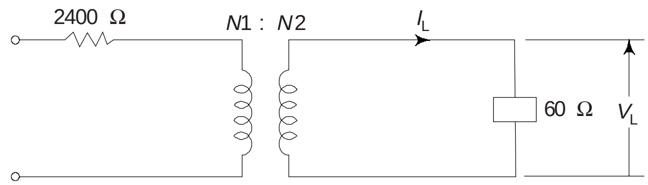


Fig. P3.12(a)

(a) 60 Ω load

When referred to primary, for maximum power transfer, the secondary impedance should be equal to the internal resistance of source, i.e., 2400 Ω

$$2400 = \left(\frac{N_1}{N_2}\right)^2 60$$

$$\therefore \quad \frac{N_1}{N_2} = \sqrt{40} = 6.325$$

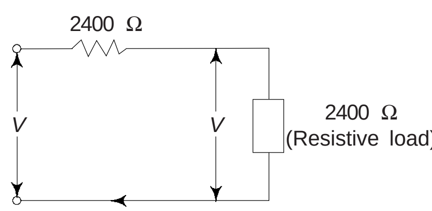


Fig. P3.12(b)

(b) For maximum power transfer, voltage drop across load

$$V_L = \frac{1}{6.325} \times \left(\frac{6 \times 2400}{2400 + 2400}\right) = 0.474 \text{ V}$$

$$I_L = 6.325 \times \frac{3}{2400} = 7.91 \text{ mA}$$

$$\text{Load power} = 0.474 \times 7.91 = 3.75 \text{ mW}$$

3.14 *An ideal transformer has a primary winding of 200 turns. On the secondary side the number of turns between A and B is 600 and between B and C is 400 turns, that between A and C being 1000. The transformer supplies a resistor connected between A and C which draws 10 kW. Further, a load of 200 ∠45° Ω is connected between A and B. The primary voltage is 2 kV. Find the primary current.*

Solution

$$V_{AC} = \frac{1000}{200} \times 2 = 10 \text{ kV}$$

$$\bar{I}_{L1} = \frac{10}{10} \angle -0° = 1 \angle 0° \text{ A}$$

$$V_{AB} = \frac{600}{200} \times 2 = 6 \text{ kV}$$

$$\bar{I}_{L2} = \frac{6 \times 1000}{2000 \angle 45°} = 3 \angle -45° \text{ A}$$

$$\bar{I}_{BA} = I_{L1} + I_{L2} = 1 + 3 \angle -45° = 3.12 - j\, 2.12 \text{ A}$$

$$\bar{I}_{CB} = I_{L1} = 1 + j\, 0$$

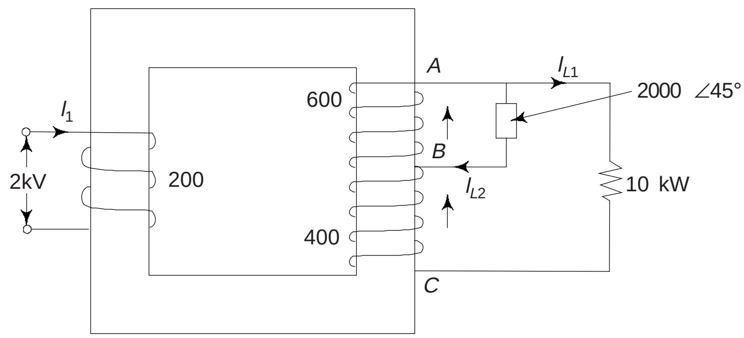


Fig. P3.14

Secondary AT = 60(3.12 – j 2.12) + 400 × 1 = 2272 – j 1272

$\therefore$ Primary current, $I_1 = \dfrac{2272 - j\,1272}{200} = 11.36 - j\, 6.36$

$\therefore$ $I_1 = 13.02$ A

3.15 *A 5 kVA, 400/80V transformer R_{eq} (HV) = 0.25 Ω and X_{eq} (HV) = 5 Ω and a lagging load is being supplied by it resulting in the following meter readings (meters are placed on the HV side).*

$$I_1 = 16 \text{ A}, \quad V_1 = 400 \text{ V}, \quad P_1 = 5 \text{ kW}$$

For this condition calculate what a voltmeter would read if connected across the load terminals. Assume the exciting current to be zero.

Solution

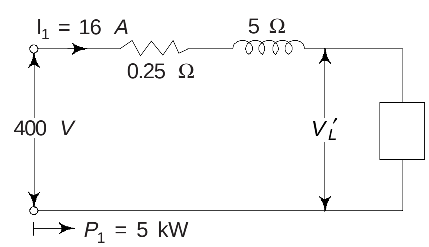


Fig. P3.15

$$\cos \phi_1 = \frac{5 \times 1000}{400 \times 16} = 0.78 \quad \therefore \phi_1 = 38.6° \text{ lagging}$$

$$\bar{I}_1 = 16 \angle -38.6°$$

$$\bar{V}_L' = 400 \angle 0° - 16 \angle -38.6° \,(0.25 + j\, 5)$$
$$= 347 - j\, 89.9$$

$$V_L' = 352 \quad \therefore \quad V_L = \frac{352 \times 80}{400} = 70.4 \text{ V}$$

3.16 *A 25 kVA, 230/115 V, 50 Hz transformer has the following data*

$$r_1 = 0.12\ \Omega \qquad r_2 = 0.04\ \Omega \qquad X_1 = 0.2\ \Omega, \quad X_2 = 0.05\ \Omega$$

Find the transformer loading which will make the primary induced emf equal in magnitude to the primary terminal voltage when the transformer is carrying the full-load current. Neglect the magnetizing current.

Solution Transformation ratio = $\frac{230}{115} = 2$

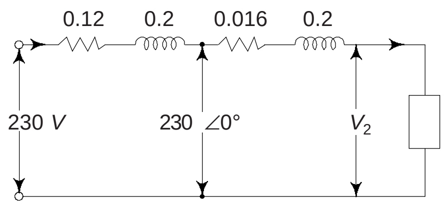


Fig. P3.16

Referring to the 230 V side

$$r_2' = 4 \times 0.04 = 0.16\ \Omega \quad x_2' = 4 \times 0.05 = 0.2\ \Omega$$

$$I_1(\text{fl}) = \frac{25 \times 1000}{230} = 108.7\ \angle\phi_1\ \text{A}$$

$$230 - 108.7\ (0.12 \cos\ \phi_1 + 0.2 \sin\ \phi_1) = 230$$

or $\tan\ \phi_1 = -0.12/0.2 = -\ 0.6$

$\text{pf} = \cos\ \phi_1 = 0.858$ leading

$\therefore$ $\phi_1 = 30.9°$

$$\bar{V}_2 = 230 - 108.7\ \angle 30.9° \times (0.66 + j\ 0.2) = 227.9\ \angle{-6.9°}$$

$$\phi_2 = 30.9° + 6.9° = 37.8° \quad \cos\ \phi_2 = 0.79 \text{ leading}$$

$\therefore$ $$\text{Load} = \frac{227.9 \times 108.7 \times 0.79}{1000} = 19.57\ \text{kW}$$

3.17 *The resistances and leakage reactances of a 10 k VA, 50 Hz, 2200/220 V distribution transformer are as follows:*

$$r_1 = 4\ \Omega \qquad r_2 = 0.04\ \Omega, \qquad x_1 = 5\ \Omega \qquad \text{and} \qquad x_2 = 0.05\ \Omega$$

Each quantity is referred to its own side of the transformer. (Suffix '1' stands for HV and '2' for LV.)

(a) Find the total leakage impedance referred to: (i) the HV side and (ii) the LV side.

(b) Consider the transformer to give its rated kVA at a pf of 0.8 lagging to a load at rated voltage. Find the HV terminal voltage and % voltage regulation.

(c) Repeat (b) for a pf of 0.8 leading.

(d) Consider the core loss to be 80 W. Find the efficiency under the conditions of part (b). Will it be different for the conditions under part (c)?

(e) If the load in part (b) gets short-circuited, find the steady-state current in the HV lines, assuming that the voltage applied to the transformer remains unchanged.

Solution

$$a = \frac{2200}{220} = 10$$

(a) $Z_{L1} = (4 + 0.04 \times 100) + j(5 + 0.05 \times 100) = 8 + j\ 10$

$$Z_{L2} = \left(\frac{4}{100} + 0.04\right) + j\left(\frac{5}{100} + 0.05\right) = 0.08 + j\ 0.1$$

(b) $I_L' = \frac{10{,}000}{2200} = 4.545\ \text{A}$

$V_H = 2{,}200 + 4.545\ (8 \times 0.8 + 10 \times 0.6)$

$= 2256.4$ V; % Regulation $= \frac{56.36}{2200} \times 100 = 2.56$

(c) $V_H = 2200 + 4.545\ (8 \times 0.8 - 10 \times 0.6)$

$= 22018$ V; % Regulation $= \frac{182}{2200} \times 100 = 0.08\%$

(d) $P_i = 80$ W

$P_c = (4.545)^2 \times 8 = 165.25$ W

$$\eta = \frac{10 \times 0.8 \times 10}{10 \times 0.8 + \dfrac{80 + 165.25}{1000}} = 97\%$$

η will be same for the condition of part (c)

(e) $I_{SC}(\text{HV}) = \dfrac{2200}{|8 + j10|} = 171.79$ A

3.18 *For Problem 3.10, assume that the load power factor is varied while the load current and secondary terminal voltage are held fixed. With the help of a phasor diagram, find the load power factor for which the regulation is zero.*

Solution The phasor diagram is drawn in Fig. P3.18

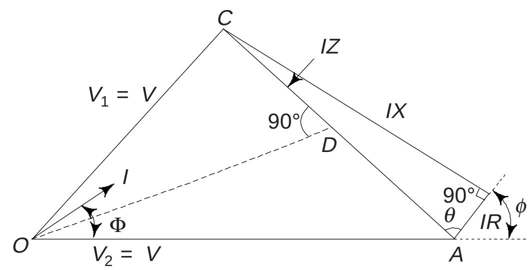


Fig. P3.18

$$\angle AOD = \sin^{-1}\frac{IZ}{2V}$$

$$\angle OAD = \frac{\pi}{2} - \sin^{-1}\frac{IZ}{2V}$$

$$\phi = \pi - \left(\frac{\pi}{2} - \sin^{-1}\frac{IZ}{2V} + \theta\right)$$

$$= \frac{\pi}{2} - \theta + \sin^{-1}\frac{IZ}{2V}$$

where $\theta = \tan^{-1}$ X/R

$$\theta = \tan^{-1}\frac{4}{2} = 63.4°; \quad Z = (2^2 + 4^2)^{0.5} = 4.47\ \Omega$$

$I = 6.25$ A $\qquad V = 2000$ V

$$\therefore \quad \phi = 90° - 63.4° + \sin^{-1}\frac{6.25 \times 4.47}{2 \times 2000} = 90° - 63.4° - 0.4° = 28.2° \text{ lead}$$

$\therefore$ pf = 0.9 leading

3.19 *A 20 kVA, 2000/200 V, single-phase transformer has the following parameters:*

HV winding $r_1 = 3\ \Omega$ $\quad x_1 = 5.3\ \Omega$

LV winding $r_2 = 0.05\ \Omega$ $\quad x_2 = 0.05\ \Omega$

(a) *Find the voltage regulation at (i) 0.8 pf lagging (ii) upf, (iii) 0.707 pf leading*

(b) *Calculate the secondary terminal voltage at: (i) 0.8 pf lagging, (ii) upf, and (iii) 0.707 pf leading when delivering full-load current with the primary voltage held fixed at 2 kV.*

Solution We will refer transformer impedance to the LV side.

$$R_{LV} = 0.05 + 3/100 = 0.08\ \Omega$$

$$X_{LV} = 0.05 + 5.3/100 = 0.103\ \Omega$$

The circuit model is drawn in Fig. 3.19(a).

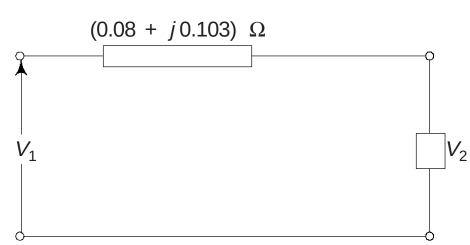


Fig. P3.19(a)

$$I_2 = \frac{20 \times 1000}{200} = 100 \text{ A}$$

(a) (i) Voltage drop = 100 (0.08 × 0.8 + 0.103 × 0.6)
= 12.58

Voltage regulation = $\dfrac{12.58}{200} \times 100 = 6.29\%$

(ii) Voltage drop = 100 (0.08 × 1 + 0.103 × 0) = 8 V

Voltage regulation = $\dfrac{8}{200} \times 100 = 4\%$

(iii) Voltage drop = 100 (0.08 × 0.707 – 0.103 × 0.707) = –1.63 V

Voltage regulation = $\dfrac{-1.63}{200} \times 100$
$= -0.815\%$

(b) The circuit model is drawn in Fig. 3.19(b).

(i) Voltage drop = 12.58 V

$\therefore \quad V_2 = 200 - 12.58 = 187.4$ V

(ii) Voltage drop = 8 V

$\therefore \quad V_2 = 200 - 8 = 192$ V

(iii) Voltage drop = –1.63 V

$\therefore \quad V_2 = 200 - (-1.63) = 201.6$ V

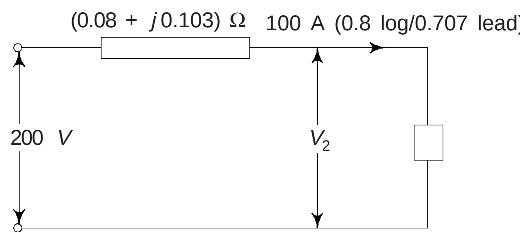


Fig. P3.19(b)

3.20 *The approximate equivalent circuit of a 4 kVA, 200/400 V single-phase transformer, referred to the LV side is shown in Fig. P3.20.*

(a) An open-circuit test is conducted by applying 200 V to the LV side, keeping the HV side open. Calculate the power input, power factor and current drawn by the transformer.

(b) A short-circuit test is conducted by passing full-load current from the HV side keeping the LV side shorted. Calculate the voltage to be applied to the transformer and the power input and power factor.

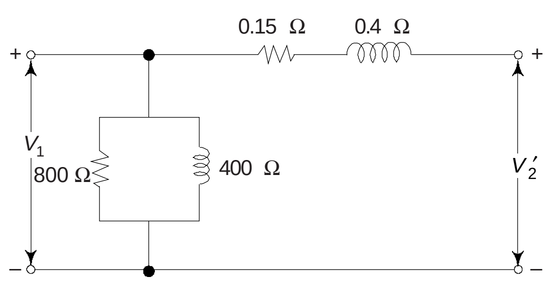


Fig. P3.20

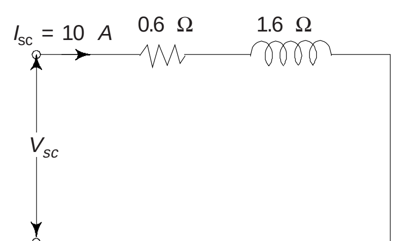


Fig. P3.20(a)

Solution

(a) $V_1 = 200$ V $\qquad \therefore P_0 = \dfrac{(200)^2}{800} = 50$ W

$I_i = \dfrac{200}{800} = 0.25$ A $\qquad I_m = \dfrac{200}{400} = 0.5$ A

$\bar{I}_0 = 0.25 + j\,0.5 \qquad I_0 = 0.56$ A $\qquad \cos\phi_0 = 0.447$ lag.

(b) Referring to HV side and neglecting the magnetizing branch (see Fig. P3.20(a)).

$$I_{fl}(\text{HV}) = \frac{4 \times 1000}{400} = 10 \text{ A}$$

$$\bar{Z} = 0.6 + j\,1.6 = 1.71 \angle 69.4° \ \Omega$$

$$V_{SC} = 10 \times 1.71 = 17.1 \text{ V}$$

$$\text{pf}_{SC} = \cos 69.4° = 0.352 \text{ lag}; \quad P_{SC} = 17.1 \times 10 \times 0.352 = 60.2 \text{ W}$$

3.21 *A 20 kVA, 2000/200 V transformer has name plate leakage impedance of 8%. What voltage must be applied on the HV side to circulate full-load current with the LV shorted?*

Solution

$$\frac{Z_{HV}(\Omega)\, I_{HV}\text{ (rated)}}{V_{HV}\text{ (rated)}} = 0.08$$

$$Z_{HV}(\Omega)\, I_{HV}\text{ (rated)} = 0.08\ V_{HV}\text{ (rated)} = 0.08 \times 2000 = 160 \text{ V} = V_{SC}$$

3.22 *Derive the condition for zero voltage regulation. Also show that the magnitude of maximum voltage regulation equals the pu value of equivalent leakage impedance.*

Solution Approximate condition for zero voltage regulation

$$IR\cos\phi - IX\sin\phi = 0$$

or

$$\tan\phi = \frac{R}{X}$$

∴

$$\cos\phi = \cos\tan^{-1}\frac{R}{X}$$

For maximum regulation $\tan\phi = \dfrac{X}{R}$

Maximum value of voltage regulation

$$\frac{IR\cos\phi + IX\sin\phi}{V_2} = \pm\frac{I(R^2+X^2)}{V_2 Z} = \frac{IZ}{V_2} = Z(\text{pu})$$

3.23 *The following test results were obtained for a 20 k VA, 50 Hz, 2400/240 V distribution transformer. OC test (LV) = 240 V, 1.066 A, 126.6 W; SC test (HV): 57.5 V, 8.34 A, 284 W.*

(a) When the transformer is operated as a step-down transformer with the output voltage equal to 240 V, supplying a load at upf, determine the maximum efficiency and the upf load at which it occurs.

(b) Determine the pf of the rated load, supplied at 240 V, such that the terminal voltage observed on reducing the load to zero is still 240 V.

Solution OC test (LV)

$$y_o = \frac{1.066}{240} = 0.0044 \ ℧$$

$$G_i = \frac{126.6}{(240)^2} = 0.0022 \ ℧$$

$$B_m = [(0.0044)^2 - (0.0022)^2]^{0.5} = 0.0038 \ ℧$$

SC test (HV)

$$Z = \frac{57.5}{8.34} = 6.89 \ \Omega, \qquad R = \frac{284}{(8.34)^2} = 4.08 \ \Omega$$

$\therefore \quad X = 5.55\ \Omega$

(a) $P_i = 126.6$ W $\quad I_{fl}(HV) = \dfrac{20 \times 1000}{2400} = 8.33$ A

$$P_{c,fl} = (8.33)^2 \times 4.08 = 283.3 \text{ W}$$

$$\text{Load at max efficiency} = 20 \times \sqrt{\frac{126.6}{183.3}} = 13.37 \text{ kVA}$$

$$\eta \text{ max (upf)} = \frac{13.37 \times 1}{13.37 \times 1 \times 2 \times 0.1266} = 98.14\%$$

(b) Voltage regulation = 0%

$$\cos \phi = \cos \tan^{-1} \frac{4.08}{5.55} = 0.805 \text{ leading}$$

3.24 *In a 25 kVA, 2000/200 V transformer, the iron and copper losses are 350 and 400 W respectively.*
(a) Calculate the efficiency on upf at (i) full load (ii) half load.
(b) Determine the load for maximum efficiency and the iron and the copper loss in this case.

Solution $P_i = 350$ W; $\quad P_{c,fl} = 400$ W

(a) $$\eta(\text{fl, upf}) = \frac{25 \times 1000 \times 1}{25 \times 1000 \times 1 + 350 + 400} = 97.08\%$$

(i) $$\eta(\text{1/2fl, upf}) = \frac{25 \times 1000 \times 1 \times 1/2}{25 \times 1000 \times 1 \ \times 1/2 + 350 + 1/4 \times 400} = 96.5\%$$

(b) $$k = \sqrt{\frac{350}{400}} = 0.935$$

Load for max $\eta = 25 \times 0.935 = 23.385$ kVA

$$P_i = 350 \text{ W} \quad P_c = (0.935)^2 \times 400 = 350 \text{ W}$$

3.25 *The efficiency of a 1000 kVA, 110/220 V, 50 Hz, single-phase transformer is 98.5% at half full-load at 0.8 pf leading and 98.8% at full-load upf.*
Determine: (a) iron loss, (b) full-load copper loss and (c) maximum efficiency at upf.

Solution

$$0.985 = \frac{500 \times 1000 \times 0.8}{500 \times 1000 \times 0.8 + P_i + 1/4 P_{c,fl}} \quad \text{(i)}$$

$$0.988 = \frac{1000 \times 1000}{1000 \times 1000 + P_i + P_{c,fl}} \quad \text{(ii)}$$

Solving Eqs (i) and (ii), we get

(a) $P_i = 4071$ W $\quad$ (b) $P_{c,fl} = 8079$ W $\quad$ (c) $k = \sqrt{\dfrac{4071}{8079}} = 0.71$

$$\eta_{max} = \frac{1000 \times 1000 \times 0.71}{1000 \times 1000 \times 0.71 + 2 \times 4071} = 98.9\%$$

3.27 *A transformer has its maximum efficiency of 0.98 at 20 kVA at upf. During the day it is loaded as follows:*

12 hours : 2 kW at pf 0.6
6 hours : 10 kW at pf 0.8
6 hours : 20 kW at pf 0.9
Find the 'all day' efficiency of the transformer.

Solution
$$\eta_{\max} = \frac{20 \times 1000 \times 1}{20 \times 1000 \times 1 + 2P_i} = 0.98$$

$$P_i = 200 \text{ W} = P_c(20 \text{ kVA})$$

2 kW, 0.6 pf, 3.33 kVA, 12 h, 2 × 12 = 24 kWh (output),

$$200 \left[1 + \left(\frac{3.33}{20}\right)^2\right] \times 12 = 2.47 \text{ kWh (loss)}$$

10 kW, 0.8 pf, 12.5 kVA, 6 h, 10 × 6 = 60 kWh (output),

$$200 \left[1 + \left(\frac{12.5}{20}\right)^2\right] \times 6 = 1.67 \text{ kWh (loss)}$$

20 kW, 0.9 pf, 22.22 kVA, 6 h, 20 × 6 = 1.20 kWh (output),

$$200 \left[1 + \left(\frac{22.22}{20}\right)^2\right] \times 6 = 2.68 \text{ kWh (loss)}$$

204 kWh (output) 6.82 kWh (loss)

$$\eta_{\text{energy}} \text{ (all day)} = \frac{204}{204 + 6.82} = 96.77\%$$

3.28 *A 20 kVA, 200/500 V, 50 Hz, single-phase transformer is connected as an auto transformer, as shown in Fig. P3.28. Determine its voltage-ratio and the kVA rating. Mark on the diagram, the magnitudes and relative directions of the currents in the winding as well as in the input and output lines when delivering the rated kVA to load.*

Solution Refer to Fig. P3.28(a)

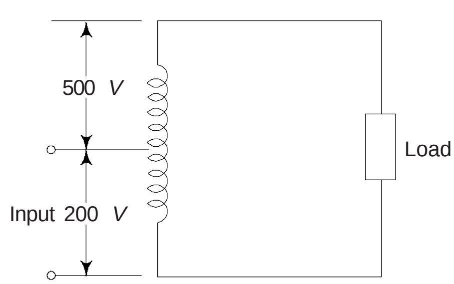


Fig. P3.28

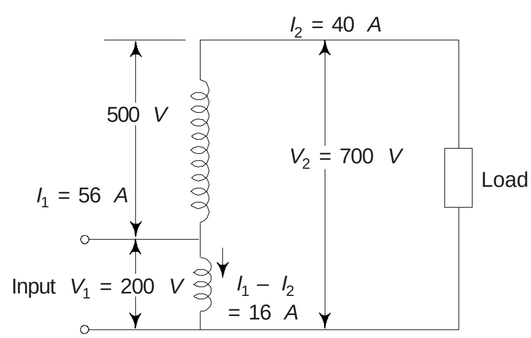


Fig. P3.28(a)

$$\frac{V_2}{V_1} = \frac{500 + 200}{200} = 3.5$$

$$I_2(\text{rated}) = \frac{20 \times 1000}{500} = 40 \text{ A}$$

$$\frac{-I_2 + I_1}{I_2} = \frac{200}{500} = \frac{2}{5} \quad \text{or} \quad \frac{-40 + I_1}{40} = \frac{2}{5}$$

$$\therefore \qquad I_1 = 56 \text{ A}$$

$$(\text{kVA})_{\text{Auto}} = 700 \times \frac{40}{1000} = 28$$

3.29 *A 400/100 V, 10 kVA, 2-winding transformer is to be employed as an auto transformer to supply a 400 V circuit from a 500 V source. When tested as a 2-winding transformer at rated load, 0.85 pf lagging, its efficiency is 0.97.*

(a) Determine its kVA rating as an auto transformer.

(b) Find its efficiency as an auto transformer.

Solution Refer to Fig. P3.29.

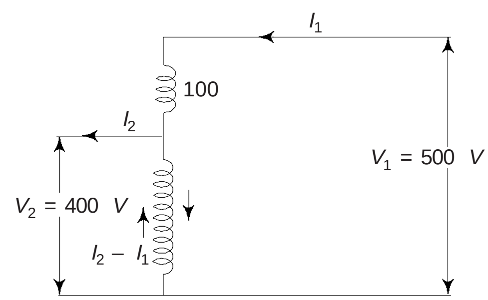


Fig. P3.29

(a) $I_1 = 10 \times \frac{1000}{100} = 100$ A

$$(\text{kVA})_{\text{Auto}} = \frac{500 \times 100}{1000} = 50$$

$$\frac{-I_1 + I_2}{I_1} = \frac{V_1 - V_2}{V_2} = \frac{100}{400}$$

$$\therefore \qquad I_2 = \frac{5}{4} I_1 = 125 \text{ A}$$

(b) As two-winding transformer

$$\eta_{\text{TW}} = \frac{10 \times 1000 \times 0.85}{10 \times 1000 \times 0.85 + P_L} = 0.97$$

$$\therefore \qquad P_{\text{L}} = 262.9 \text{ W}$$

Full load output as auto (0.85 pf) = 50 × 0.85 = 42.4 kW

$$\eta_{\text{Auto}} = \frac{42.5}{42.5 + 0.263} = 99.38\%$$

3.30 *A 20 kVA, 2000/200 V, two-winding transformer is to be used as an auto transformer, with a constant source voltage of 2000 V. At full-load of unity power factor, calculate the power output, power transformed and power conducted. If the efficiency of the two-winding transformer at 0.7 pf is 97%, find the efficiency of the auto transformer.*

Solution Refer to Fig. P3.30.

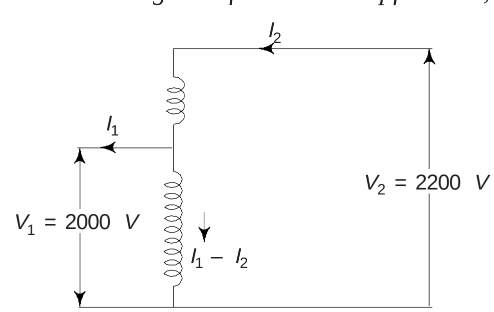


Fig. P3.30

$$I_2 = \frac{20 \times 1000}{200} = 100 \text{ A}$$

Power output = 2200 × 100 × 1 = 220 kW

Power transformed = 200 × 100 × 1 = 20 kW

Power conducted = 200 kW

$$\eta_{\text{TW}} = 0.97 = \frac{20 \times 1000 \times 0.7}{20 \times 1000 \times 0.7 + P_{\text{L}}}$$

$$\therefore \qquad P_L = 433 \text{ W}$$

$$\eta_{\text{Auto}} = \frac{220 \times 100}{220 + 0.433} = 99.8\%$$

3.31 *A 200/400 V, 20 kVA, and 50 Hz transformer is connected as an auto transformer to transform 600 V to 200 V.*

(a) Determine the auto transformer ratio a.

(b) Determine the kVA rating of the auto transformer.

(c) With a load of 20 kVA, 0.8 pf lagging connected to 200 V terminals, determine the currents in the load and the two transformer windings.

Solution Refer to Fig. P3.31.

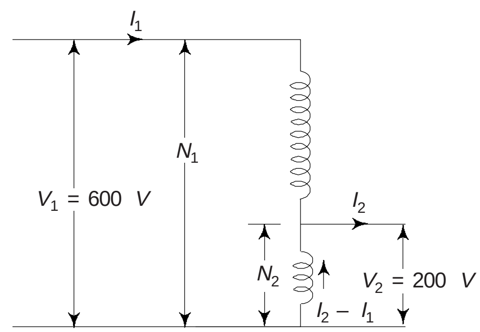


Fig. **P3.30**

(a) $a = \dfrac{N_1}{N_2} = \dfrac{600}{200} = 3$

(b) $I_1 = \dfrac{20 \times 1000}{400} = 50 \text{ A}$

$$(\text{kVA})_{\text{Auto}} = \frac{600 \times 50}{1000} = 30$$

(c) $I_2 = \dfrac{20 \times 1000}{200} = 100 \text{ A}$

$$I_1 = 50 \text{ A}$$

$$\therefore \qquad I_2 - I_1 = 100 - 50 = 50 \text{ A}$$

3.33 *A 20 kVA, 4400/220 V transformer with an equivalent impedance of 0.01 Ω is to operate in parallel with a 15 kVA, 4400/220 V transformer with an equivalent impedance of 0.015 Ω. The two transformers are connected in parallel and made to carry a load of 25 kVA. Assume both the impedances to have the same angle.*

(a) Find the individual load currents.

(b) What per cent of the rated capacity is used in each transformer?

Solution $Z_1 = 0.01\ \Omega, \qquad Z_2 = 0.015\ \Omega$

Since the impedances have the same angle

$$\bar{Z}_1 + \bar{Z}_2 = 0.01 + 0.015 = 0.025\ \Omega$$

$$S_1 = \frac{\bar{Z}_2}{|\bar{Z}_1 + \bar{Z}_2|} S_L = \frac{0.015}{0.025} \times 25 = 15 \text{ kVA}$$

$$S_2 = \frac{\bar{Z}_1}{|\bar{Z}_1 + \bar{Z}_2|} S_L = \frac{0.01}{0.025} \times 25 = 10 \text{ kVA}$$

(a) $I_1 = \dfrac{15 \times 1000}{220} = 68.2 \text{ A}; \quad I_2 = \dfrac{10 \times 1000}{220} = 45.6 \text{ A}$

(b) % rated capacity used in transformer 1 $= \dfrac{15}{20} = 75\%$

% rated capacity used in transformer 2 $= \dfrac{10}{15} = 66.7\%$

3.34 *Two single-phase transformers, rated 1000 kVA and 500 kVA respectively, are connected in parallel on both HV and LV sides. They have equal voltage ratings of 11 kV/400 V and their per unit impedances are (0.02 + j 0.07) and (0.025 + j 0.0875) respectively. What is the largest value of the unity power factor load that can be delivered by the parallel combination at the rated voltage?*

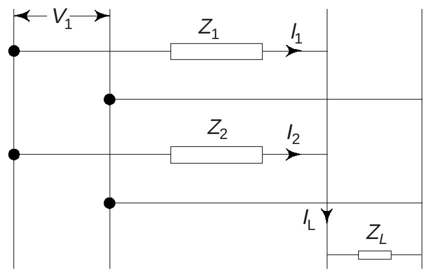


Fig. P3.34

Solution Refer to Fig. P3.34

$$S_1 \text{ (rated)} = 1000 \text{ kVA}; \quad S_2 \text{ (rated)} = 500 \text{ kVA}$$

Choose a kVA base of 1000.

$$\bar{Z}_1 = 0.02 + j\,0.07 = 0.0728\ \angle 74°$$

$$\bar{Z}_2 = (0.025 + j\,0.0875) \times 2 = 0.05 + j\,0.175 = 0.182\ \angle 74°$$

$$\bar{Z}_1 + \bar{Z}_2 = 0.07 + j\,0.245 = 0.255\ \angle 74°$$

$$S_1 = \frac{\bar{Z}_2}{|\bar{Z}_1 + \bar{Z}_2|}\, S_L \qquad \text{(i)}$$

$$S_2 = \frac{\bar{Z}_1}{|\bar{Z}_1 + \bar{Z}_2|}\, S_L \qquad \text{(ii)}$$

From (i),

$$S_L = 1000 \times \frac{0.255}{0.182} = 1400 \text{ kVA}$$

From (ii),

$$S_L = 500 \times \frac{0.255}{0.0728} = 3500 \text{ kVA}$$

As total load is increased the 1000 kVA transformer will be the first to reach its full load.

$$\therefore \qquad S_L \text{ (max)} = 1400 \text{ kVA}$$

3.35 *Two single-phase transformers rated 600 kVA and 500 kVA respectively, are connected in parallel to supply a load of 1000 kVA at 0.8 lagging power factor. The resistance and reactance of the first transformer are 3% and 6.5% respectively, and of the second transformers 1.5% and 8% respectively. Calculate the kVA loading and the power factor at which each transformer operates.*

Solution Refer to Fig. P3.35.

S_1 = 600 kVA $\qquad S_2$ = 500 kVA

S_L = 1000 kVA; $\qquad$ 0.8 pf lagging

Choose kVA base of 1000 kVA

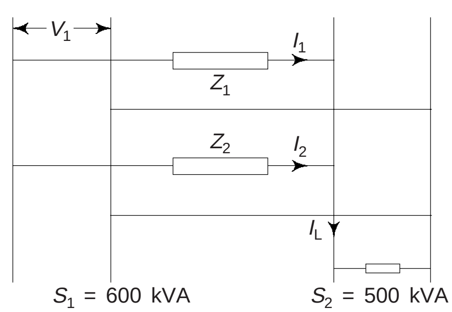


Fig. P3.35

$$\bar{Z}_1 \text{ (pu)} = (0.03 + j\,0.065) \times \frac{1000}{600} = 0.05 + j\,0.108$$

$$\bar{Z}_2 \text{ (pu)} = (0.015 + j\,0.08) \times \frac{1000}{500} = 0.03 + j\,0.16$$

$$\bar{S}_1 = \frac{\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}\,\bar{S}_L = \frac{0.03 + j\,0.16}{0.08 + j\,0.268} \times 1000\ (0.8 - j\,0.6)$$

$$= 584.2\ \angle -30.9° \text{ kVA}$$

= 584.2 kVA at 0.858 pf lagging

$$\bar{S}_2 = \frac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2}\bar{S}_L = \frac{0.05 + j\,0.108}{0.08 + j\,0.268} \times 1000\,(0.8 - j\,0.6)$$

$$= 426.5 \angle -45.15° \text{ kVA}$$

$$= 426.5 \text{ kVA at } 0.705 \text{ lagging pf}$$

3.36 *An ideal 3-phase step-down transformer, connected delta/star delivers power to a balanced 3-phase load of 120 kVA at 0.8 power factor. The input line voltage is 11 kV and the turns ratio of the transformer, phase-to-phase is 10. Determine the line voltage, line currents, phase voltages and phase currents on both the primary and the secondary sides.*

Solution Refer to Fig. P3.36.

Output kVA = 120 at 0.8 pf

$$V_{PY} = \frac{V_{P\Delta}}{10} = \frac{11000}{10V} = 1.1 \text{ kV}$$

$$V_{LY} = 1.1 \times \sqrt{3} = 1.9 \text{ kV}$$

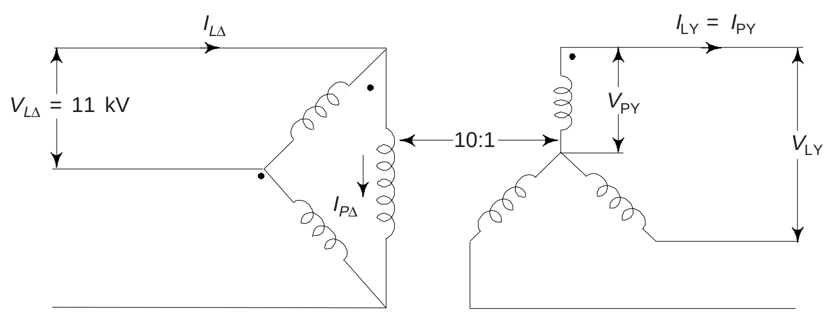


Fig. P3.36

Output kVA = 120 = $\sqrt{3} \times 1.9 \times I_{LY}$

$$\therefore \qquad I_{LY} = \frac{120}{\sqrt{3} \times 1.9} = 36.4 \text{ A} = I_{PY}$$

$$\therefore \qquad I_{P\Delta} = \frac{36.4}{10} = 3.64 \text{ A}; \qquad I_{L\Delta} = \sqrt{3} \times 3.64 = 63 \text{ A}$$

3.37 *A Δ/Y connected bank of three identical 60 kVA 2000/100 V, 50 Hz transformers is fed with power through a feeder whose impedance is 0.75 + j 0.25 Ω per phase. The voltage at the sending end of the feeder is held fixed at 2 kV line-to-line. The short circuit test when conducted on one of the transformers with its LV terminals short-circuited gave the following results:*

$$V_{HV} = 40 \text{ V} \qquad f = 50 \text{ Hz} \qquad I_{HV} = 35 \text{ A} \qquad P = 800 \text{ W}$$

(a) *Find the secondary line-to-line voltage when the bank delivers rated current to a balanced 3-phase upf load.*

(b) *Calculate the currents in the transformer primary and secondary windings and in the feeder wires on the occurrence of a solid 3-phase short-circuit at the secondary line terminals.*

Solution Refer Fig. P3.37.

Each transformer

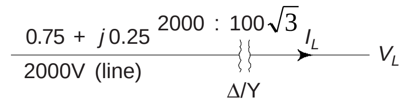


Fig. P3.37

$$Z_{HV} = \frac{40}{35} = 1.143\ \Omega$$

$$R_{HV} = \frac{800}{(35)^2} = 0.653\ \Omega$$

$$\therefore \quad X_{HV} = [(1.143)^2 - (0.653)^2]^{0.5} = 0.938\ \Omega$$

Transformer impedance on HV side on equivalent star basis

$$\bar{Z}_T = (0.653 + j\ 0.938)$$

$$\bar{Z}_{total} = (0.75 + j\ 0.25) + (0.653 + j\ 0.938) = 1.403 + j\ 1.188$$

$$= 1.838\ \angle 40.25° \qquad \therefore\ \phi = 40.25°$$

(a) $$I_L(\text{HV}) = \frac{60 \times 1000}{2000/\sqrt{3}} = 52\ \angle 0°$$

$$I_L(R \cos\ \phi + X \sin\ \phi) = 57.425\ \text{V}$$

$$V_L\ (\text{line-to-line}) = \sqrt{3}\left(\frac{2000}{\sqrt{3}} - 57.425\right) = \frac{1900.54\ \text{V}}{2000/100\sqrt{3}} = 164.59\ \text{V}$$

(b) 3-phase short-circuit on secondary terminals

$$I_{SC}^{F} = \frac{2000/\sqrt{3}}{1.84} = 627.56\ \text{A}$$

$$I_{SC}\ (\text{transformer primary}) = 627.56\sqrt{3} = 1087\ \text{A (line current)}$$

$$I_{SC}\ (\text{transformer secondary}) = 1087 \times \frac{2000}{100\sqrt{3}} = 12551.2\ \text{A}$$

3.42 *A single-phase, 50 Hz, three-winding transformer is rated at 2200 V on the HV side with a total of 250 turns. Of the two secondary windings, each can handle 200 kVA, one is rated at 550 V and the other at 220 V. Compute the primary current when the rated current in the 220 V winding at upf and the rated current in the 550 V winding is 0.6 pf lagging. Neglect all leakage impedance drops and magnetizing current.*

Solution Refer Fig. P3.42.

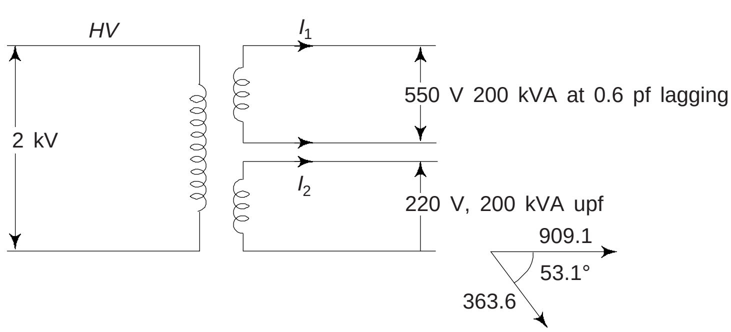


Fig. P3.42

$$I_1 = \frac{200}{0.55} = 363.6 \angle{-53.1°} \text{ A}$$

$$I_2 = \frac{200}{0.22} = 909.1 \angle 0° \text{ A}$$

Current in HV side due to $I_1 = \dfrac{363.6\angle -53.1° \times 0.55}{2.2} = 90.9 \angle{-53.1°}$ A

Current in HV side due to $I_2 = \dfrac{909.1 \times 0.220}{2.2} = 90.9 \angle 0°$ A

$\therefore$ HV current $= 90.9 \angle{-53.1°} + 90.9 \angle 0° = 162.6 \angle{-26.6°}$ A

3.43 *A small industrial unit draws an average load of 100 A at 0.8 lagging pf from the secondaries of its 2000/200 V, 60 kVA Y/Δ transformer bank. Find:*

(a) the power consumed by the unit in kW

(b) the total kVA used

(c) the rated line currents available from the transformer bank

(d) the rated transformer phase currents of the Δ-secondaries

(e) per cent of rated load on transformers

(f) primary line and phase currents

(g) the kVA rating of each individual transformer

Solution Refer Fig. P3.43.

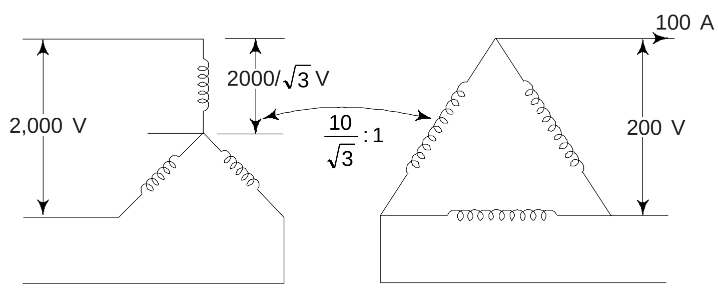


Fig. P3.43

(a) Power consumed by unit $= \sqrt{3} \times 100 \times 200 \times 0.8$ W $= 27.7$ kW

(b) Total kVA used $= \sqrt{3} \times 100 \times 200 = 34.6$ kVA

(c) Rated line current $= \dfrac{60 \times 1000}{\sqrt{3} \times 200} = 173.2$ A

(d) Rated phase current of Δ secondaries $= \dfrac{173.2}{\sqrt{3}} = 100$ A

(e) % of rated load $= \dfrac{34.6}{60} \times 100 = 57.7\%$

(f) Primary phase current $= \dfrac{100 \times \sqrt{3}}{10} = 17.32$ A

Primary line current = 17.32 A

(g) kVA rating of each individual transformer = $\frac{60}{3}$ = 20 kVA

3.44 *The HV terminals of a 3-phase bank of three single-phase transformers are connected to a 3-wire, 3-phase 11 kV (line-to-line) system. The LV terminals are connected to a 3-wise, 3-phase load rated of 1000 kVA and 2200 V line-to-line. Specify the voltage, current and kVA ratings of each transformer (both HV and LV windings) for the following connections:*

(a) HV-Y, LV-Δ (b) HV-Δ, LV-Y (c) HV-Y, LV-Y (d) HV-Δ, LV-Δ

Solution

(a) 11 kV, Y/Δ 2.2 kV, 1000 kVA load

Rating of each transformer = $\frac{1000}{3}$ = 333.3 kVA (in each case)

$$\text{Transf. ratio} = \frac{11}{\sqrt{3}/2.2} = 2.88$$

$$3\ V_{P\Delta}\ I_{P\Delta} = 1000 \text{ kVA}$$

$$\therefore \quad I_{P\Delta} = \frac{1000}{3 \times 2.2} = 1515 \text{ A} \qquad V_{P\Delta} = 2.2 \text{ kV}$$

$$I_{PY} = \frac{1515}{2.88} = 52.6 \text{ A} \qquad V_{PY} = \frac{11}{\sqrt{3}} = 6.35 \text{ kV}$$

(b) 11 kV Δ/Y 2.2 kV

$$I_{PY} = \frac{1000}{\sqrt{3} \times 2.2} = 262.4 \text{ A} \qquad V_{PY} = \frac{2.2}{\sqrt{3}} = 127 \text{ kV}$$

$$\text{Turns ratio} = \frac{11}{2.2/\sqrt{3}} = 8.66$$

$$I_{P\Delta} = \frac{262.4}{8.66} = 30.3 \text{ A} \qquad V_{P\Delta} = 11 \text{ kV}$$

(c) 11 kV Y/Y 2.2 kV

$$\text{Turns ratio} = 11/2.2 = 5$$

$$I_{PY}(\text{LV}) = \frac{1000}{\sqrt{3} \times 2.2} = 262.4 \text{ A} \qquad V_{PY}(\text{LV}) = 1.27 \text{ kV}$$

$$I_{PY}(\text{HV}) = \frac{262.4}{5} = 52.6 \text{ A} \qquad V_{PY}(\text{HV}) = \frac{11}{\sqrt{3}} = 6.35 \text{ kV}$$

(d) 11 kV Δ/Δ 2.2 kV Turns ratio = 11/2.2 = 5

$$I_{P\Delta}(\text{LV}) = \frac{1000}{3 \times 2.2} = 151.5 \text{ A} \qquad V_{P\Delta} = 2.2 \text{ kV}$$

$$I_{P\Delta}(\text{HV}) = \frac{151.5}{5} = 30.3 \text{ A} \qquad V_{P\Delta}(\text{HV}) = 11 \text{ kV}$$

3.45 *A 3-phase bank consisting of three single-phase 3-winding transformers (Y/Δ/Y) is employed to step down the voltage of a 3-phase, 220 kV transmission line. The data pertaining to one of the transformers is given below:*

Ratings

Primary 1: 20 MVA, 220 kV; Secondary 2: 10 MVA, 33 kV

Tertiary 3: 10 MVA, 11 kV.

Short-circuit reactances on 10 MVA base : X_{12} *= 0.15 pu,* X_{23} *= 0.1 pu,* X_{13} *= 0.2 pu.*

Resistances are to be ignored. The Δ-connected secondaries supply their rated current to a balanced load at 0.85 pf lagging, whereas the tertiaries provide the rated current to a balanced load at upf (constant resistance).

(a) Compute the primary line-to-line voltage to maintain the rated voltage at the secondary terminals.

(b) For conditions of part (a) find the line-to-line voltage at the tertiary terminals.

(c) If the primary voltage is held fixed as in part (a), to what value will the tertiary voltage increase when the secondary load is removed?

Solution Refer Fig. P3.45.

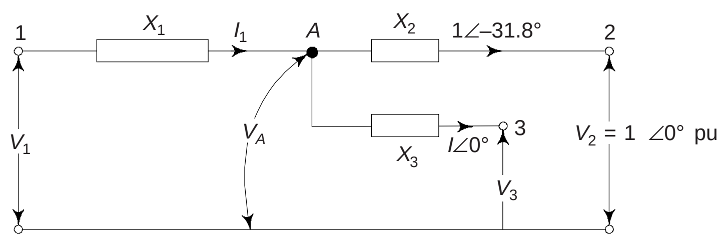


Fig. P3.45

$$X_1 = \frac{1}{2}\,(0.15 + 0.2 - 0.1) = 0.125 \text{ pu}$$

$$X_2 = \frac{1}{2}\,(0.1 + 0.15 - 0.2) = 0.025 \text{ pu}$$

$$X_3 = \frac{1}{2}\,(0.2 + 0.1 - 0.15) = 0.075 \text{ pu}$$

All computations are carried out in pu.

Assumption To simplify calculation, we shall assume that the phase angle of V_3 with respect to the reference voltage V_1 is 0°. It actually has a small angle which must otherwise be determined. The error caused is negligible.

(a) $\bar{V}_A = 1 + 1\ \angle{-31.8°} \times j\,0.025 = 1.013 + j\,0.0212$

$\bar{I}_1 = 1\ \angle{-31.8°} + 1 = 1.85 - j\,0.527$

$\bar{V}_1 = (1.013 + j\,0.0212) + j\,0.125\,(1.85 - j\,0.527) = 1.08 + j\,0.252$

$V_1 = 1.109 \times 220 = 243.98$ kV

(b) $\bar{V}_3 = (1.013 + j\,0.0212) - j\,0.075 \times 1\ \angle 0° = 1.013 - j\,0.0538$

$V_3 = 1.014 \times 11 = 11.16$ kV

(c) Secondary load removed

$\bar{V}_3 = (1.08 + j\,0.252) - j\,0.2 \times 1\ \angle 0° = 1.08 + j\,0.052$

$V_3 = 1.081 \times 11 = 11.89$ kV

3.46 *A 500 kVA, 11/0.43 kV, 3-phase delta/star-connected transformer has on rated load an HV copper-loss of 2.5 kW and an LV loss of 2 kW. The total leakage reactance is 0.06 pu. Find the ohmic values of the equivalent resistance and leakage reactance on the delta side.*

Solution Refer Fig. P3.46.

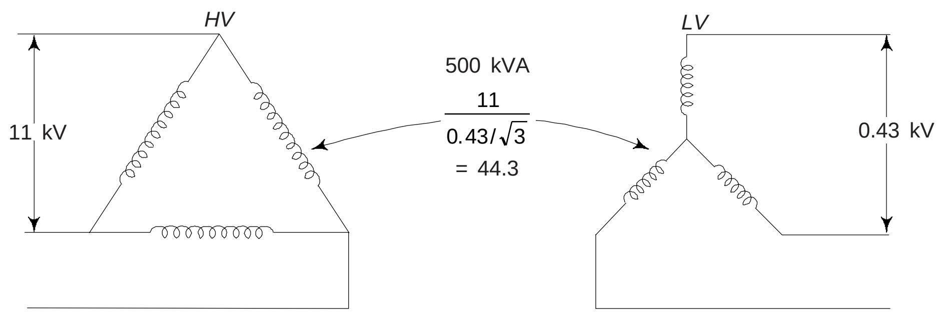


Fig. P3.46

$$I_{PY}\text{(rated)} = \frac{500}{\sqrt{3}\times 0.43} = 671.4 \text{ A}$$

$$I_{P\Delta}\text{(rated)} = \frac{500}{3\times 11} = 15.2 \text{ A}$$

$$R_{LV} = \frac{2000}{3\times(671.4)^2} = 1.48 \times 10^{-3}\ \Omega;\quad R_{HV} = \frac{2500}{3\times(15.2)^2} = 3.6\ \Omega$$

$$R_{eq}\text{ HV} = 3.6 + 1.48 \times 10^{-3} \times (44.3)^2 = 6.5\ \Omega \text{ (per phase } \Delta)$$

$$X\text{ (pu)} = 0.06$$

$$X_{Base}\text{(HV)} = \frac{11000}{15.2} = 723.7\ \Omega$$

$$X_{eq}\text{ HV} = 0.06 \times 723.7 = 43.4\ \Omega \text{ (per phase } \Delta)$$

3.47 *Two transformers, each rated 250 kVA, 11/2 kV and 50 Hz, are connected in open delta on both the primary and secondary.*

(a) Find the load kVA that can be supplied from this transformer connection.

(b) A delta-connected three-phase load of 250 kVA, 0.8 pf, 2 kV is connected to the LV terminals of this open-delta transformer. Determine the transformer currents on the 11 kV side of this connection.

Solution Refer Fig. P3.47.

(a) $$I_{ph(secondary)} = \frac{250}{2} = 125 \text{ A}$$

$$S_{open\ delta} = \sqrt{3}\, VI_{ph}$$

$$= \sqrt{3} \times 2 \times 125 = 433 \text{ kVA}$$

(b) $$\sqrt{3}\ VI_{ph(secondary)} = 250$$

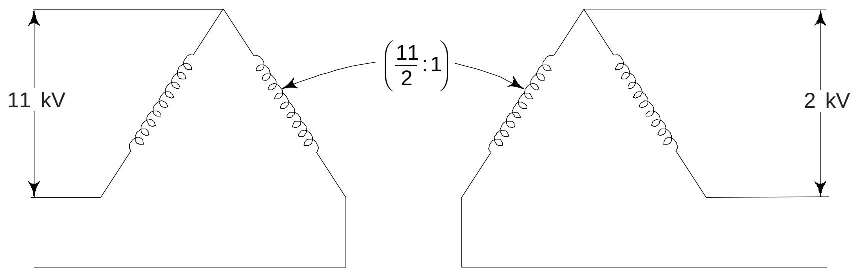


Fig. P3.47

$$I_{\text{ph(secondary)}} = \frac{250}{\sqrt{3} \times 2} = 72.2 \text{ A}$$

$$I_{\text{line}}(11 \text{ kV side}) = \frac{72.2}{11} \times 2 = 13.12 \text{ A}$$

$$I_{\text{phase}}(11 \text{ kV side}) = 13.12 \text{ A}$$

3.48 *Two 110 V, single-phase furnaces take loads of 500 kW and 800 kW respectively at a power factor of 0.71 lagging and are supplied from 6600 V, 3-phase mains through a Scott-connected transformer combination. Calculate the currents in the 3-phase line, neglecting transformer losses. Draw the phasor diagram.*

Solution Refer Figs P3.48(a) and P3.48(b).

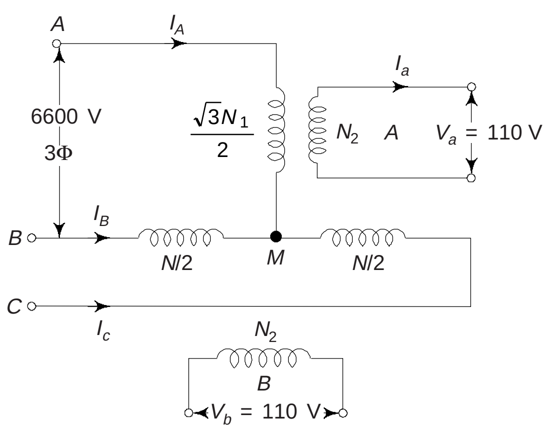


Fig. P3.48(a)

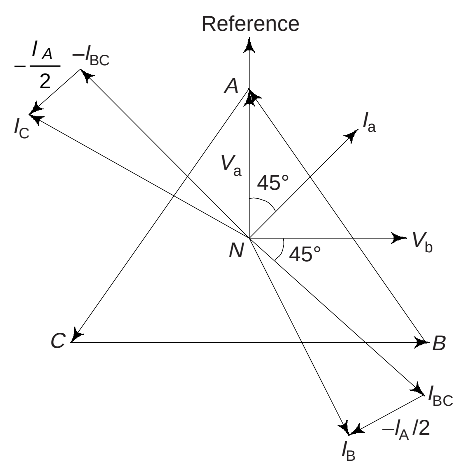


Fig. P3.48(b)

Furnace *A*: 500 kW at 0.71 pf lag; Furnace *B*: 800 kW at 0.71 pf lag

$$\frac{N_1}{N_2} = \frac{6600}{110} = 60$$

$$\therefore \qquad \frac{\sqrt{3}}{2}\frac{N_1}{N_2} = 51.96$$

$$\left.\begin{array}{ll} I_a = \dfrac{500 \times 1000}{110 \times 0.71} = 6402\text{A} & \phi_a = \cos^{-1}(0.71) = 45° \\ I_b = \dfrac{800 \times 1000}{110 \times 0.71} = 10243\text{A} & \phi_b = \cos^{-1}(0.71) = 45° \end{array}\right\} \text{With } V_b \text{ as reference}$$

$$\therefore \qquad \bar{I}_A = \frac{2}{\sqrt{3}} \frac{N_2}{N_1} \bar{I}_a = \frac{6402}{51.96} = 123.2 \angle 45°$$

$$\bar{I}_{BC} = \frac{N_2}{N_1} \bar{I}_b = \frac{10243}{60} = 170.7 \angle{-45°}$$

$$\bar{I}_B = \bar{I}_{BC} - \frac{I_A}{2} = 170.7\,(0.71 - j\,0.71) - \frac{123.2}{2}\,(0.71 + j\,0.71)$$

$$= 77.46 - j\,164.93$$

$$\therefore \qquad \bar{I}_B = 182.2 \text{ A}$$

$$\bar{I}_C = -\left(I_{BC} + \frac{\bar{I}_A}{2}\right) = -170.7\,(0.71 - j\,0.71) - \frac{123.2}{2}\,(0.71 + j\,0.71)$$

$$= -164.93 + j\,77.46$$

$$\therefore \qquad I_C = 182.2 \text{ A}$$

3.49 *Figure P3.49 shows a Scott-connected transformer, supplied from 11 kV, 3-phase, 50 Hz mains. Secondaries, series-connected as shown, supply 1000 A at a voltage of* $100\sqrt{2}$ *to a resistive load. The phase sequence of the 3-phase supply is ABC.*

(a) *Calculate the turns-ratio of the teaser transformer.*

(b) *Calculate the line current* I_B *and its phase angle with respect to the voltage of phase A to neutral on the 3-phase side.*

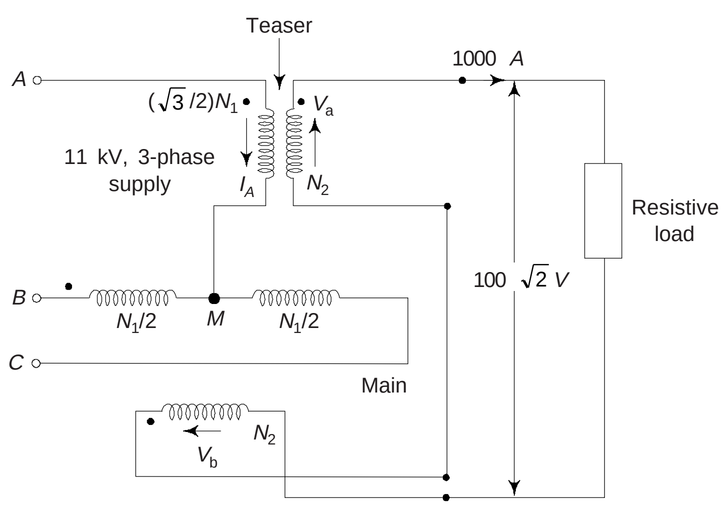


Fig. P3.49

Solution Refer Fig. P3.49(a).

(a) $V_a = V_b = \dfrac{100\sqrt{2}}{\sqrt{2}} = 100$ V

$$\frac{N_1}{N_2} = \frac{V_{BC}}{V_b} = \frac{11000}{100} = 110$$

$$\text{Turn ratios} = \frac{\sqrt{3}}{2}\frac{N_1}{N_2} = \frac{\sqrt{3}}{2} \times 100 = 95.26$$

(b) $I_A = \dfrac{2}{\sqrt{3}}\dfrac{N_2}{N_1} \times 1000 \text{ A} = \dfrac{1000}{95.26} = 10.5\text{A}$

$$I_{BC} = \frac{N_2}{N_1} \times 1000 = \frac{1000}{110} = 9.1 \text{ A}$$

Note V and I are in phase because of a resistive load.

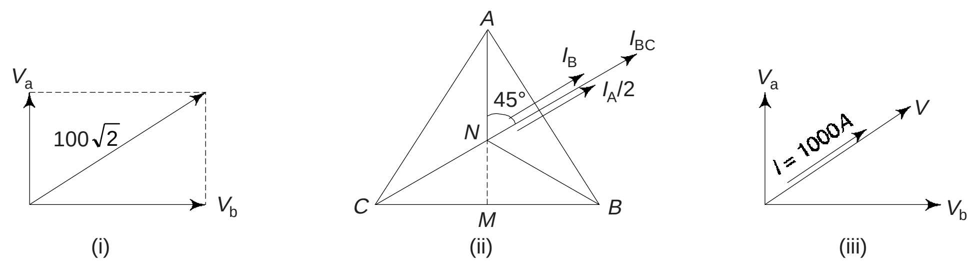


Fig. P3.49(a)

$$\bar{I}_B = \bar{I}_{BC} - \frac{\bar{I}_A}{2}$$

$$= 9.1 - \frac{10.5}{2} = 9.1 - 5.25 = 3.85 \text{ A}$$

(lagging V_{AN} by 45°)

3.50 A 15 kVA, 2200/220 V, 50 Hz transformer gave the following test results:

OC (LV side) V = 220 V I = 2.72 A
P = 185 W

SC (HV side) V = 112 V I = 6.3 A
P = 197 W

Compute the following:

(a) Core loss
(b) Full-load copper loss
(c) Efficiency at full-load 0.85 lagging pf
(d) Voltage regulation at full-load 0.8 lagging / leading pf

<u>Solution:</u>
Turns ratio = 10

a) Core loss P_i = 185W

b) I (FL) = 15000/2200 = 6.82A
Full – load copper loss = $(6.82 / 6.3)^2$ X 197 = 231 W

c) P(out) = 15 X 0.85 = 12.75 kW
$P_L = P_i + P_c(FL)$ = 185 + 231 = 416 W
η = 12.75 / (12.75 + 0.416) = 96.8 %

d) Z (HV) = 112 / 6.3 = 17.78 Ω
R (HV) = 197 / (1.3)2 = 4.96 Ω

$X(HV) = \sqrt{((17.78)^2 - (4.96)^2)} = 17.07\ \Omega$

Voltage drop = 6.82 (4.96 X 0.8 ± 17.07 X 0.6)
= 6.82 (3.97 ± 10.24) = 96.92V- 42.76

% Voltage regulation = + (96.92 / 2200) X 100 = + 4.41 % (0.8 lag pf)
= - (42.76 / 2200) X 100 = -1.94 % (0.8 lag pf)

3.51 A transformer of rating 20kVA, 2000/200V has the following parameters:

R_{eq} (HV side) = 2.65 Ω
Z_{eq} (HV side) = 4.23 Ω
Core loss at rated voltage = 95 W

(a) Calculate transformer efficiency when delivering 20 k VA at 200 V at 0.8 pf lagging.
(b) What voltage must be applied on the HV side for load as in part (a).
(c) Find the percentage voltage regulation.

(a) $P_a = 95$ W
$I_2 = (20 \times 1000) / 2000 = 10$ A
P_c (FL) $= (10)^2 \times 2.65 = 265$ w
$P_{(out)} = 20 \times 0.8 = 16$ kW
$P_L = 95 + 265 = 360$ w
$\eta = (16/16.36) \times 100 = 97.8\%$

(b) $Xeq = \sqrt{(\ (4.23)^2 - (2.65)^2} = 3.3\ \Omega$
Voltage drop $= 10 \times (2.65 \times 0.8 + 3.3 \times 0.6)$
$= 41$ V

HV side applied voltage $= 2000 + 41 = 2041$ V

(c) % voltage regulation $= (41/2041) \times 100 = 2.05$ %

3.52 A 100kVA, 11 kV/231 V transformer has HV and LV winding resistances of 8.51 Ω and 0.0038 Ω respectively. It gave the following test results:

OC (LV side)	**231 V**	**15.2 A**	**1.25 kW**
SC (HV side)	**440 V**	**9 A**	**Not measured**

Calculate
(a) Equivalent leakage reactance of the transformer
(b) Full load copper loss
(c) Efficiency at full – load and half full-load at 0.85 lagging power factor.

Turns ratio a = 11000 / 231 = 47.6

(a) $r_1 = 8.51\ \Omega$ $\quad r_2 = 0.0038\ \Omega$
$r_2' = (47.6)^2 \times 0.0038 = 8.61\ \Omega$

$R_1 = 8.51 + 8.61 = 17.12\ \Omega$
$Z_1 = 440 / 9 = 48.9\ \Omega$
$X_1 = \sqrt{(\ (48.9)^2 - (17.12)^2\ )} = 45.81\ \Omega$

(b) $I_1(FL) = 100/11 = 9.09$ A
$P_c(FL) = (9.09)^2 \times 17.12 = 1.41$ kW

(c) $P_i = 1.25$kW
$P_c(FL) = 1.41$
$P_L(FL) = 1025 + 10141 = 2.66$kW

$P_{(out)}$(FL) = 100 X 0.85 = 85kW
η(FL) = 85/(85 + 2.66) = 96.7 %

P_c(1/2 FL) = ¼ X 1.41 = 0.705kW
P_L(1/2FL) = 1.25 + 0.705 = 1.955kW
$P_{(out)}$(1/2FL) = 50 X 0.85 = 42.5kW

η = 42.5/(42.5 + 1.955) = 95.6 %

3.53 A 100 kVA, 2200 V/220 V transformer has the following circuit parameters.

$R_1 = 0.23\ \Omega$ $\quad$ $R_2 = 0.0023\ \Omega$
$X_1 = 1.83\ \Omega$ $\quad$ $X_2 = 0.0183\ \Omega$
R_1 (HV side) = 5.6 k Ω
X_m (HV side) = 1.12 k Ω

The transformer is subjected to the following daily load cycle = 4 hrs on no load, 8 hrs on 1/4th full-load at 0.8 pf, 8 h on ½ full-load at upf, and 4 hrs on full-load at 0.9 pf. Determine the all-day energy efficiency of the transformer.

$P_i = (2200)^2/(5.6 \text{ X } 1000) = 846.3$ w
$R_1 = r_1 + r_2 = 0.23 + (10)2 \text{ X } 0.0023 = 0.46\ \Omega$

I_i(FL) = (100 X 1000)/2200 = 45.5A
$P_c(FL) = (45.5)^2 \text{ X } 0.46 = 952.3$ w

P_o	Time(i)	w_0	$P_i = P_o + P_i + k^2P_c$	wi
0	4	0	0.846	3.38
(100/4) X 0.8 = 20	8	160	20+0.846+1/16X0.952 = 20.906	167.24
50	8	400	50+0.846+1/4 X 0.952 = 51.084	408.67
90	4	360	90 + 0.846 + 0.952 = 91.798	367.19
		920		946.48

η_e = 920/946.5 = 97.2 %

3.54 A 400/200 V, 50 Hz transformer has a primary impedance of 1.2 + j 3.2 Ω and secondary impedance of 0.4 + j 1.0 Ω. A short – circuit occurs on the secondary side with 400 V applied to the primary. Calculate the primary current and its power factor.

$Z_1 = (1.2 + j\,3.2) + (400/200)^2\,(0.4 + j1.0)$
$\quad = (1.2 + j\,3.2) + (1.6 + j\,4.0)$
$\quad = 2.8 + j7.2 = 7.725 / 68.7^\circ$

$I_{1(sc)} = 400 / 7.725 = 51.78$ A
Pf = cos 68.7° = 0.363 lagging

3.55 A 50 Hz, 3-winding transformer can be considered as an ideal transformer. The primary is rated 2400 V and has 300 turns. The secondary winding is rated 240 V, 400 kVA and supplies full-load at UPF. The tertiary is rate 600 V, 200 kVA and supplies full-load at 0.6 pf lagging. Determine the primary current.

Primary	: 2400V, 300turns	
Secondary	: 240 V, 400kVA,	upf
Tertiary	: 600 V, 200 kVA	0.6 pf lagging

$I_2 = (400 \text{ X } 1000) / 240 = 1667$ A, upf
$\bar{I}_2 = 1667 \angle 0^\circ$

$I_3 = (200 \text{ X } 1000) / 600 = 333$ A, 0.6 pf lag
$\bar{I}_3 = 333 \angle -53.1^\circ$

$\bar{V}_1 \bar{I}_1 = \bar{V}_2 \bar{I}_2 + \bar{V}_3 \bar{I}_3$
$2.4 \angle 0^\circ \bar{I}_1 = 400 \angle 0^\circ + 200 \angle -53.1^\circ$
$\quad = 400 + 120 - j\,160$
$\quad = 520 - j\,160$

$\bar{I}_1 = 216.7 - j66.7 = 226.7 = 226.7 \angle -17.11^\circ$

3.56 An ideal transformer has 200 primary turns and 360 secondary turns, the primary being excited at 600 V. The full secondary has a resistive load of 8 kW. The secondary is also tapped at 240 turns which supplies a pure inductive load of 10 kVA. Find the primary current and its pf.

$V_{ca} = 600 \text{ X } (360/200) = 1080$ V
$I_c = (8 \text{ X } 1000) / 1080 = 7.41 \angle 0^\circ$ A
$V_{ba} = 600 \text{ X } (240/200) = 720$ V
$I_b = (10 \text{ X } 1000) / 720 = 13.89 \angle (-90^\circ)$

$I_1 \text{ X } 200 = 7.41 \text{ X } 0^\circ \angle 360 + 13.89 \text{ X } 240 \angle (-90^\circ)$

$I_1 = 13.34 \text{ X} -j\ 16.67 = 21.35 \angle (-51.3^\circ)$

$I_1 = 21.35$ A, bf = cos 51.3° = 0.625 lagging

3.57 A 50 kVA, 2300 V/230 V transformer draws power of 750 W at 0.5 A at no load when 2300 V is applied to the HV side. The HV winding resistance and leakage reactance are 1.8 Ω and 4 Ω respectively. Calculate:
(a) the no load pf
(b) the primary induced emf
(c) the magnetizing current and
(d) the core loss component of current.

(a) No load pf = (750/(2300 X 0.5)) = 0.652 lagging
$I_0 = 0.5 \angle (-49.3^\circ)$

(b) $E_1 = V_1 - I_0 (r_1 + jx_1)$
$= 2300 - 0.5 \angle (-49.3^\circ)(1.8 + ju)$
$= 2300 - 0.5 \text{ X } 4.39 \angle (65.8 - 49.3^\circ)$
$= 2300 - 2.195 \angle (16.5^\circ)$
$= 2300 - 2.105 - j \text{ X } 0.623$
$= 2300\text{V} \angle 0^\circ$

(c) $I_i = 750/2300 = 0.033$ A

$Im = \sqrt{((0.5)^2-(0.033)^2)}$
$= 0.499$ A

(d) $I_0 = 0.033$ A

3.58 Two single-phase transformers operate in parallel to supply a load of 44 + j 18.6 Ω. The transformer A has a secondary emf of 600V on open circuit with an internal impedance on 1.8 + j 5.6 Ω referred to the secondary. The corresponding figures for transformer B are 610 V and 1.8 + j 7.4 Ω. Calculate the terminal voltage, current and power factor of each transformer.

$\overline{I_A} = (\overline{E_A}\,\overline{Z_B} + (\overline{E_A} - \overline{E_B})\, Z_L)/(\overline{Z_A}\overline{Z_B} + \overline{Z_L}(\overline{Z_A}+\overline{Z_B})$

$I_B = (E_B Z_A + (E_B - E_A)Z_L / Z_A Z_B + Z_L(Z_A + Z_B)$

$Z_L = 44 + j\ 18.6\ \Omega$
$= 47.77 \angle (22.9^\circ)$

$Z_A = 1.8 + j5.6\ \Omega$
$= 5.88 \angle (72.2^\circ)$

$Z_B = 1.8 + j7.4\ \Omega$

$= 7.62 \angle (76.3^\circ)$

$Z_A Z_B = 44.8 \angle (148.5^\circ)$
$= -38.2 + j\,23.4$

$Z_A + Z_B = 3.6 + j\,13$
$= 13.49 \angle (74.5^\circ)$

$Z_L(Z_A + Z_B) = 6444.4 \angle (97.40)$
$= -83 + j\,639$

$$
\begin{array}{lr}
Z_A Z_B + Z_L (Z_A + Z_B) = & -38.2 + j\ \ 23.4 \\
 & -83 \quad + j\,639 \\
\hline
 & -121.2 + j\,662.4 = 673.4 \angle (100.4^\circ) \\
\hline
\end{array}
$$

$E_A = 600 \angle 0^\circ$
$E_B = 610 \angle 0^\circ$
$E_A E_B = 600(1.8 + j\,7.4)$
$= 1080 + j\,4440$

$E_B Z_A = 610\,(1.8 + j\,5.6)$
$= 1098 + j\,3416$

$(E_A - E_B)\,Z_L = -10(44 + j\,18.6) = -\,440 - j\,186$

$$
\begin{array}{lr}
E_A Z_B + (E_A - E_B) Z_L = & 1080 + j\,4440 \\
 & -\,440 - j\quad 186 \\
\hline
 & 640 + j\,4254 \quad = 4302 \angle (81.40) \\
\hline
\end{array}
$$

$$
\begin{array}{lr}
E_B Z_A + (E_B - E_A)\,Z_L = & 1098 + j\,3416 \\
 & 440 + j\quad 186 \\
\hline
 & 1538 + j\,3602 \quad = 3917 \angle (66.90^\circ) \\
\hline
\end{array}
$$

$I_A = 4302\,(81.40) / 673.4\,(100.400^\circ) = 6.39(-190^\circ) = \quad 6.04 - j\,2.08$

$I_B = 3917\,(66.90^\circ) / 673.4(100.40^\circ) = \ 5.82(-33.50^\circ = \quad 4.85 - \ j\,3.21$

$$
\begin{array}{r}
\hline
10.89 - j\,5.29 \\
\hline
\end{array}
$$

$I_A + I_B = 10.8 - j5.29 = 12.03\,(-26.1^\circ)$
$V_t = (I_A + I_B) Z_L = 12.03\,(-26.1^\circ \times 44.77\,(22.9^\circ)$
$= 538.6\,(-3.2^\circ\,)V$

V_t, I_A = -19° + 3.2° = 15.8° $\quad$ Pf_A = 0.962 lag
V_t, I_B = -33.5° + 3.2° = -30.3° $\quad$ Pf_B = 0.863 lag

CHAPTER 4: PRINCIPLES OF ELECTROMECHANICAL ENERGY CONVERSION

4.1 *In the electromagnetic relay of Fig. 4.11, the exciting coil has 1000 turns. The cross-sectional area of the core is A = 5 cm × 5 cm. The reluctance of the magnetic circuit may be assumed to be negligible. Also neglect fringing effects.*

(a) *Find the coil inductance for an air-gap of x = 1 cm. What is the field energy when the coil carries a current of 2 A? What is the force on the armature under these conditions?*

(b) *Find the mechanical energy output when the armature moves from x_a = 1 cm to x_b = 0.5 cm assuming that the coil current is maintaind constant at 2.0 A.*

(c) *With constant coil current of 2.0 A, derive an expression for the force on armature as a function of x. Find the work done by the magnetic field when x changes from x_a = 1 cm to x_b = 0.5 cm from $\int_{x_a}^{x_b} F_f \, dx$ · Verify the result of part (b).*

(d) *Find the mechanical energy output in part (b) if the flux linkages are maintained constant corresponding to a coil current of 2.0 A.*

Solution

(a) $R = \dfrac{2x}{\mu_0 A}$

$$L(x) = \frac{\mu_0 N^2 A}{2x} = \frac{4\pi \times 10^{-7} \times (1,000)^2 \times (0.05)^2}{2x}$$

$$L(x) = \frac{\pi \times 10^{-3}}{2x}$$

$$x = 0.01 \text{ m}$$

$$\therefore \qquad L = \frac{\pi \times 10^{-3}}{2 \times 0.01} = 0.157 \text{ H}$$

Field energy for coil current of 2.0 A

$$= \frac{1}{2} \times 0.157 \times (2)^2 = 0.314$$

$$W_f' = \frac{1}{2} L(x) i^2$$

$$F_f = \frac{\partial W_f'}{\partial x} = \frac{1}{2} i^2 \frac{\partial}{\partial x}\left(\frac{\pi \times 10^{-3}}{2x}\right) = -\frac{\pi \times 10^{-3}}{x^2} = \frac{-\pi \times 10^{-3}}{(0.01)^2} = -314 \text{ N}$$

(b) Electrical energy input, $\Delta W_e = (\lambda_2 - \lambda_1) i$

$$= (L\,(x = 0.005) - L(x = 0.01)) i^2$$

$$= \frac{\pi \times 10^{-3}}{2}\left(\frac{1}{0.005} - \frac{1}{0.01}\right) \times 4$$

$$= 0.2\,\pi \text{ J}$$

Mechanical output,

$$\Delta W_\text{m} = \frac{1}{2}\ \Delta W_\text{e} = 0.1\pi = 0.314 \text{ J}$$

(c) Mechanical work done $= \int_{0.01}^{0.005} F_\text{f}\ dx\ = -\ (\pi \times 10^{-3}) \int_{0.01}^{0.005} \frac{1}{x^2}\,dx$

$$= \pi \times 10^{-3} \left[\frac{1}{x}\right]_{0.01}^{0.005}$$

$$= 0.314 \text{ J}$$

(This agrees with the value obtained in part (b))

(d) $\Delta W_\text{m} = \frac{1}{2}\ \lambda_0\ (i_1 - i_2)$

$$i_1 = 2 \text{ A}, \qquad x_\text{a} = 0.01 \text{ cm}$$

$$\lambda_0 = L(x = 0.01) \times 2 = \frac{\pi \times 10^{-3}}{2 \times 0.01} \times 2 = 0.1\pi$$

$$\lambda_0 = L(x = 0.005) \times i_2$$

or $$i_2 = 0.1\pi \times \frac{2 \times 0.005}{\pi \times 10^{-3}} = 1\text{A}$$

$$4\ W_\text{m} = \frac{1}{2} \times 0.1\pi\ (2 - 1) = 0.05\pi$$

$$= 0.157 \text{ J}$$

4.2 *In Fig. P4.7(b) if the i-λ curve ab is assumed to be a straight line, find an expression for the mechanical energy output. If this figure pertains to the electromagnetic relay of Fig. 4.11, find the value of the mechanical energy output, given that i_1 = 2.0 A, i_2 = 1.5 A, x_a = 1 cm and x_b = 0.5 cm.*

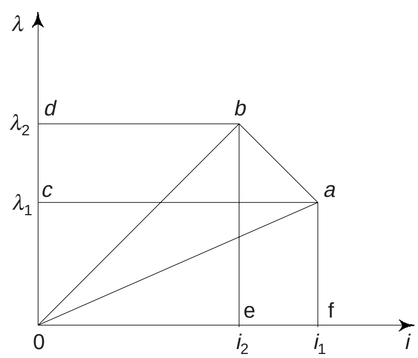


Fig. P4.2(a)

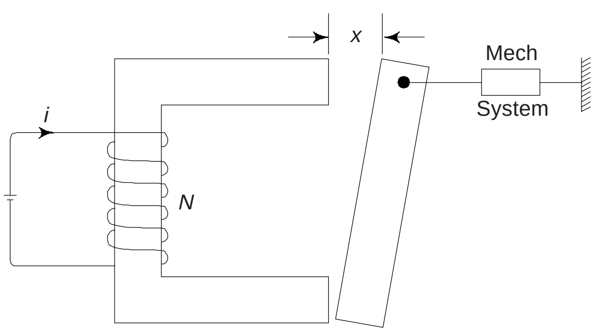


Fig. P4.2(b)

Solution Electrical energy input

ΔW_e = Area *cabd*

$$= (\lambda_2 - \lambda_1)i_2 + \frac{1}{2}\ (\lambda_2 - \lambda_1)\ (i_1 - i_2)$$

Increase in field energy

$$\Delta W_f = \text{Area } obd - \text{Area } oca$$

$$= \frac{1}{2}\lambda_2 i_2 - \frac{1}{2}\lambda_1 i_1$$

Mechanical output

$$\Delta W_m = \Delta W_e - \Delta W_f$$

$$= (\lambda_2 - \lambda_1)i_2 + \frac{1}{2}(\lambda_2 - \lambda_1)(i_1 - i_2) - \frac{1}{2}\lambda_2 i_2 + \frac{1}{2}\lambda_1 i_1$$

$$= \frac{1}{2}(\lambda_2 i_1 - \lambda_1 i_2)$$

From Prob 4.1,

$$L(x) = \frac{\pi \times 10^{-3}}{2x}$$

$$L(x = 0.01) = 0.157 \text{ H}, \qquad L(x = 0.005) = 0.314 \text{ H}$$

$$\lambda_1 = L(x = 0.01)i_1 \qquad \lambda_2 = L(x = 0.005)\, i_2$$

$$= 0.314 \text{ WbT} = 0.314 \times 1.5 = 0.471 \text{ WbT}$$

Mechanical output $= \frac{1}{2}(0.471 \times 2 - 0.314 \times 15)$

$= 0.2355$ J

4.4 *For the cylindrical iron-clad solenoid magnet of Fig. 4.9.*

$$MMF = 120 \text{ AT}$$

$$Flux = 0.00175 \text{ Wb}$$

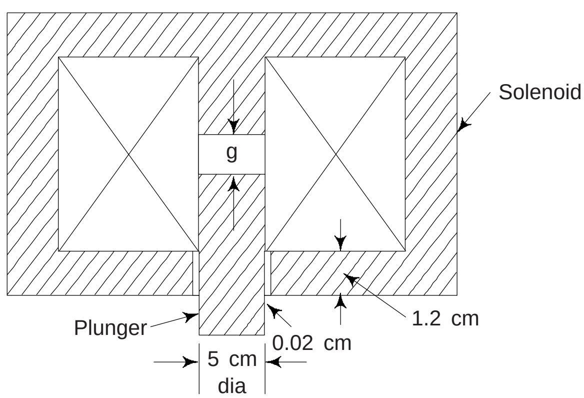


Fig. P4.4

assume that the magnetic path reluctance remains constant at a value corresponding to the linear part of the magnetization curve.

(a) Derive an expression for the force in terms of g for a constant coil current of 2.25 A. Calculate the value of the force for g = 1 and 0.2 cm.

(b) *What is the electrical energy input to the system when g changes from 1 to 0.2 cm, while the coil current is maintained constant at 2.25 A.*

(c) *Calculate the work done on the plunger during the movement specified in part (b).*

(d) *With the coil current maintained constant at 2.25 A what is the direction and magnitude of the electrical energy flow if the plunger is made to move from g = 0.2 to 1 cm?*

Solution

(a) $R_{\text{iron}} = \dfrac{120}{0.00175} = 68{,}570 = 68.57 \times 10^3$

Reluctance of annular air-gap $= 84.4 \times 10^3$

Reluctances of circular air-gap $(g) = \dfrac{g}{4\pi \times 10^{-2} \times (\pi/4) \times (0.05)^2}$

$= 4{,}053\ g \times 10^5$

Total reluctance $R = 153 \times 10^3 + 4{,}053\ g \times 10^5 = 153\,(1 + 26.5 \times 10^2\ g) \times 10^3$

$$L(g) = \frac{N^2}{R} = \frac{(1{,}200)^2}{153\,(1 + 26.5 \times 10^2\ g) \times 10^3} = \frac{9.41}{(1 + 26.5 \times 10^2\ g)}$$

$$W_f'\ (i,\ g) = \frac{1}{2}\ i\lambda = \frac{4.71\, i^2}{(1 + 26.5 \times 10^2\ g)}$$

$$F_f = \frac{\partial W_f'}{\partial g} = -\frac{124.8 \times 10^2\ i^2}{(1 + 26.5 \times 10^2\ g)^2}$$

$g = 0.2$ cm, $\quad i = 2.25$ A

$$F_f = -\frac{124.8 \times 10^2\ (2.25)^2}{(1 + 26.5 \times 0.2)^2} = -1592 \text{ N}$$

$g = 1$ cm, $\quad i = 2.25$ A

$$F_f = -\frac{124.8 \times 10^2 \times (2.25)^2}{(1 + 26.5 \times 1)^2} = -83.5 \text{ N}$$

(b) $\Delta W_e = (\lambda_2 - \lambda_1)\ i_o$

$= (L(g_2) - L(g_1))\ i_0^2$

$$L(g) = \frac{9.41}{(1 + 26.5 \times 10^2\ g)}$$

$$\Delta W_e = 9.41 \left(\frac{1}{1 + 26.5 \times 0.2} - \frac{1}{1 + 26.5 \times 1}\right) \times (2.25)^2$$

$= 5.81$ J

(c) As per Eq. (4.8)

$$\Delta W_m = \frac{1}{2}\ \Delta W_e = 2.91 \text{ J*}$$

* For derivation of this relation, the reader may refer to Nagrath, I.J. and D.P.Kothari, *Electric Machines*, 2nd edn. Tata McGraw-Hill, New Delhi, 1997, Ch. 4, p.156.

(d) $\Delta W_e = -5.81$ J

The electrical energy flows out of the solenoid coil.

4.6 *For the electromagnetic relay of Fig. 4.11, calculate the maximum force on the armature if saturation flux density in the iron part is 1.8 T. Given: cross-sectional area of core = 5 cm × 5 cm, coil turns = 1000.*

Solution

$$\omega_f = \frac{1}{2}\left(\frac{B^2}{\mu_0}\right)$$

In Fig. 4.2(a), field energy in the two air-gaps is

$$W_f(B, x) = 2 \times \frac{1}{2}\left(\frac{B^2 Ax}{\mu_0}\right)$$

$$F_f = -\frac{\partial W_f}{\partial x} = -\frac{B^2 A}{\mu_0}$$

$$F_f(\max) = -\frac{B^2 (\text{saturation}) A}{\mu_0}$$

$$= -\frac{(18)^2 \times (0.05 \times 0.05)}{4\pi \times 10^{-7}} = -6.446 \times 10^3 \text{ N}$$

4.7 *For the electromagnetic device shown in Fig. P4.7, assume the reluctance of the iron part of the magnetic circuit to be negligible. Determine the time average force on the movable member at any fixed position of the moving member, if*

(a) $i = I \cos \omega t$

(b) $\upsilon = V \cos \omega t$

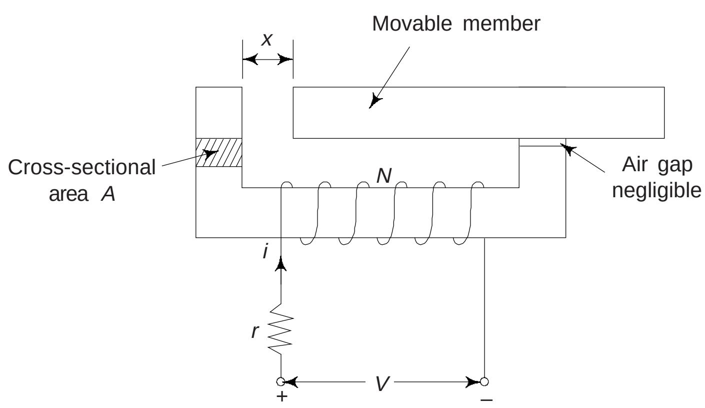


Fig. P4.7

Solution

$$R = \frac{x}{\mu_0 A}$$

$$\lambda = N\phi = \frac{N^2 i}{R}\left(\frac{N^2 \mu_0 A}{x}\right) i$$

$$L(x) = \frac{N^2 \mu_0 A}{x} = \frac{a}{x}$$

(a) $W'_{\mathrm{f}}(i, x) = \dfrac{1}{2} L(x) i^2$

$$F_{\mathrm{f}} = \frac{\partial W'_f}{\partial x} = \frac{1}{2} i^2 \frac{\partial L}{\partial x}$$

$$= -\frac{1}{2} i^2 \left(\frac{a}{x^2}\right)$$

$$F_{\mathrm{f}} = -\frac{1}{2} I^2 \left(\frac{a}{x^2}\right) \cos^2 \omega t$$

$$F_{\mathrm{f}}(\mathrm{av}) = -\frac{1}{4} I^2 \left(\frac{a}{x^2}\right) \text{ (in a direction to reduce } x)$$

(b) $\upsilon = ri + L \dfrac{di}{dt}$

$$V(j\omega) = (r + j\omega L)\, I(j\omega)$$

or

$$I(j\omega) = \frac{V(j\omega)}{(r + j\omega L)}$$

$\therefore$

$$i = -\frac{1}{4} \frac{V}{\sqrt{r^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{r}\right)$$

$$F_{\mathrm{f}}(\mathrm{av}) = -\frac{1}{4}\left(\frac{V^2}{r^2 + \omega^2 L^2}\right)\left(\frac{a}{x^2}\right)$$

Substituting $L(x) = a/x$, we get

$$F_{\mathrm{f}}(\mathrm{av}) = -\frac{aV^2}{(r^2 x^2 + a^2 \omega^2)}$$

4.8 *Two coils have self and mutual-inductances of*

$$L_{11} = L_{22} = \frac{2}{(1 + 2x)}$$

$$L_{12} = (1 - 2x)$$

The coil resistances may be neglected.

(a) *If the current I_1 is maintained constant at 5 A and I_2 at –2 A, find the mechanical work done when x increases from 0 to 0.5 m. What is the direction of the force developed?*

(b) *During the movement in part (a), what is the energy supplied by sources supplying currents I_1 and I_2?*

Solution

$$W'_f(i_1, i_2, x) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$= \frac{i_1^2}{(1+2x)} + i_1 i_2 (1-2x) + \frac{i_2^2}{(1+2x)}$$

$$F_f = \frac{\partial W'_f}{\partial x} = \frac{-2i_1^2}{(1+2x)^2} - 2i_1 i_2 + \frac{-2i_2^2}{(1+2x)^2}$$

(a) $I_1 = 5$ A $\quad I_2 = -2$ A

$$F_f = -\frac{58}{(1+2x)^2} + 20$$

$$\text{Mechanical work done} = \int_0^{0.5} F_f\, dx = -\int_0^{0.5} \frac{58}{(1+2x)^2} dx + 20 \int_0^{0.5} dx$$

$$= \left.\frac{58}{2(1+2x)}\right|_0^{0.5} + 20 \times 0.5 = -4.5 \text{ J}$$

Force becomes zero for

$$(1 + 2x)^2 = \frac{58}{20} = 2.9$$

$$1 + 2x = 17 \quad \text{or} \quad x = 0.35 \text{ m}$$

If F_f is negative $0 < x < 0.35$, i.e., tends to decrease x.
If F_f is positive $x > 0.35$, i.e. tends to increase x.

(b) $\lambda_1 = L_{11} i_1 + L_{12} i_2$

$$= \frac{2i_1}{1+2x} + (1 - 2x) i_2$$

$$= \frac{10}{1+2x} - 2(1 - 2x)$$

$$\lambda_1(x = 0.5) - \lambda_1(x = 0) = \left[\frac{10}{2} - 2(1-1)\right] - [10 - 2] = -3$$

$$\text{Energy input to coil } 1 = \frac{1}{2} \times 5 \times (-3) = -7.5 \text{ J}$$

$$\lambda_2 = L_{12} i_1 + L_{22} i_2$$

$$= (1-2x) i_1 + \frac{2i_2}{1+2x} = 5(1 - 2x) - \frac{4}{1+2x}$$

$$\lambda_2(x = 0.5) - \lambda_2(x = 0) = (0 - 2) - (5 - 4) = -3$$

$$\text{Energy input to coil 2} = \frac{1}{2} \times (-2x) - 3 = 3 \text{ J}$$

4.9 *Two coils have self- and mutual-inductances of*

$$L_{11} = L_{22} = \frac{2}{(1+2x)}$$

$$L_{12} = \frac{1}{(1+2x)}$$

Calculate the time-average force and coil currents at x = 0.5 m if:

(a) both the coils are connected in parallel across a voltage source 100 cos 314t

(b) coil 2 is shorted while coil 1 is connected across a voltage source of 100 cos 314t

(c) the two coils are connected in series across a voltage source of 100 cos 314t.

Solution

$$W'_f(i_1, i_2, x) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$= \frac{1}{1+2x} i_1^2 + \frac{1}{1+2x} i_1 i_2 + \frac{1}{1+2x} i_2^2$$

$$= \left(\frac{1}{1+2x}\right)\left(i_1^2 + i_1 i_2 + i_2^2\right)$$

$$F_f = \frac{\partial W'_f}{\partial x} = -\frac{2}{(1+2x)^2}\left(i_1^2 + i_1 i_2 + i_2^2\right)$$

$$x = 0.5 \text{ m}$$
$$L_{11} = L_{22} = 1$$
$$L_{12} = 0.5$$

$$F_f = -\frac{1}{2}\left(i_1^2 + i_1 i_2 + i_2^2\right)$$

(a) $\bar{V}(j\omega) = (j\omega)\ \bar{I}_1(j\omega) + 0.5\ (j\omega)\ \bar{I}_2(j\omega)$

$\bar{V}(j\omega) = 0.5\ (j\omega)\ \bar{I}_1(j\omega) + (j\omega)\ \bar{I}_2(j\omega)$

Solving we get

$$\bar{I}_1(j\omega) = \bar{I}_2(j\omega) = \frac{\bar{V}(j\omega)}{1.5(j\omega)}$$

Therefore

$$i_1 = i_2 = \frac{100}{1.5 \times 314} \cos(314t - 90°) = \frac{100}{1.5 \times 314} \sin 314t$$

$$F_f = -\frac{1}{2}\left(\frac{100}{1.5 \times 314}\right)^2 \times 3 \sin^2 314t$$

$$F_f(\text{av}) = -\frac{3}{4}\left(\frac{100}{1.5 \times 314}\right)^2 = -0.034 \text{ N}$$

(b) $\bar{V}(j\omega) = (j\omega)\ \bar{I}_1(j\omega) + 0.5\ (j\omega)\ \bar{I}_2(j\omega)$

$0 = 0.5\ (j\omega)\ \bar{I}_1(j\omega) + (j\omega)\ \bar{I}_2(j\omega)$

Solving we get

$$\bar{I}_1(j\omega) = \frac{\bar{V}(j\omega)}{0.75(j\omega)}$$

$$\bar{I}_2(j\omega) = -\frac{\bar{V}(j\omega)}{1.5(j\omega)}$$

Therefore

$$i_1 = \frac{1400}{3 \times 314}\ \sin 314t$$

$$i_2 = -\frac{200}{3 \times 314}\ \sin 314t$$

Substituting in F_f

$$F_f = -\frac{1}{2} \times \left(\frac{200}{3 \times 314}\right)^2 (4 - 2 + 1) \sin^2 314t$$

$$= -\frac{3}{2} \times \left(\frac{200}{3 \times 314}\right)^2 \sin^2 314t$$

$$F_f(\text{av}) = -\frac{3}{4} \times \left(\frac{200}{3 \times 314}\right)^2 = -0.034 \text{ N}$$

(c) $\bar{V}(j\omega) = (j\omega)\ \bar{I}(j\omega) + 0.5\ (j\omega)\ \bar{I}(j\omega)$

or
$$\bar{I}(j\omega) = \frac{\bar{V}(j\omega)}{1.5(j\omega)}$$

Therefore,

$$i_1 = i_2 = i = \frac{100}{1.5 \times 314}\ \sin 314t$$

$$F_f = -\frac{1}{2} \times \left(\frac{100}{1.5 \times 314}\right)^2 (1 + 1 + 1) \sin^2 314t$$

$$F_f(\text{av}) = -\frac{3}{4} \times \left(\frac{100}{1.5 \times 314}\right)^2 = -0.034 \text{ N}$$

4.10 *The doubly-excited magnetic field system of Fig. 4.15 has coil self- and mutual-inductances of*

$$L_{11} = L_{22} = 2 + \cos 2\theta$$
$$L_{12} = \cos\ \theta$$

where θ is the angle between the axes of the coils. The coils are connected in series and carry a current of $i = \sqrt{2} I \sin\ \omega t$. Derive an expression for the time average torque as a function of angle θ.

Solution

$$W'_f = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

or $$W'_f(i_1, i_2, \theta) = \frac{1}{2}(2 + \cos 2\theta) i_1^2 + (\cos\theta) i_1 i_2 + \frac{1}{2}(2 + \cos 2\theta) i_2^2$$

$$i = i_1 = i_2 \text{ (given)}$$

$\therefore$ $$W'_f(i, \theta) = [(2 + \cos 2\theta) + \cos\theta] i^2$$

$$T_f = \frac{\partial W'_f}{\partial \theta} = (-2 \sin 2\theta - \sin\theta) i^2$$

$$= -(2 \sin 2\theta + \sin\theta) \times 2I^2 \sin^2 \omega t$$

$$T_f(\text{av}) = -I^2 (2 \sin 2\theta + \sin\theta)$$

4.11 *In the rotary device of Fig. 4.15, when the rotor is in the region of $\theta = 45°$, the coil inductances can be approximated as*

$$L_{11} = L_{22} = 2 + \frac{\pi}{2}\left(1 - \frac{\theta}{45}\right)$$

$$L_{12} = L_{21} = \frac{\pi}{2}\left(1 - \frac{\theta}{90}\right)$$

where θ is in degrees.

Calculate the torque of field origin if the rotor is held in position $\theta = 45°$ with

(a) $i_1 = 5$ A $i_2 = 0$

(b) $i_1 = 0$ $i_2 = 5$ A

(c) $i_1 = 5$ A $i_2 = 5$ A

(d) $i_1 = 5$ A $i_2 = -5$ A

(e) Find the time-average torque if coil 1 carries a current of 5 sin 314t while coil 2 short-circuited.

Solution

$$W'_f(i_1, i_2, \theta) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$= \left[1 + \frac{\pi}{4}\left(1 - \frac{\theta}{45}\right)\right] i_1^2 + \frac{\pi}{2}\left(1 - \frac{\theta}{90}\right) i_1 i_2 + \left[1 + \frac{\pi}{4}\left(1 - \frac{\theta}{45}\right)\right] i_2^2$$

$$T_f = \frac{\partial W'_f}{\partial \theta} = \left(-\frac{\pi}{180} i_1^2 - \frac{\pi}{180} i_1 i_2 - \frac{\pi}{180} i_2^2\right) \times \frac{180}{\pi}$$

$$= -\left(i_1^2 + i_1 i_2 + i_2^2\right) \text{ (independent of } \theta)$$

(a) $T_f = -(25 + 0 + 0) = -25$ N

(b) $T_f = -(0 + 0 + 25) = -25$ N

(c) $T_f = -(25 + 25 + 25) = -75$ N

(d) $T_f = -(25 - 25 + 25) = -25$ N

(e) For the shorted coil

$$0 = L_{21} \frac{di_1}{dt} + L_{22} \frac{di_2}{dt}$$

At θ = 45°,

$$L_{11} = L_{22} = 2 + \frac{\pi}{2}\left(1 - \frac{45}{45}\right) = 2$$

$$L_{12} = L_{21} = \frac{\pi}{2}\left(1 - \frac{45}{90}\right) = \frac{\pi}{4}$$

$$0 = \frac{\pi}{4}\,(j\omega)\ \bar{I}_1(j\omega)\ \bar{I}_1(j\omega) + 2\,(j\omega)\ \bar{I}_2(j\omega)$$

$\therefore$ $$\bar{I}_2(j\omega) = -\frac{\pi}{8}\ \bar{I}_1(j\omega)$$

Now $$i = 5 \sin 314t$$

$\therefore$ $$i_2 = -\frac{\pi}{8} \times 5 \sin 314t$$

Hence

$$T_f = -\left(25 - 5 \times \frac{\pi}{8} \times 5 + \frac{\pi^2 \times 25}{64}\right) \sin^2 314t$$

$$= -19.04 \sin^2 314t$$

$$T_f\,(\mathrm{av}) = -9.52 \text{ N}$$

4.12 *Figure P4.12 shows the cross-sectional view of a cylindrical plunger magnet. The position of the plunger when the coil is unexcited is indicated by the linear dimension D. Write the differential equations describing the dynamics of the electromagnetic system. Determine the equilibrium position of the plunger and linearize the describing equation for incremental changes about the equilibrium point. Assume the iron to be infinitely permeable.*

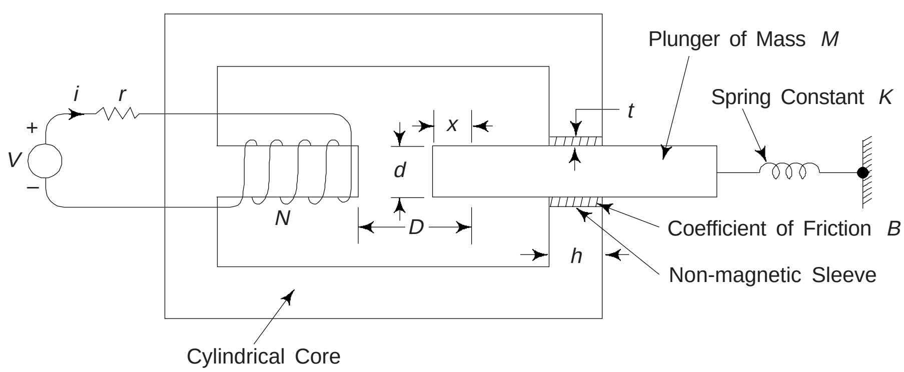


Fig. P4.12

Solution Reluctance of circular air gap,

$$R_1 = \frac{(D - x)}{\mu_0\left(\pi \frac{d^2}{4}\right)} = \frac{4(D - x)}{\pi \mu_0 d^2}$$

Reluctance of annular air gap,

$$R_2 = \frac{t}{\mu_0 \pi (d+t)h}$$

Coil inductance
$$L = \frac{N^2}{R_1 + R_2}$$

$$L = \frac{N^2}{\dfrac{4(D-x)}{\pi\mu_0 d^2} + \dfrac{t}{\pi\mu_0 (d+t)h}} = \frac{N^2}{a-bx}$$

where

$$a = \frac{t}{\pi\mu_0 (d+t)h} + \frac{4D}{\pi\mu_0 d^2}; \; b = \frac{4}{\pi\mu_0 d^2}$$

Electrical circuit equation

$$v = ir + \left(\frac{d}{dt}\lambda = \frac{d}{dt}(L(x))i\right)$$

$$= Ir + L(x)\frac{di}{dt} + i\frac{dL}{dx}\cdot\frac{dx}{dt}$$

or
$$v = ir + \frac{N^2}{a-bx}\frac{di}{dt} + i\frac{bN^2}{(a-bx)^2}\frac{dx}{dt} \tag{i}$$

$$W'_f(i, x) = \frac{1}{2} i^2 L(\mathrm{x})$$

$$F_f = \frac{dW'_f}{\partial x} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

$$= \frac{1}{2} i^2 \frac{bN^2}{(a-bx)^2}$$

Mechanical circuit equation

$$\frac{(b/2)N^2}{(a-bx)^2} i^2 = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \tag{ii}$$

Under equilibrium conditions

$$v = V_0 \qquad i = I_0 \qquad \frac{di}{dt} = 0 \qquad \frac{dx}{dt} = 0, \qquad x = X_0 \tag{iii}$$

We get from Eqs (i) and (ii)

$$V_0 = I_0 r$$

$$\frac{(b/2)N^2}{(a-bX_0)^2} I_0^2 = KX_0$$

Let the incremental values be expressed as

$$v_1, \; i_1, \; x_1$$

Substituting in (i) and (ii)

$$V_0 + v_1 = (I_0 + i_1)r + \frac{N^2}{a - bX_0 - bx_1}\frac{di_1}{dt}(I_0 + i_1)\frac{bN^2}{(a - bX_0 - bx_1)^2}\frac{dx_1}{dt} \quad \text{(iv)}$$

$$\frac{(b/2)N^2(I_0 + i_1)^2}{(a - bX_0 - bx_1)^2} = M\frac{d^2x_1}{dt^2} + B\frac{dx_1}{dt} + K(X_0 + x_1) \quad \text{(v)}$$

Cancelling out equilibrium terms in (iv) and (v) and neglecting the product of incremental values, we get

$$v_1 = i_1 r + \left(\frac{N^2}{a - bX_0}\right)\frac{di_1}{dt} + \left[\frac{b^2 NI_0}{(a - bX_0)^2}\right]\frac{dx_1}{dt} \quad \text{(vi)}$$

$$\frac{(b/2)N^2 I_0 i_1}{(a - bX_0)^2} = M\frac{d^2x_1}{dt^2} + B\frac{dx_1}{dt} + Kx_1 \quad \text{(vii)}$$

4.13 *For the electromagnet of Fig. P4.13 write the dynamical equation. Assume the cross-sectional area of each limb of the magnet to be A and the coupling between the two coils to be tight. Iron is to be taken as infinitely permeable.*

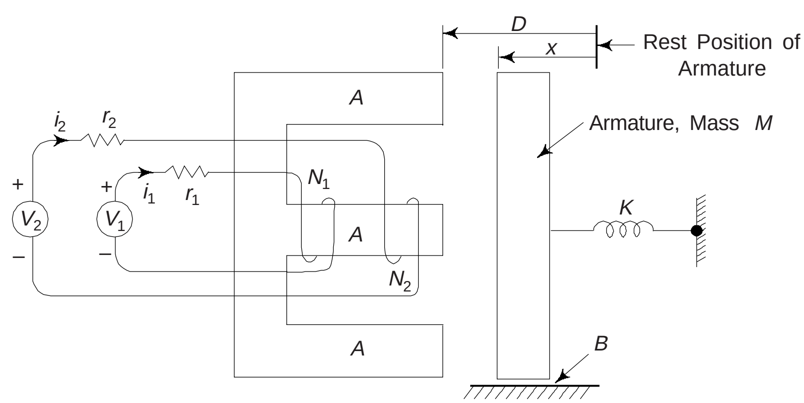


Fig. P4.13

Solution

$$R = \frac{D - x}{\mu_0 A} + \frac{D - x}{\mu_0(2A)} = \frac{3}{2\mu_0 A}(D - x) = a(D - x); \qquad a = \frac{3}{2\mu_0 A}$$

$$L_{11} = \frac{N_1^2}{a(D - x)}, \quad L_{22} = \frac{N_2^2}{a(D - x)}, \quad L_{12} = \frac{N_1 N_2}{a(D - x)}$$

$$v_1 = i_1 r_1 + \frac{d\lambda_1}{dt}$$

$$= i_1 r_1 + \frac{d}{dt}[L_{11} i_1 + L_{12} i_2]$$

$$= i_1 r_1 + i_1 \frac{dL_{11}}{dx}\frac{dx}{dt} + L_{11}(x)\frac{di_1}{dt} + i_2 \frac{dL_{12}}{dx}\frac{dx}{dt} + L_{12}(x)\frac{di_2}{dt}$$

or $$v_1 = i_1 r_1 + \frac{N_1^2 i_1}{a(D-x)}\frac{dx}{dt} + \frac{N_1^2}{a(D-x)}\frac{di_1}{dt} + \frac{N_1 N_2 i_2}{a(D-x)}\frac{dx}{dt} + \frac{N_1 N_2}{a(D-x)}\frac{di_2}{dt} \quad \text{(i)}$$

Similarly,

$$v_2 = i_2 r_2 + \frac{N_2^2 i_2}{a(D-x)}\frac{dx}{dt} + \frac{N_2^2}{a(D-x)}\frac{di_2}{dt} + \frac{N_1 N_2 i_1}{a(D-x)}\frac{dx}{dt} + \frac{N_1 N_2}{a(D-x)}\frac{di_1}{dt} \quad \text{(ii)}$$

$$W'_f(i_1, i_2, x) = \frac{1}{2} L_{11}(x) i_1^2 + L_{12}(x) i_1 i_2 + \frac{1}{2} L_{22}(x) i_2^2$$

$$F_f = \frac{\partial W'_f}{\partial x} = \frac{N_1^2}{2a(D-x)^2} i_1^2 + \frac{N_1 N_2}{a(D-x)^2} i_1 i_2 + \frac{N_2^2}{2a(D-x)^2} i_2^2 \quad \text{(iii)}$$

$$\frac{N_1^2}{2a(D-x)^2} i_1^2 + \frac{N_1 N_2}{a(D-x)^2} i_1 i_2 + \frac{N_2^2}{2a(D-x)^2} i_2^2 = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \quad \text{(iv)}$$

Equations (i) and (iv) describe the system dynamics.

4.14 *For the electromechanical system shown in Fig. P4.14, the air-gap flux density under steady operating conditions is*

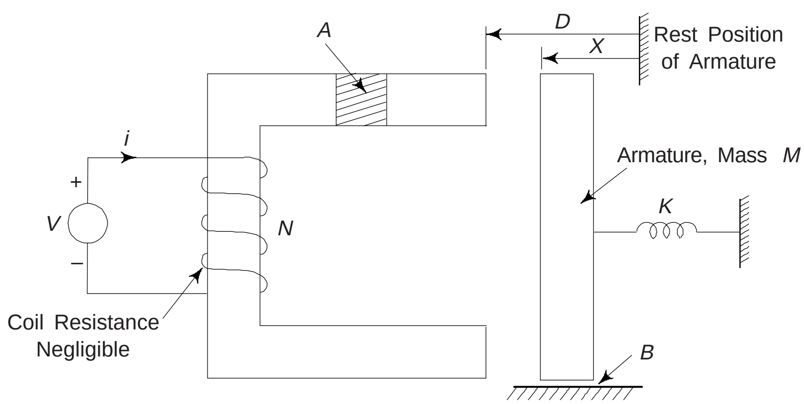


Fig. P4.14

$$B(t) = B\text{m} \sin \omega t$$

Find

(a) the coil voltage

(b) the force of field origin as a function of time

(c) the motion of armature as a function of time.

Solution

$$R = \frac{2(D-x)}{\mu_0 A}$$

$$L(x) = \frac{N^2}{R} = \frac{\mu_0 N^2 A}{2(D-x)}$$

$$\lambda = NAB_m \sin \omega t$$

(a) $e = \dfrac{d\lambda}{dt} \; NA\, B_m\, \omega \cos \omega t$

(b) $W_f(\lambda, x) = \dfrac{1}{2}\dfrac{\lambda^2}{L} = \dfrac{1}{2}\dfrac{N^2 A^2 B_m^2 \sin^2 \omega t}{(\mu_0 N^2 A)/2(D - x)}$

$$= \mu_0^{-1} A B_m^2 \; (D - x) \sin^2 \omega t$$

$$F_f = -\frac{\partial W_f}{\partial x} = \mu_0^{-1} A B_m^2 \; \sin^2 \omega t$$

$$= \frac{1}{2} \; \mu_0^{-1} A B_m^2 \; (1 - \cos 2\omega t)$$

(c) $\dfrac{1}{2} \; \mu_0^{-1} A B_m^2 - \dfrac{1}{2} \; \mu_0^{-1} A B_m^2 \; \cos 2\omega t = M \dfrac{d^2 x_1}{dt^2} + B \dfrac{dx_1}{dt} + K(X_0 + x_1)$

$$X_0 = \frac{\mu_0^{-1} A B_m^2}{2K}$$

x can be obtained from

$$-\frac{1}{2} \; \mu_0^{-1} A B_m^2 \; \cos 2\omega t \quad = \mathrm{M} \; \frac{d^2 x_1}{dt^2} + B \frac{dx_1}{dt} + K x_1$$

$$H(j\omega) = \frac{1}{M(j\,2\omega)^2 + B(j\,2\omega) + K} = \frac{1}{\sqrt{(K - 4M\omega)^2 + 4B^2 \; \omega^2}}$$

$$-\tan^{-1} \frac{k - 4M\omega}{2B\omega} = -\psi$$

$$\therefore \qquad x_1(t) = \frac{-\mu_0^{-1} A B_m^2}{2\sqrt{(K - 4M\omega)^2 + 4B^2 \omega^2}} \; \cos(2\omega t - \psi)$$

Net movement is

$$x(t) = X_0 + x_1(t)$$

CHAPTER 5: BASIC CONCEPTS IN ROTATING MACHINES

5.1 *Determine the breadth and pitch factors for a 3-phase winding with two slots per pole per phase. The coil span is five slot pitches.*

If the flux density wave in the air-gap consists of the fundamental and a 24% third harmonic, calculate the percentage increase in the rms value of phase voltage due to the harmonic.

Solution

$$m = 2$$

$$\gamma = \frac{60°}{2} = 30°$$

$$K_b = \frac{\sin \dfrac{2 \times 30°}{2}}{2 \dfrac{\sin 30°}{2}} = 0.966$$

$$\text{Full pitch} = 2 \times 3 = 6 \text{ slot pitches}$$

$$\text{Coil pitch} = 5 \text{ slot pitches}$$

$$\theta_{sp} = 1 \text{ slot pitch or } 30°$$

$$\therefore \quad K_p = \cos \frac{\theta_{sp}}{2} = \cos 15° = 0.966$$

For the third harmonic

$$K_{b3} = \frac{\sin 90°}{2 \sin 45°} = 0.707$$

$$K_{p3} = \cos 45° = 0.707$$

Let

$$\phi_1 \text{ (fundamental flux/pole)} = 1 \text{ unit}$$

$$\phi_2 \text{ (third harmonic flux)} = \frac{0.24}{3} = 0.08 \text{ unit}$$

Now

$$E_1 = K \times 1 \times 0.966 \times 0.966 = 0.933\ K$$

$$E_3 = 3K \times 0.08 \times 0.707 \times 0.707 = 0.12\ K$$

$$\text{rms value of phase voltage} = \sqrt{E_1^2 + E_3^2} = K\sqrt{(0.933)^2 + (0.12)^2} = 0.94\ K$$

$$\%\text{ increase in rms value} = \frac{0.94 - 0.933}{0.933} \times 100 = 0.75\%$$

5.2 *A 50 Hz, 6-pole synchronous generator has 36 slots. It has a two-layer winding with full-pich coils of eight turns each. The flux per pole is 0.015 Wb (sinusoidally distributed). Determine the induced emf (line-to-line) if the coils are connected to form (a) 2-phase winding (b) star-connected 3-phase winding.*

Solution

(a) 2-phase winding

$$m = \frac{36}{2 \times 6} = 3$$

$$\gamma = \frac{180° \times 6}{36} = 30°$$

$$K_b = \frac{\sin(2 \times 30°)/2}{3 \sin(30°/2)} = 0.911$$

$$N_{ph} = \frac{36 \times 8}{2} = 144$$

$$\begin{aligned} E_p &= 4.44 K_b f \Phi N_{ph} \\ &= 4.44 \times 0.911 \times 50 \times 0.015 \times 144 \\ &= 436.84 \end{aligned}$$

$$E_L = 436.84 \sqrt{2} = 617.78 \text{ V}$$

(b) 3-phase winding

$$m = \frac{36}{3 \times 6} = 2$$

$$\gamma = 30°$$

$$K_b = \frac{\sin(2 \times 30°)/2}{2 \sin(30°/2)} = 0.966$$

$$N_{ph} = \frac{36 \times 8}{3} = 96$$

$$\begin{aligned} E_p &= 4.44 \times 0.966 \times 50 \times 0.015 \times 96 \\ &= 308.81 \text{ V} \end{aligned}$$

$$E_L = \sqrt{3} \times 308.81 = 534.86 \text{ V}$$

5.3 *The air-gap flux density distribution of a 6-pole, 50 Hz synchronous generator is*

$$B(\theta) = B_1 (\sin \theta + 0.3 \sin 2\theta + 0.15 \sin 5\theta)$$

The total flux/pole is 0.015 Wb. Find the fundamental, third and fifth harmonic flux/ pole.

Solution From Eq. (5.9)

$$\Phi_1 = \frac{4}{P_1} B_1 lr = \frac{2}{3} B_1 lr$$

$$\Phi_3 = \frac{4}{P_3} \times 0.3 \ B_1 lr = \frac{0.2}{3} B_1 lr$$

$$\Phi_5 = \frac{4}{P_5} \times 0.1 \ B_1 lr = \frac{0.2}{15} B_1 lr$$

$$\Phi = \Phi_1 + \Phi_3 + \Phi_5 = \left(\frac{2}{3} + \frac{0.2}{3} + \frac{0.2}{15}\right) B_1 lr = 0.015$$

$\therefore$
$$B_1 lr = \frac{0.015 \times 15}{11.2} = 0.02$$

It now follows

$$\Phi_1 = \frac{2}{3} \times 0.02 = 0.0133 \text{ Wb}$$

$$\Phi_3 = \frac{0.2}{3} \times 0.02 = 0.0013 \text{ Wb}$$

$$\Phi_5 = \frac{0.2}{15} \times 0.02 = 0.0002 \text{ Wb}$$

5.4 *Show that the limiting value of the breadth factor for the fundamental is*

$$K_b = \frac{\sin 1/2\sigma}{1/2\sigma}$$

where $\sigma = m\gamma =$ *phase spread*

and m, the slots per pole per phase tends to be large.

Solution For a given phase spread σ (usually 60°) as m tends to be large, γ tends to be small.

The phasor diagram of coil voltage now becomes the arc of a circle whose chord AB is the resultant voltage, as shown in Fig. P5.4.

$$K_b = \frac{\text{chord } AB}{\text{arc } AB} = \frac{2OA \sin(\sigma/2)}{(OA)\sigma} = \frac{\sin(\sigma/2)}{\sigma/2}$$

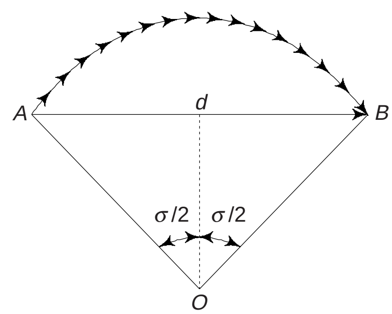


Fig. P5.4

Note: σ must be expressed in radians.

5.5 *A 50 Hz synchronous salient pole generator is driven by a hydroelectric turbine at a speed of 125 rpm. There are 576 stator slots with two conductors per slot. The air-gap diameter is 6.1 m and the stator length is 1.2 m. The sinusoidally distributed flux density has a peak value of 1.1 T. (a) Calculate the maximum rms single-phase voltage that can be produced by suitably connecting all the conductors. (b) Find the per phase emf if the conductors are connected in a balanced 3-phase winding.*

Solution

$$P = \frac{120 f}{N} = \frac{120 \times 50}{125} = 48$$

$$\Phi = \frac{4}{P} B_p lr; \quad \text{Eq. (5.9)}$$

$$= \frac{4}{48} \times 11 \times 12 \times \frac{6.1}{2} = 0.336 \text{ Wb}$$

(a) Slots/pole $$m = \frac{576}{48} = 12$$

$$\gamma = \frac{180° \times 48}{576} = 15°$$

In a single-phase connection emfs of all the coils under a pole pair are added, therefore

$$K_b = \frac{\sin\left(\frac{12 \times 15°}{2}\right)}{12 \sin\left(\frac{15°}{2}\right)} = 0.638$$

Total series turns, $$N = \frac{576 \times 2}{2} = 576$$

$$E = 4.44\ K_b\ f\Phi\ N$$
$$= 4.44 \times 0.638 \times 50 \times 0.336 \times 576 = 27{,}412\ \text{V}$$

(b) Slots/pole/phase, $$m = \frac{12}{3} = 4$$

$$K_b = \frac{\sin \frac{4 \times 15°}{2}}{4 \sin \frac{15°}{2}} = 0.958$$

$$N_{ph} = \frac{576}{3} = 192$$

$$E_p = 4.44 \times 0.958 \times 50 \times 0.336 \times 192$$
$$= 13{,}720\ \text{V}$$

5.6 *Find the number of series turns required for each phase of a 3-phase, 50 Hz, 10-pole alternator with 90 slots. The winding is to be star-connected to give a line voltage of 11 kV. The flux/pole is 0.16 Wb.*

Solution

Slots/pole/phase, $$m = \frac{90}{3 \times 10} = 3$$

$$\gamma = \frac{180° \times 10}{90} = 20°$$

$\therefore$ $$K_b = \frac{\sin \frac{3 \times 20°}{2}}{3 \sin \frac{20°}{2}} = 0.96$$

Winding is assumed to be full-pitched.

$$E_p = 4.44\ K_b\ f\Phi\ N_{ph}$$

$$\frac{11,000}{\sqrt{3}} = 4.44 \times 0.96 \times 50 \times 0.16 \times N_{ph}$$

or
$$N_{ph} = \frac{11,000}{\sqrt{3} \times 4.44 \times 0.96 \times 50 \times 0.16} = 186 \approx 180$$

$$\text{Conductors/slot} = \frac{180 \times 3 \times 2}{90} = 12$$

5.7 *A dc armature is built up of laminations having an external diameter of 80 cm and internal diameter of 42 cm. The length of the armature is 32 cm. The flux density in the armature core is 0.85 T. The armature is wave-connected with 72 slots, with 3 conductors/slot. If the number of poles is 6, find the emf induced when the armature is rotated at a speed of 600 rpm.*

Hint: (see page 267)

Solution Cross-sectional area of armature case

$$= 32 \times \left(\frac{80 - 42}{2}\right) \times 10^{-4} = 0.0608 \text{ m}^2$$

Flux through armature core = 0.85 × 0.0608 = 0.0517
Flux/pole = 2 × 0.0517 = 0.1034 Wb
For wave winding, $A = 2$

$$Z = 72 \times 8 = 576$$

$$E_a = \frac{\Phi NZ}{60}\left(\frac{P}{A}\right)$$

$$= \frac{0.1034 \times 600 \times 576}{60} \times \left(\frac{6}{2}\right) = 1{,}787 \text{ V}$$

5.8 *A 6-pole, wave-connected dc armature has 250 conductors and runs at 1200 rpm. The emf generated is 600 V. Find the useful flux/pole.*

Solution

$$A = 2$$

$$E_a = \frac{\Phi NZ}{60}\left(\frac{P}{A}\right)$$

$$600 = \frac{\Phi \times 1,200 \times 250}{60}\left(\frac{6}{2}\right)$$

or
$$\Phi = 0.04 \text{ Wb}$$

Note: Useful flux links armature coils and induces emf. Some of the pole flux does not link armature coil and is called leakage flux. The total flux in the pole body is the sum of these two fluxes and is therefore more than the useful flux/pole.

5.9 *A 4-pole, dc machine has a lap-connected armature having 60 slots and eight conductors per slot. The flux per pole is 30 mWb. If the armature is rotated at 1000 rpm, find the emf available across its armature terminals. Also calculate the frequency of emf in the armature coils.*

Solution

$$E_a = \frac{\Phi NZ}{60}\left(\frac{P}{A}\right)$$

$$Z = 60 \times 8 = 480$$

$$E_a = \frac{30 \times 10^{-3} \times 1{,}000 \times 480}{60} \times \left(\frac{4}{4}\right)$$

$$= 240 \text{ V}$$

$$f = \frac{NP}{120} = \frac{1{,}000 \times 4}{120} = 33\frac{1}{3} \text{ Hz}$$

5.10 *Trace out the variations in mmf due to a belt of current-carrying conductors representing one phase of a 2-pole, 3-phase winding. The belt may be assumed to be a current sheet with uniform current density. What is the peak amplitude of the mmf wave if the total current in the belt is A amperes?*
(Hint: The mmf wave is trapezoidal)

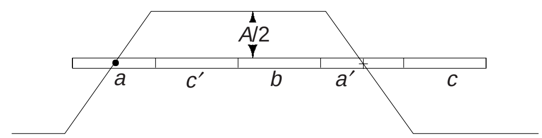


Fig. P5.10

Solution (see Fig. 5.10)

5.11 *Each phase belt of a 2-pole, 3-phase winding carrying balanced 3-phase currents can be assumed to be a current sheet with uniform density. Sketch the resultant mmf wave at $\omega t_1 = 0$, $\omega t_2 = \pi/3$ and $\omega t_3 = 2\pi/3$.*

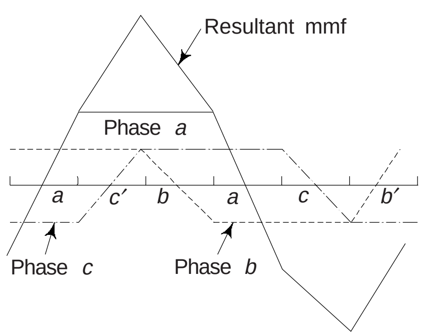


Fig. P5.11(a)

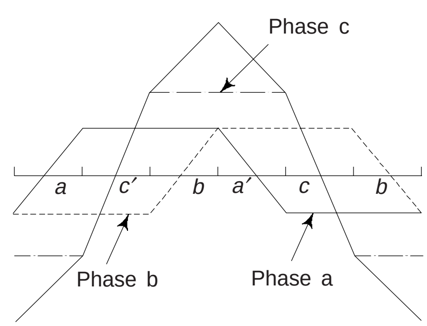


Fig. P5.11(b)

Solution

	$\omega t_1 = 0$	$\omega t_2 = \pi/3$	$\omega t_3 = 2\pi/3$
$i_a = I_m \cos \omega t$	I_m	$I_m/2$	$-I_m/2$
$i_b = I_m \cos (\omega t - 120°)$	$-I_m/2$	I_m	I_m
$i_c = l_m \cos (\omega t - 240°)$	$-I_m/2$	$-I_m$	$-I_m/2$

current corresponding to I_m in a phase belt = 1 unit (AC) say. The resultant mmf waves at $\omega t_1 = 0$ and $\omega t_2 = \pi/3$ are drawn in Figs P5.11(a) and P5.11(b).

The third case can be similarly drawn.

5.12 *Phase a of a 3-phase stator at the instant of carrying maximum current has 60 A/ conductors in the phase belt. Sketch the mmf wave of the phase when the slots/pole/phase are 1, 2, 3, 4 and 5 respectively. Comment upon the change in the shape of the mmf wave with the number of slots/pole/phase.*

Solution The mmf waves for the five cases are drawn in Fig. P5.12. As SPP increases, the mmf wave has more stip and its shape becomes progressively closer to a sine wave.

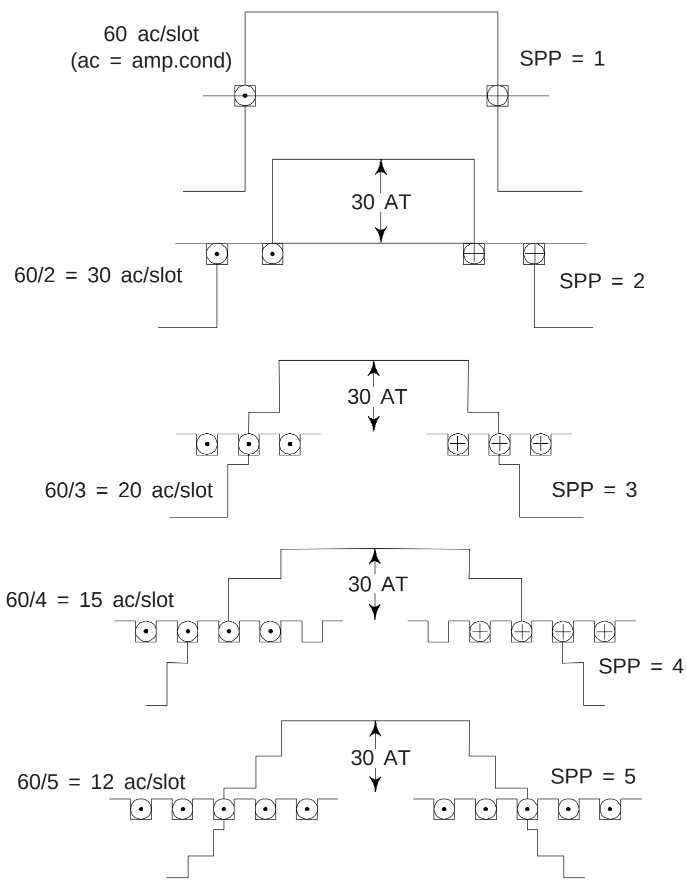


Fig. P5.12

5.13 *A 2-pole, 3-phase ac winding is housed in 18 slots, each slot having 12 conductors. Consider the time instant at which the current in phase a has its maximum value 10.0 A.*

(a) Sketch all the 18 slots on a horizontal axis. Mark the direction of currents in the conductors occupying the slots relevant to phase a. Make a proportional sketch of the mmf wave of phase a only.

(b) Mark the maximum value of mmf wave on the sketch.

(c) Calculate the peak value of the fundamental of the mmf of phase a.

Solution

(a) Ampere-conductors/slot = 12 × 10 = 120
MMF wave of phase *a* is sketched in Fig. P5.13.

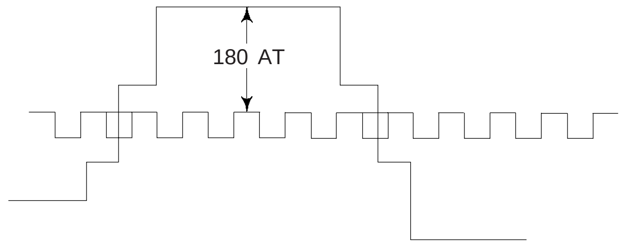


Fig. P5.13

(b) Peak value of mmf wave = 180 AT

(c) $m = 3, \quad \gamma = \dfrac{2 \times 180°}{18} = 20°, \quad K_p = \dfrac{\sin(3 \times 20°/2)}{3 \sin(20°/2)} = 0.96$

$$F_{a1}(\text{peak}) = \frac{4}{\pi} \times 0.96 \times \left(\frac{12 \times 18}{2 \times 3 \times 2}\right) \times 10 = 220 \text{ AT}$$

5.14 *A 4-pole, 50 Hz induction motor has 24 stator slots with 2-layer winding. It has a 16-turn coil chorded (short-pitched) by one slot. The machine is delta-connected and has a 440 V, 3-phase supply. If the stator resistance and leakage reactance are assumed negligible, find the flux/pole of the rotating flux density wave.*

Solution

$$S = 24 \text{ C (coils)}$$

$$\text{SPP} = \frac{24}{4 \times 3} = 2$$

$$\gamma = \frac{180° \times 4}{24} = 30°$$

$$\theta_{sp} = \text{one slot pitch} = 30°$$

$$K_b = \frac{\sin m\gamma/2}{m \sin \gamma/2} = \frac{\sin(2 \times 30°)}{2 \sin 30°/2} = 0.966$$

$$K_p = \cos \frac{\phi_{sp}}{2} = \cos 15° = 0.966$$

$$N_{ph}(\text{series}) = \frac{16 \times 24}{3} = 128$$

Phase voltage $E_p = 440$ V

$$E_p = 4.44\ K_b K_p\ f \Phi\ N_{ph}(\text{series})$$

$$440 = 4.44 \times 0.966 \times 0.966 \times 50 \times \Phi \times 128$$

or $\Phi = 0.0166$ Wb/pole

5.15 *The induction machine of Prob. 5.14 has a stator length of 28 cm and a mean air-gap diameter of 18 cm. The machine air-gap is 1 mm. What line current will it draw when running at no-load? (Hint At no-load the machine draws only the magnetizing current to establish flux/pole, as calculated in Prob. 5.14).*

Solution As calculated in Prob. 5.14, Φ = 0.0166 Wb/pole

$$\text{Pole area} = 28 \times \frac{\pi \times 18}{4} \times 10^{-4} = 0.0396 \text{ m}^2$$

$$B_{av} = \frac{0.0166}{0.0396} = 0.419 \text{ Wb/m}^2$$

$$B_{peak} = \frac{\pi}{2} \times 0.419 = 0.658 \text{ Wb/m}^2$$

Neglecting reluctance of air-gap,

$$B_{peak} = \frac{3\mu_0 F_m}{2g}$$

or

$$F_m = \frac{2g B_{peak}}{3\mu_0}$$

$$= \frac{2 \times 1 \times 10^{-3} \times 0.658}{3 \times 4\pi \times 10^{-7}} = 349 \text{ AT/pole}$$

Now

$$F_m = \frac{4}{\pi} K_W \left(\frac{N_{ph}\,(\text{series})}{P}\right) I_m$$

K_w= 0.966 × 0.966 = 0.933 (from Prob. 5.14)

$$I_m = \frac{\pi F_m P}{4K_w N_{ph(\text{series})}}$$

$$= \frac{\pi \times 349 \times 4}{4 \times 0.933 \times 128} = 9.18 \text{ A}$$

$$I_p(\text{rms}) = \frac{9.18}{\sqrt{2}} = 6.5 \text{ A}$$

$$I_L = 6.5\sqrt{3} = 11.26 \text{ A}$$

5.16 *In Problem 5.6 what will be the peak value of resultant mmf/pole if the winding is chorded by one slot?*

Solution

$$\gamma = 12°$$

$$\theta_{sp} = 12°$$

$$K_p = \cos\frac{\theta_{sp}}{2} = \cos 6° = 0.994$$

$\therefore$

$$F_{peak} = 10{,}425 \times 0.994 = 10{,}368 \text{ AT/pole}$$

5.17 *A 3-phase induction motor runs at a speed of 1485 rpm at no-load and at 1,350 rpm at full-load when supplied from a 50 Hz, 3-phase line.*

(a) How many poles does the motor have?

(b) What is the % slip at no-load and at full-load?

(c) What is the frequency of rotor voltages at no-load and at full-load?

(d) What is the speed at both no-load and full-load of: (i) the rotor field with respect to rotor conductors (ii) the rotor field with respect to the stator and (iii) the rotor field with respect to the stator field.

Solution

(a) At 50 Hz, the nearest synchronous speed to no-load speed (1,490) is 1,500 rpm. Therefore the number of motor poles = 4.

(b) $$s(\text{no-load}) = \frac{N_s - N_1}{N_s} = \frac{1,500 - 1,485}{1,500} \times 100 = 1\%$$

$$s(\text{full-load}) = \frac{1,500 - 1,350}{1,500} \times 100 = 10\%$$

(c) $f_2 = sf$

$= 0.01 \times 50 = 0.5$ Hz (no-load)

$= 0.1 \times 50 = 5$ Hz (full-load)

(d) (i) N (rotor field wrt rotor conductors)

$$= \frac{120 \times 0.5}{4} = 15 \text{ rpm (no-load)}$$

$$= \frac{120 \times 5}{4} = 150 \text{ rpm (full-load)}$$

(ii) N (rotor field wrt stator) = 1,500 rpm (no-load and full-load)

(iii) N (rotor field wrt stator) = 0 (no-load and full-load)

5.18 *A 4-pole, 3-phase synchronous motor fed from 50 Hz mains is mechanically coupled to a 24-pole, 3-phase synchronous generator. At what speed will the set rotate? What is the frequency of the emf induced in the generator?*

Solution

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1,500 \text{ rpm (set speed)}$$

$$f = \frac{NP}{120} = \frac{1,500 \times 24}{120} = 300 \text{ Hz (gen. frequency)}$$

5.19 *A 20-pole synchronous generator running at 300 rpm feeds a 6-pole induction motor which is loaded to run at a slip of 5%. Find the speed at which the induction motor runs and the frequency of the currents induced in its rotor.*

Solution

$$f = \frac{NP}{120} = \frac{300 \times 200}{120} = 50 \text{ Hz}$$

For induction motor

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$s = 0.05$$

$$\therefore \quad N = (1 - 0.05) \times 1,000 = 950 \text{ rpm}$$

$$f_2 = sf = 0.05 \times 50 = 2.5 \text{ Hz}$$

5.20 *A slip ring induction motor runs at 285 rpm on full-load when connected to 50 Hz supply. Calculate: (a) the number of poles (b) the slip and (c) the slip for full-load torque if total resistance of the rotor circuit is doubled. Assume the rotor leakage reactance to be negligible in the range of slips being considered.*

Solution

(a) Nearest synchronous speed = 300 rpm

$$P = \frac{120f}{N} = \frac{120 \times 50}{300} = 20$$

(b) slip = $\dfrac{300 - 285}{300} \times 100 = 5\%$

(c) Since the torque remains constant at full-load value when rotor resistance is doubled, the rotor current must remain constant. It means that the rotor voltage (sv_2) must double. Hence

$$s = 2 \times 5 = 10\%$$

5.27 The outside diameter of the rotor of an alternation is 0.74 and the axial length is 1.52 m. The machine has four poles and the flux density at the rotor surface is given by 1.12 cos θ_e where θ_e = elect.angle.
(a) Find the flux/pole
(b) If the peak value of Fr, is 18000 AT, calculate the permeance/pole

Rotor area of each pole = ((π X 0.74) / 4) X 1.52 = 3.534m^2
Brotaor = 1.12 Cos θe

$Bav = 1/\pi \times \int^{\pi/2}_{\pi/2} 1.12 \cos \theta_e \, d\theta_e$

$= 1/\pi \times 1.12 [\sin \theta_e]^{\pi/2}_{-\pi/2}$

$= 1/\pi \times 1.12 \times (1+1) = 2/\pi \times 1.12 = 0.713$ T

(a) Flux pole = 0.713 X 3.534 = 2.52 wb
(b) Fr = 18000 T
Φ = P Fr
P = Φ/Fr = (2.52 X 1000)/18000 = 0.14 wb/AT

5.28 A synchronous generator of 50 Hz with 6 poles has a flux / pole of 0.15 Wb. Each stator coil has two turns and a coil pitch of 150° elect. Calculate the coil voltage (rms).

Φ = 0.15 wb
$E_c = 4.44\, K_p\, f\, \Phi\, Nc$
$K_p = \cos \theta_{sp} = \cos (180° - 150°)/2 = \cos 30°/2 = 0.966$
Therefore = E_C = 4.44 X 0.966 X 50 X 2 = 64.34 V

5.29 Calculate the short-pitching angle to eliminate the fifth harmonic in the induced emf of a synchronous generator. What is the corresponding reduction in the fundamental and the thirteenth harmonic?

$K_p(n) = \cos n\, \theta_{sp} = 0$
$n\, \theta_{sp}/2 = 90°$, n= 5
$\theta_{sp/2} = 90°/5$
$= 18°$
$\theta_{sp} = 36°$

Kp(1) = Cos 18° = 0.95
Reduction in fundamental = 5%
Kp(13) = Cos 13 X 18° = -0.588
Reduction in 13^{th} harmonic = 1- 0.588 = 41.2 %

5.30 A 50 Hz, 8-pole, pole 3-phase synchronous generator has 48 slots. Calculate the % reduction in the fundamental, third and fifth harmonic strengths on account of distributed windings.

S = 48

SPP = 48/(3X8) = 2, γ = (18° X 8)/48 = 30°
$K_b(1)$ = (sin m r/2)/(m sin r/2) = sin (3 X 30/2)/(3sin 15°) = 0.707/0.7765 = 0.91

% reduction in fundamental = 100 X (1 – 0.91) = 9

$K_b(3)$ = (sin m (3r/2))/(msinr/2) = sin3 X (3/2 X 30°) / 3 sin ((3 X 30°)/2) = 0.707/2.121
= 0.333

% reduction in 3rd harmonic = 66.7%

$K_b(5)$ = (sin 3 X 5 X 15°) / 3 sin 5 X 15° = -0.707 / 2.898 = -0.245

% reduction in 5th harmonic = 75.5%

5.31 A synchronous generator has 12 poles and 3-phase winding placed in 144 slots; the coil span is 10 slots. Determine the distribution factor, pitch factor and winding factor.

S = 144, SDP = 144/(3 X 12) = 4 γ = (180° X 12) / 144 = 15°

K_p = sin(4 X 15/2) / (4 sin 15/2) = 0.5 / 0.522 = 0.958

Coil span = 10 X 15 = 150° elect
θ_{sp} = 30°
K_p = cos 30° = 0.866
$K_w = K_bK_p$ = 0.958 X 0.866 = 0.83

5.32 The phase voltage of a 50 Hz synchronous generator is 3.3 kV at a field current of 10 A. Determine the open-circuit voltage at 50 Hz with a field current of 8 A. Neglect saturation.

V = 4.44 Kw F Φ N
Or V α f X If ; Φ α I_f (field current)

3.3 α 60 X 8

Dividing (ii) by (i)
V / 3.3 = (60 X 8) / (50 X 10) = 3.168 Kv

5.33 A 50 Hz, 3-phase hydroelectric generator has a rated speed of 100 rpm. There are 540 stator slots with two conductors per slot. The air-gap dimensions are: D = 6.25 m, L = 1.16 m. The maximum flux density B_m = 1.2 T. Calculate the generated voltage/phase.

n = 100 rpm P = 120f/n = (120 X 50)/100 = 60
S = 540 N_c = 2
γ = 60 X 180° / 540 = 20°

m = 540 / (3 X 60) = 3

$K_p = \sin (3 \text{ X } 20^\circ/2)/3 \sin (20^\circ/2)$
$= 0.5 / 0.521 = 0.96$

Pole area = $(\pi \text{ X } 6.25) / 60 \text{ X } 1.16 = 0.38$

Flux pole = $2/\pi \text{ X } 1.2 \text{ X } 0.38 = 0.29$w
N_{ph}(series) = (540 X 2)/(2 X 3) = 180
V = 4.44 Kw f Φ N_{ph}(series)
= 4.44 X 0.96 X 50 X 0.29 X 180
= 11.125 kV

5.34 Calculate the voltage induced in the armature of a 4-pole lap-wound dc machine having 728 conductors and running at 1600 rpm. The flux/pole is 32 m Wb.
If this armature carries a current of 100 A, what is the electromagnetic power and torque developed?

$E_a = \Phi nZ / 60 \ (P/A)$

$E_a = [(32 \text{ X } 10^{-3} \text{ X } 1600 \text{ X } 728) / 60] \text{ X } 1$

= 621.2 V
Power developed = 621.2 X 100 / 1000 = 62.12 kw
Torque developed = $E_a I_a / \ \omega$
= (621.2 X 1000) / [(2 π X 1600)/60]
= 370.75 Nm

5.35 A 240 V dc motor takes 25 A when running at 945 rpm. The armature resistance is 0.24Ω. Determine the no-load speed assuming negligible losses. Flux/pole is constant.

E_a = 240 – 25 X 0.24 = 234
At no-load   Ia ≈ 0
Therefore E_a = V = 240
no / n = 240 / 234 X 945 = 969 rpm

5.36 A 4-pole dc motor has a lap-connected armature with 60 slots and 8 conductors/slot. The armature has an applied voltage of 240 V. It draws a current of 50 A when running at 960 rpm. The resistance of the armature is 0.1 Ω. Find the flux/pole that would be necessary for this operation.

E_a = 240 – 50 X 0.1 = 235 V
$E_a = (\Phi \text{ n } Z / 60)(P/A)$
240 = (Φ X 960 X 60 X 8) / 60 X 1
or Φ = 31.25 mwb

5.37 In a given machine F2 (rotor mmf) 850 AT and F1(stator mmf) 400 AT, α (included angle) = 123.6° and P (permeance/pole) 1.408 X 10^{-4} Wb/AT. Find the value of the resultant air-gap flux/pole.

Fr = $\sqrt{(400)^2 + (850)^2 - 2 \times 400 \times 850 \cos 56.40}$
= 711.5 AT
Φ = P Fr
= 1.408 X 10^{-4} X 711.5
= 0.1 wb

5.38 A P-pole machine has a sinusoidal field distribution as shown in Fig. P5.38. The armature carries a uniform current sheet of value JA /m causing a triangular mmf distribution as shown in the figure.

The machine as an axial length of l and a mean air-gap diameter of D.

(a) Find the peak value of the armature mmf.
(b) Derive an expression for the electromagnetic torque developed.

(a) F(peak) = J X D/2 X (2/P X π/2) = πJD / 2P

Current in elemental strip dθ

$$di = \pm J\,(D/P)\,d\theta$$

$$dT = D/2\,(B_p \sin\theta)\, l\, J(D/P)\, d\theta$$

$$= [(JB_pD^2 l)/ 2P] \sin\theta\, d\theta$$

$$T = JB_pD^2l/2P \left[\int_{\delta}^{\pi/2+\delta} \sin\theta\, d\theta - \int_{\pi/2+\delta}^{\pi+\delta} \sin\theta\, d\theta\right]$$

$$T = K\left[-\cos\theta\right]_{\delta}^{\pi/2+\delta} + \left[\cos\theta\right]_{\pi/2+\delta}^{(\pi+\delta)}$$

$$= K[-\cos(\pi/2+\delta) + \cos\delta + \cos(\pi+\delta) - \cos(\pi/2+\delta)]$$

$$= K[\sin\delta + \cos\delta - \cos\delta + \sin\delta]$$

$$= 2K \sin\delta\ ;\ K = JBpD^2l/2P$$

5.39 A 3-phase, 50 Hz, 4-pole, 400 V wound rotor induction motor has a stator winding Δ-connected and a rotor winding Y-connected. Assume effective turn ratio speed of 1440 rpm, calculate:
(a) the slip
(b) the standstill rotor induced emf/phase
(c) the rotor induced emf/phase at this speed
(d) the rotor frequency in (b) and (c)

(a) n_s = 1500rpm
slip = (1500 – 1440) / 1500 X 100 = 4%

(b) Per phase rotor voltage = 400 / √3 = 231 V

(standstill)

(c) Per phase rotor induced voltage (remaining) = 115.5 X 0.04
= 4.62V

(d) Stanstill rotor frequency = f = 50 Hz
Rotor frequency running = 50 X 0.04 = 2 Hz

5.40 A 50 Hz induction motor runs at 576 rpm at full load. Determine:
(a) the synchronous speed and the number of poles.
(b) the frequency of rotor currents
(c) the rotor speed relative to the revolving field

(a) n_s = 600 rpm , P = (120 X 50) / 600 = 10
(b) s = (600 – 576) / 600 X 100 = 4
F2 = 0.04 X 50 = 2 Hz

(c) Rotor speed relative to the revolving field = 6000 – 576 = 24 rpm

5.41 A 3-phase induction motor runs at a speed of 940 rpm at full-load when supplied with power at 50 Hz, 3-phase.

(a) How many poles does the motor have?
(b) What is its slip at full-load?
(c) What is the corresponding speed of :
(i) the rotor field wrt the rotor surface
(ii) the rotor field wrt the stator
(iii) what is the rotor speed at twice full-load slip?

(a) n_s = 1000 rpm, P = (120 X 50) / 1000 = 6
(b) s = (1000 – 940)/1000 X 100 = 6%
(c) (i) 1000 – 940 = 60 rpm
(ii) 960 + 40 = 1000 rpm
(iii) 2s = 12 % n = 1000 – (12 X 1000)/100 = 880 rpm

CHAPTER 6: ARMATURE WINDINGS

6.1 *Draw a single-layer unbifurcated winding for a 3-phase, 4-pole machine having 24 armature slots. Assume one coil-side. Clearly show the end connection if a continuous chain arrangement is used.*

Solution

pole pitch = 24/4 = *6* slots

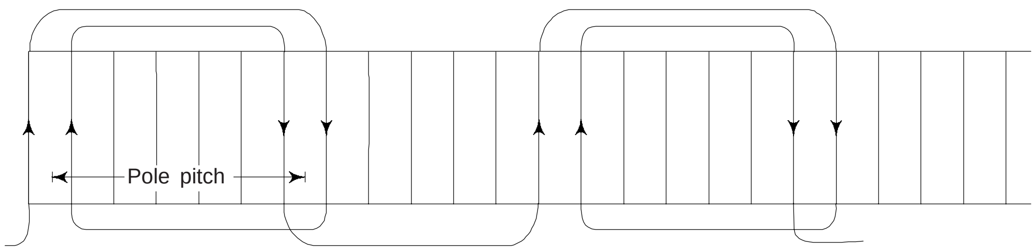


Fig. P6.1

6.2 *For the same number of slots and poles as in P6.1 draw a bifurcated winding. If the number of slots is changed from 24 to 36, is it possible to have bifurcated winding? If not, why; if yes how?*

Solution

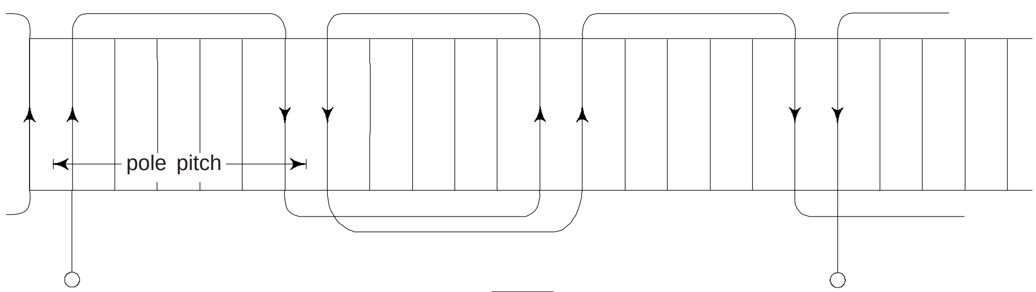


Fig. P6.2

$$m = \frac{24}{3 \times 4} = 2$$

$m = \dfrac{36}{3 \times 4}$ = 3, odd number does not permit bifurcation as can be seen from Fig. 6.3.

6.5 *The armature of a 3-phase machine with 16 poles and 180 slots is wound with fractional slot winding. Construct the winding table for one basic unit of poles. Indicate the start of each phase. For the basic unit determine the distribution of coil groups and phase sequence.*

Solution

$$\text{SPP} = \frac{180}{3 \times 6} = 3\frac{3}{4}$$

$$= \frac{16 \times 180^\circ}{180} = 16^\circ$$

$$\frac{S}{P} = \frac{180}{16} = \frac{45}{4} = \frac{S'}{P'}$$

P' = 4 poles $\quad S'$ = 45

Pole-pitch 1

Slot No.	1	2	3	4	5	6	7	8	9	10	11	12
Angle	0	16	32	48	64	80	96	112	128	144	160	176
Phase	[a]	a	a	a	c′	c′	c′	c′	b	b	b	b

Pole-pitch 2

Slot No.	13	14	15	16	17	18	19	20	21	22	23
Angle	12	28	44	60	76	92	108	124	140	156	172
Phase	a′	a′	a′	[c]	c	c	c	b′	b′	b′	b′

Pole-pitch 3

Slot No.	24	25	26	27	28	29	30	31	32	33	34
Angle	8	24	40	56	72	88	104	120	136	152	168
Phase	a	a	a	a	c′	c′	c′	[b]	b	b	b

Pole-pitch 4

Slot No.	35	36	37	38	39	40	41	42	43	44	45
Angle	4	20	36	52	68	84	100	116	132	148	164
Phase	a′	a′	a′	a′	c	c	c	c	b′	b′	b′

Phase grouping a(4, 3, 4, 4) = 15 coils
c(4, 4, 3, 4) = 15 coils
b(4, 4, 4, 3) = 15 coils

Phase sequence ABC

Winding layout for the basic unit is shown in Fig. 6.5.

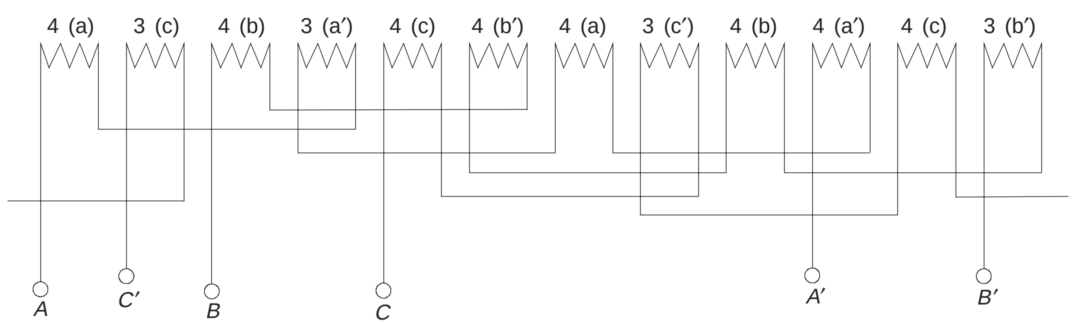


Fig. P6.5

6.6 *A 3-phase, 50 Hz, 10-pole machine has 72 slots. Construct the winding table for fractional slot winding. Draw the winding diagram with a coil span of seven slots.*

Solution

$$m = \frac{72}{3 \times 10} = 12/5 = 2\frac{2}{5} \ (\textit{fractional})$$

$$\frac{S}{P} = \frac{72}{10} = \frac{36}{5} = \frac{S'}{P'}$$

$P' = 5$ (basic unit) $\quad S' = 36$

$$\gamma = \frac{10 \times 180}{72} = 25°$$

Coil span = 7 slots

pole pitch 1

Slot No.	1	2	3	4	5	6	7	8
Angle	0	25	50	75	100	125	150	175
Phase	a	a	a	c′	c′	b	b	b
	[a]							

pole pitch 2

Slot No.	9	10	11	12	13	14	15
Angle	20	45	70	95	120	145	170
Phase	a′	a′	c	c	b′	b′	b′
					[b]		

pole pitch 3

Slot No.	16	17	18	19	20	21	22
Angle	15	40	65	90	115	140	165
phase	a	a	c′	c′	c′	b	b

pole pitch 4

Slot No	23	24	25	26	27	28	29
Angle	10	35	60	85	110	135	160
Phase	a′	a′	c	c	c	b′	b′
			[c]				

pole pitch 5

Slot No.	30	31	32	33	34	35	36
Angle	5	30	55	80	105	130	155
Phase	a	a	a	c′	c′	b	b

phase *a*	(3, 2, 2, 2, 3) = 12
phase *b*	(2, 2, 3, 3, 2) = 12
phase *b*	(3, 3, 2, 2, 2) = 12
phase sequence *abc*	coils 36

CHAPTER 7: DC MACHINES

7.1 *A compensated dc machine has 20,000 AT/pole. The ratio of pole arc to pole pitch is 0.8. The interpolar air-gap length and flux density are respectively 1.2 cm and 0.3 T. For rated I_a = 1,000 A, calculate the compensating winding AT per pole and number of turns on each interpole.*

Solution

$$\text{AT}_{cw}/\text{pole} = \text{AT}_a(\text{peak}) \times \left(\frac{\text{pole arc}}{\text{pole pitch}}\right)$$

$$= 20{,}000 \times 0.8 = 16{,}000$$

$$\text{AT}_i = \text{AT}_a(\text{peak}) + \frac{B_i}{\mu_0} l_{gi}$$

$$\text{AT}_a(\text{peak})_{\text{interpolar region}} = 20{,}000 - 16{,}000 = 4{,}000$$

$$= 4{,}000 + \frac{0.3}{4\pi \times 10^{-7}} \times 12 \times 10^{-2}$$

$$= 6{,}865$$

$$N_i = \frac{6{,}865}{1000} = 7 \text{ (say)}$$

7.2 *The no-load saturation curve for a generator operating at 1,800 rpm is given by the following data:*

E_g	8	40	74	113	152	213	234	248	266	278
I_f	0	0.5	1.0	1.5	2.0	3.0	3.5	4.0	5.0	6.0

(a) *Plot the no-load saturation curve for 1,500 rpm.*

(b) *Calculate the generated voltage, when the generator is operating on no-load with a field current of 4.6 A and at a speed of 1,000 rpm.*

(c) *What is the field current required to generate 120 V on no-load, when the generator is operating at 900 rpm?*

(d) *This machine is operated as a shunt generator at 1,800 rpm with a field current of 4.6 A. What is the no-load voltage, when the generator is operating at 1,500 rpm?*

Solution

(a) OCC is drawn in Fig. P7.2.

(b) From OCC

n_1 = 1,800 rpm; I_f = 4.6 A; $E_a \approx V_{oc}$ = 260 V (no-load)

$$\therefore \quad E_a(1{,}000 \text{ rpm}) \ \frac{260 \times 1{,}000}{1{,}800} = 144.4 \text{ V}$$

(c) V_{OC}(900 rpm) = 120 V

V_{OC}(1,800 rpm) = 240 V

From OCC(1,800 rpm), I_f = 3.7 A

(d) V_{OC}(1,800 rpm) = 260 V at I_f = 4.6 A

$$R_f = \frac{260}{4.6} = 56.5 \ \Omega$$

56.5 Ω line is drawn in Fig. 7.2 and OCC is translated to 1,500 rpm.
From the intersection of these

$$V_{OC} \ (1{,}500 \text{ rpm}, \quad R_f = 56.5 \ \Omega) = 195 \text{ V}$$

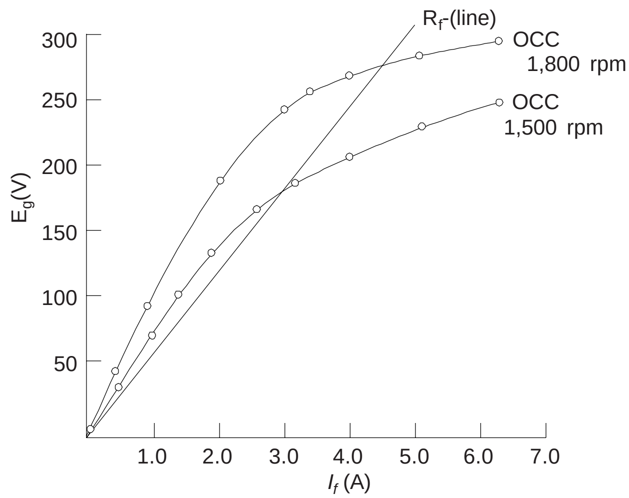


Fig. P7.2

7.3 *The accompanying data are given for the saturation curve of an 80 kW, 220 V, 1,200 rpm shunt generator, the data being for 1,200 rpm:*

I_f*(A)*	*0*	*0.4*	*0.8*	*1.2*	*1.6*	*2.0*	*2.5*	*3.2*	*4.0*	*4.5*	*5.0*	*5.5*
E_g *(V)*	*10*	*38*	*66*	*96*	*128*	*157*	*188*	*222*	*248*	*259*	*267*	*275*

(a) The shunt field resistance is adjusted to 50 Ω and the terminal voltage is found to be 250 V, at a certain load at 1,200 rpm. Find the load supplied by the generator and the induced emf. Assume that the flux is reduced by 4% due to armature reaction. Armature resistance is 0.1 Ω.

(b) For the same field resistance and an armature current of 250 A obtain the values of E_g, V_t *and* I_f.

Solution

(a) $I_f = \dfrac{25T}{50} = 5$ A

Corresponding $E_g = 267$ V (read from OCC)

Accounting for the effect of armature reaction

$$E_g = 0.96 \times 267 = 256.3 \text{ V}$$

$$\therefore \qquad I_a = \frac{E_g - V}{R_a} = \frac{256.3 - 250}{0.1} = 63 \text{ A}$$

$$I_L = I_a - I_f = 63 - 5 = 58 \text{ A}$$

$$\text{Load} = \frac{250 \times 58}{1{,}000} = 14.5 \text{ kW}$$

(b) $I_a = 250$ A, $\quad\therefore\quad I_aR_a = 25$ V

OCC characteristic with 4% reduction caused by armature reaction is drawn dotted in Fig. 7.2. Drawing a line parallel to the R_f-line and 25 V above it, we read the values of terminal voltage as (Points P', Q')

$V = 224$ V, 50 V

$E_g = 249$ V, 75 V

$I_f = 4.48$ A, 1 A (read corresponding to points P', Q' or $I_f = V_t/50$)

7.4 *Find the resistance of the load which takes a power of 5 kW from a shunt generator whose external characteristic is given by the equation $V = (250 - 0.5I_L)$.*

Solution

$$VI_L = 5{,}000 \text{ W}$$

$$(250 - 0.5I_L)\, I_L = 5{,}000$$

$$0.5I_L^2 - 250I_L + 5{,}000 = 0$$

Solving the quadratic

$$I_L = 20.87 \text{ A}$$

$$\therefore \quad \text{Load Resistance} = \frac{V}{I_L} = \frac{250}{20.87} = 1198\ \Omega$$

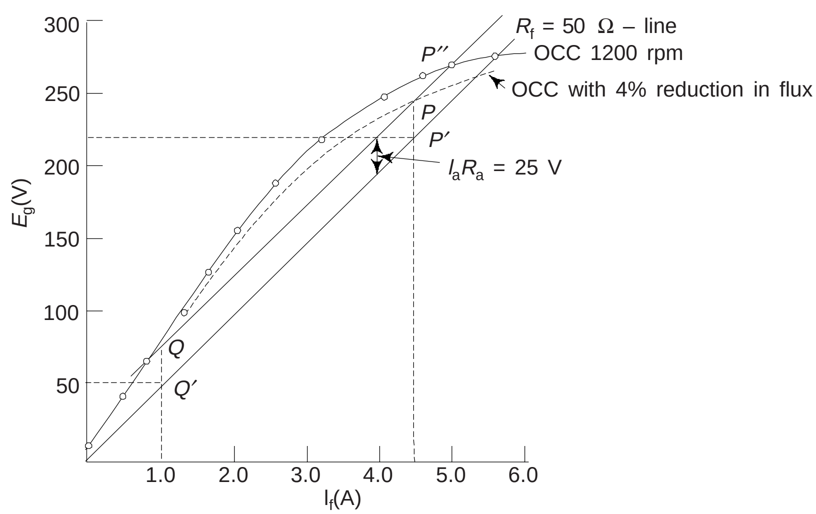


Fig. P7.4

7.6 *In a 110 V compound generator, the armature, shunt and series windings are 0.06 Ω, 25 Ω and 0.04 Ω respectively. The load consists of 200 lamps each rated at 55 W, 110 V. Find the total emf and armature current, when the machine is connected for (a) long shunt (b) short shunt. How will the ampere-turns of the series windings be changed, if in (a), a diverter of resistance 0.1 Ω is connected across the series field. Ignore the armature reaction and brush voltage drop.*

Solution

$$I_L = \frac{200 \times 55}{110} = 100 \text{ A}$$

(a) Long shunt: $\qquad I_f = \dfrac{110}{25} = 4.4 \text{ A}$

$$I_a = I_L + I_f = 100 + 4.4 = 104.4 \text{ A}$$

$$E_a = V + I_a (R_a + R_{se})$$

$$= 110 + 104.4\,(0.06 + 0.04)$$

$$= 120.4 \text{ V}$$

(b) Short shunt:

$$V_a = 110 + I_L R_{se} = 110 + 100 \times 0.04 = 114 \text{ V}$$

$$I_f = \frac{V}{R_f} = \frac{114}{25} = 4.56 \text{ A}$$

$$I_a = I_L + I_f = 100 + 4.56 = 104.\ 56 \text{ A}$$

$$E_a = V_a + I_a R_a = 114 + 104.56 \times 0.06 = 120.3 \text{ V}$$

(c) Now with diverter

$$I_{se}^{d} = 104.4 \times \frac{0.1}{0.14} = 74.57 \text{ A}$$

$$I_{se} = \text{(original)} = I_a = 104.4 \text{ A}$$

Series field AT reduces to

$$= \frac{74.57}{104.4} \times 100 = 71.4\%$$

7.7 *A dc shunt generator has the following open-circuit characteristic when separately excited:*

Field current, A	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*	*1.4*	*2.0*
EMF, V	*80*	*135*	*178*	*198*	*210*	*228*	*246*

The shunt winding has 1,000 turns per pole and a total resistance of 240 Ω.. Find the turns per pole of a series winding that will be needed to make the terminal voltage the same at 50 A output as on no-load. The resistance of the armature winding, including the series compounding winding, can be assumed to be 0.36 Ω and constant. Ignore armature reaction.

Solution

Assume long-shunt compound connections. The OCC is drawn in Fig. P7.7 and its intersection with R_f = 240 Ω line, gives a no-load voltage of 200 V. I_f = 0.85 A.

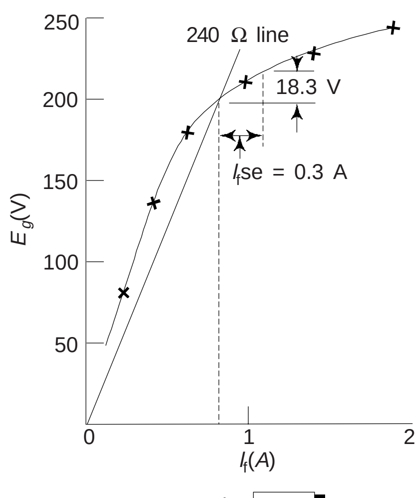


Fig. P7.7

$$I_a = 50 + 0.85 = 50.85 \text{ A}$$

Armature circuit voltage drop = 50.85 × 0.36 = 18.3 V.
From the figure:

$$I_f^{se} = 0.3 \text{ A}$$

$$0.3 \times 1{,}000 = 50.85 \times N_{se}$$

or $\qquad N_{se} = 5.9 \quad \text{or} \quad 6 \text{ turns}$

7.8 *A dc compound generator has a shunt field winding of 3,600 turns per pole and a series field winding of 20 turns per pole. Its open-circuit magnetization characteristic when it is separately excited by its shunt field winding and driven at its no-load rated speed is given below:*

AT/pole	*3,120*	*4,680*	*6,240*	*7,800*	*9,360*
EMF,V	*289*	*361*	*410*	*446*	*475*

The full-load armature current is 100 A and the ohmic drop in the armature circuit for this current is 20 V including brush drop. At no-load the ohmic drop may be ignored, and the terminal voltage is 415 V. The fall in speed from no-load to full-load is 8%; the shunt field circuit is connected across the output terminals of the machine, and its resistance is kept constant.

Determine the terminal voltage and power output for the full-load armature current of 100 A. Neglect the effects of armature reaction.

Solution

The OCC at rated speed is drawn in Fig. P7.8. The generator is connected long shunt. The OCC at 0.92 rated speed is also drawn. The R_f-line is drawn to give 415 V at no-load.

$$R_f = \frac{415}{6,400/3,600} = 233.4\ \Omega$$

$$\text{AT}_{\text{series}} = 100 \times 20 = 2{,}000$$

$$\frac{V}{R_f} \times 3{,}600 + 2{,}000 \quad \Rightarrow \quad (V + 20)$$

The terminal voltage is read corresponding to point Q in the R_f-line

$$V = 403 \text{ V}$$

$$I_f = \frac{403}{233.4} = 1.73 \text{ A}$$

$$I_L = 100 - 1.73 = 98.27 \text{ A}$$

$$\text{Power output} = 403 \times 98.27 = 39.6 \text{ kW}$$

7.9 *A 250 kW, 6 pole, dc compound generator is required to give 500 V on no-load and 550 V on full-load.*

The armature is lap-connected and has 1,080 conductors; the total resistance of the armature circuit is 0.037 Ω.

The open-circuit characteristic for the machine at rated speed is given by:

Armature voltage (V)	*500*	*535*	*560*	*580*
Field ampere-turns/pole	*6,000*	*7,000*	*8,000*	*9,000*

The field ampere-turns per pole to compensate for armature reaction are 10% of the armature ampere-turns per pole.

The shunt field winding is connected across the output terminals and has a resistance of 85 Ω. Determine the required number of series turns per pole.

Solution

At no-load (500 V): $\quad \text{AT}_f(\text{no-load}) = 6{,}000$

$$I_f(\text{no-load}) = \frac{500}{85} = 5.88 \text{ A}$$

$$\therefore \quad N_f = \frac{6,000}{5.88} = 1{,}020$$

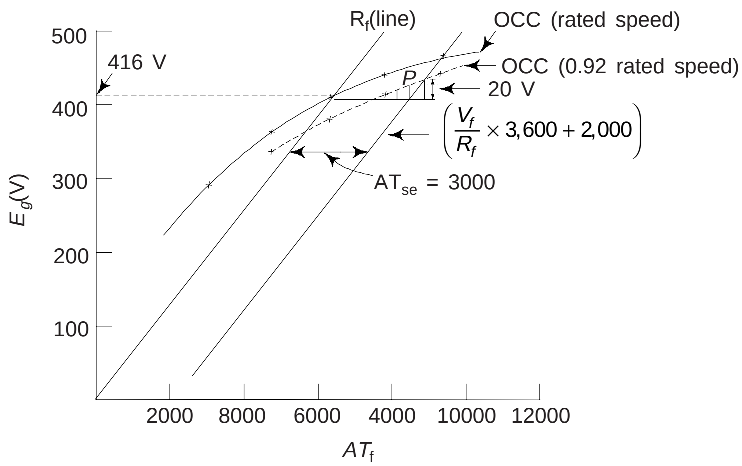


Fig. P7.9(a)

At full-load (550 V):

$$I_f\,(\text{fl}) = \frac{550}{85} = 6.47 \text{ A}$$

$$AT_f\,(\text{fl}) = 6.47 \times 1,020 = 6,600$$

$$I_L = \frac{250 \times 1,000}{550} = 454.5 \text{ A}$$

$$I_a = I_L + I_f = 454.5 + 6.5 = 461 \text{ A}$$

Armature circuit voltage drop = 461 × 0.037 = 17 V
Induced emf (fl) = 550 + 17 = 567 V
From Fig. 7.9(b)

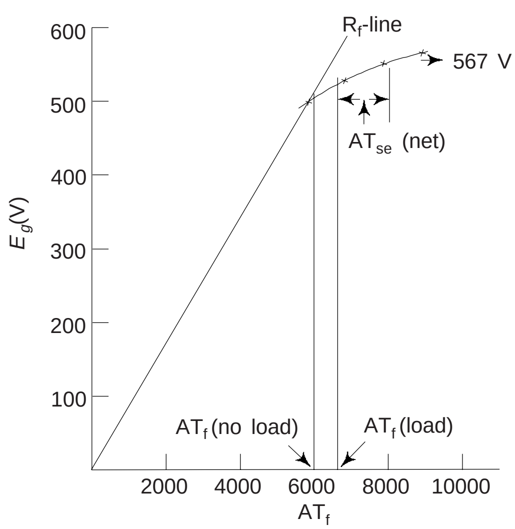


Fig. P7.9(b)

Corresponding to this induced emf, $AT_{se}(\text{net}) = 1{,}660$

$$AT_a = \frac{454.5}{6} \times \frac{1{,}080}{2 \times 6} = 6{,}817.5$$

Armature demagnetizing ampere-terms, $AT_d = 6{,}817.5 \times 0.1 = 682$
Hence

$$AT_{se} = 1{,}660 + 682 = 2{,}342$$

$$N_{se} = \frac{2{,}342}{461} = 5.08 \text{ or } 5$$

7.10 *A 10 kW, 250 V shunt motor has an armature resistance of 0.5 Ω and a field resistance of 200 Ω. At no-load, and rated voltage, the speed is 1,200 rpm and the armature current is 3 A. At full-load and rated voltage, the line current is 47 A and because of armature reaction, the flux is 4% less than its no-load value.*
(a) What is the full-load speed?
(b) What is the developed torque at full-load?

Solution

At no-load

$$I_f = \frac{250}{200} = 1.25 \text{ A}$$

$$I_{a0} = 3 \text{ A}$$

$$E_{a0} = 250 - 3 \times 0.5 = 248.5 \text{ V}$$

$$T_0 = \frac{248.5 \times 3}{(2\pi \times 1{,}200)/60} = 5.93 \text{ Nm}$$

At full-load

$$\Phi_{fl} = 0.96\ \Phi_0; \qquad I_{a,\,fl} = 47 - 1.25 = 45.75$$

$$E_{a,fl} = 250 - 45.75 \times 0.5 = 227.1 \text{ V}$$

(a) $n_{fl} = 1{,}200 \times \left(\frac{227.1}{248.5}\right) \times \left(\frac{1}{0.96}\right) = 1{,}142$ rpm

(b) $T_{fl} = 5.93 \times \left(\frac{0.96}{1}\right) \times \left(\frac{45.75}{3}\right) = 86.8$ Nm

7.12 *A 200-V shunt motor has $R_a = 0.1$ Ω, $R_f = 240$ Ω and rotational loss = 236 W. On full load the line current is 9.8 A with the motor running at 1450 rpm. Determine*
(a) the mechanical power developed
(b) the power output
(c) the load torque
(d) the full-load efficiency.

Solution

$$I_f = \frac{200}{240} = 0.833 \text{ A}$$

$$I_a = 9.8 - 0.833 = 8.97 \text{ A}$$

$$E_a = 200 - 8.97 \times 0.1 = 199.1 \text{ V}$$

(a) P_{mech} developed $= 199.1 \times 8.97 = 1.786$ kW

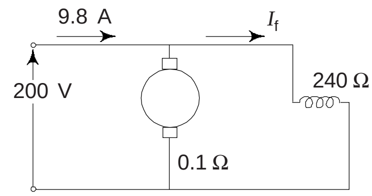


Fig. P7.12

(b) $P_{out} = 1.786 - 0.236 = 1.55$ kW

(c) $n = 1450$ rpm $\qquad \omega = \dfrac{2\pi \times 1450}{60} = 96.67$ rad/s

$$\text{Load torque} = \frac{1.55}{96.67} = 16.03 \text{ Nm}$$

(d) Input $= 200 \times 9.8 = 1.96$ kW

η (fl) $= 1.55/1.96 = 79.1\%$

7.13 *A 220 V unsaturated shunt motor has an armature resistance (including brushes and interpoles) of 0.04 Ω and a field resistance of 100 Ω. (a) Find the value of resistance to be added to the field circuit to increase the speed from 1,200 to 1,600 rpm, when the supply current is 200 A; (b) with the field resistance as in (a), find the speed when the supply current is 120 A. If the machine is run as a generator to give 200 A at 220 V, find (c) the field current at 1,300 rpm, and (d) the speed when the field current is 2 A.*

Solution

(a) $I_{fl} = \dfrac{220}{100} = 2.2$ A

$E_{al} = 220 - (200 - 2.2) \times 0.04 = 212.1$

$E_{a2} = 220 - (200 - I_{f2}) \times 0.04 = 212 + 0.04 I_{f2}$

$E_{al} \propto n_1 I_{f1}$

$E_{a2} \propto n_2 I_{f2}$

$212.1 \propto 1{,}200 \times 2.2$ (i)

$212 + 0.04 I_{f2} \propto 1600 \times I_{f2}$ (ii)

Dividing Eq. (ii) by (i)

$$\frac{212 + 0.04 I_{f2}}{212.1} = \frac{1{,}600}{1{,}200 \times 2.2} I_{f2}$$

$$I_{f2} = 1.65 \text{ A}$$

$$R_{f2} = 133.3 \ \Omega$$

$$R_f(\text{ext}) = 133.3 - 100 = 33.3 \ \Omega$$

Note No difference will be made by assuming $I_a \approx I_L$

(b) $E_{a3} = 220 - (120 - 1.65) \times 0.04 = 215.3$ V

$$215.3 \propto n_3 \times 1.65 \qquad \text{(iii)}$$

Dividing Eq. (iii) by (i)

$$\frac{215.3}{212.1} = \frac{n_3}{1{,}200} \times \frac{1.65}{2.2}$$

or $\qquad n_3 = 1{,}624$ rpm

(c) Generator

$$I_a = 200 + I_{fg}$$

$$E_{eg} = 220 + (200 + I_{fg}) \times 0.04 \propto 1{,}300 \times I_{fg} \qquad \text{(iv)}$$

Dividing Eq. (iv) by (i)

$$\frac{220 + (200 + I_{fg}) \times 0.04}{212.1} = \frac{1{,}300}{1{,}200} \times \frac{I_{fg}}{2.2}$$

$$1.037 + 0.038 + 1.89 \times 10^{-4} I_{fg} = 0.492 I_{gf}$$

or $$I_{fg} = 2.185 \text{ A}$$

(d) $I_f = 2$ A

$$I_a = 200 + 2 = 202$$

$$E_a = 220 + 202 \times 0.04 = 228.08 \propto n_g \times 2 \qquad \text{(v)}$$

Dividing Eq. (v) by (i)

$$\frac{228.08}{212.1} = \frac{n_g}{1{,}200} \times \frac{2}{2.2}$$

$$n_g = 1{,}419.3 \text{ rpm}$$

7.14 *A 4-pole series motor has 944 wave-connected armature conductors. At a certain load the flux per pole is 34.6 mWb and the total mechanical power developed is 4 kW. Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 500 V. Total motor resistance is 3 Ω.*

Solution

$$E_a = \frac{\Phi n z}{60}\left(\frac{P}{A}\right) \qquad \text{(i)}$$

$$E_a I_a = 4 \times 10^3 \qquad \text{(ii)}$$

$$I_a = \frac{V - E_a}{R_a} \qquad \text{(iii)}$$

The unknowns are n, E_a and I_a. Substituting values in (iii)

$$I_a = \frac{500 - E_a}{3} \qquad \text{(iv)}$$

Substituting in (ii)

$$E_a = \left(\frac{500 - E_a}{3}\right) = 4 \times 10^3$$

$$E_a^2 - 500 E_a + 12 \times 10^3 = 0$$

$$E_a = \frac{500 \pm \sqrt{25 \times 10^4 - 4.8 \times 10^4}}{2}$$

$$= 474.7 \text{ V},\ 25.3 \text{ (rejected, } \eta \text{ will be too low)}$$

$$I_a = \frac{500 - 474.4}{3} = 8.43 \text{ A}$$

Substituting values in (i)

$$474.7 = \frac{34.6 \times 10^{-3} \times n \times 944}{60} \times \left(\frac{4}{2}\right)$$

$\therefore$ $$n = 436 \text{ rpm}$$

7.15 *The following data pertain to a 250 V dc series motor;*

$$Z = 180, \quad \frac{P}{A} = 1$$

Flux/pole = 3.75 mWb/field amp

Total armature circuit resistance = 1 Ω

The motor is coupled to a centrifugal pump whose load torque is

$$T_L = 10^{-4}\, n^2 Nm$$

where n = speed in rpm.

Calculate the current drawn by the motor and the speed at which it will run.

Solution

$$E_a = \frac{3.75\times 10^{-3}\, I_a \times n \times 180}{60} \times 1 = 11.25 \times 10^{-3}\, nI_a \quad \text{(i)}$$

$$T = \frac{1}{2\pi} \times 3.75 \times 10^{-3} I_a \times 180 \times I_a = 107.4 \times 10^{-3}\, I_a^2 \quad \text{(ii)}$$

$$\frac{250 - E_a}{1} = I_a \quad \text{(iii)}$$

Under steady conditions

$$T = T_L$$

$$107.4 \times 10^{-3}\, I_a^2 = 10^{-4}\, n^2$$

or
$$I_a = \frac{n}{32.77} \quad \text{(iv)}$$

Substituting (iii) and (iv) in (i)

$$250 - \frac{n}{32.77} = 11.25 \times 10^{-3}\, \frac{n^2}{32.77}$$

$$n^2 + \frac{10^3}{1125}\, n - \frac{250 \times 32.77 \times 1{,}000}{11.25} = 0$$

$$n^2 + 88.9n - 72.8 \times 10^4 = 0$$

$$n = \frac{-89 \pm \sqrt{0.79 \times 10^4 + 291.2 \times 10^4}}{2}$$

$$= 810 \text{ rpm}$$

$$I_a = \frac{810}{32.77} = 24.7 \text{ A}$$

7.16 *A dc shunt motor is being operated from 300 V mains. Its no-load speed is 1,200 rpm. When fully loaded, it delivers a torque of 400 Nm and its speed drops to 1,100 rpm. Find its speed and power output when delivering the same torque if operated with an armature voltage of 600 V. Excitation is assumed unchanged, i.e., the motor field is still excited at 300 V. State any assumption you are required to make.*

Solution

At no-load the $I_a R_a$ drop can be neglected

$$300 = \frac{\Phi \times 1{,}200 \times Z}{60}\left(\frac{P}{A}\right) \times K_N \times 1200 \quad \text{(i)}$$

$$400 = \frac{1}{2\pi} = \Phi_Z I_a \left(\frac{P}{A}\right) = K_T \times \Phi \quad \text{(ii)}$$

Dividing Eq. (ii) by (i)

$$\frac{400}{300} = \frac{1}{2}\,\frac{60}{1,200}\,I_a$$

or $$I_a = 167.6 \text{ A}$$

Since speed drops to 1,100 rpm, on-load

$$E_a = 300 \times \frac{1,100}{1,200} = 275 \text{ V}$$

$$R_a = \frac{300 - 275}{167.6} = 0.15 \ \Omega$$

Under new operating conditions (600 V):

For same torque with no change in ϕ,

$$I_a = 167.6 \text{ A}$$

$$E_a = 600 - 167.6 \times 0.15 = 574.9 \text{ V}$$

$$\text{Power output} = E_a I_a = 574.9 \times 167.6 \times 10^{-3} = 96.35 \text{ kW}$$

From Eq. (i)

$$K_N = 0.25 \ (\Phi \text{ remaining fixed})$$

$$574.9 = 0.25 \times n$$

$$n = \frac{574.9}{0.25} = 2,300 \text{ rpm}$$

Another method for speed

$$\frac{96.35 \times 10^3}{(2\pi \times n)/60} = 400$$

or $$n = 2,300 \text{ rpm}$$

7.17 *A 50 kW, 230 V dc shunt motor has an armature resistance of 0.1 Ω and a field resistance of 200 Ω. It runs on no-load at a speed of 1,400 rpm, drawing a current of 10 A from the mains. When delivering a certain load, the motor draws a current of 200 A from the mains. Find the speed at which it will run at this load and the torque developed. Assume that the armature reaction causes a reduction in flux/pole of 4% of its no-load value.*

Solution

Field resistance is not changed, so that the field current remains constant. But there is a change of flux/pole due to armature reaction:

$$(230 - (10 - 1.15) \times 0.1) \propto 1,400 \ \Phi_1 \qquad I_f = \frac{230}{200} = 1.15 \text{ A}$$

$$(230 - (200 - 1.15) \times 0.1) \propto n_2 \Phi_2$$

Dividing,

$$\frac{210.1}{229.1} = \frac{n_2}{1,400} \times 0.96$$

or $$n = 1.337 \text{ rpm}$$

$$\text{Torque developed} = \frac{210.1 \times (200 - 1.15)}{(2\pi \times 1,337)/60} = 298.4 \text{ Nm}$$

7.18 *A 250 V dc series motor has the following OCC at 1,200 rpm:*

$I_f(A)$	5	10	15	20	25	30
$V_{OC}(V)$	100	175	220	240	260	275

R_a = 0.3 Ω and series field resistance is 0.3 Ω.

Find the speed of the machine when (a) I_a = 25 A (b) the developed torque is 40 Nm.

Solution

(a) $I_f = I_a = 25$ A

$$E_a\ (1{,}200\ \text{rpm}) = 260\ \text{V}$$

$$E_a(\text{actual}) = 50 - 25 \times 0.6 = 235\ \text{V}$$

$$n = \frac{1{,}200}{260} \times 235 = 1{,}085\ \text{rpm}$$

(b) $T = \dfrac{E_a I_a}{\omega_m}$

$$\omega_m = \frac{1{,}200 \times 2\pi}{60} = 40\pi\ \text{rad/s} = 125.66371\ \text{rad/s}$$

$I_f = I_a$	5	10	15	20	25	30
$E_a = V_{OC}$	100	175	220	240	260	275
T	3.98	13.92	26.26	38.20	51.73	65.65

From Fig. 7.6, at T = 40 Nm,

$$I_a = 20.7\ \text{A}$$

$$E_a I_a = \left(\frac{2\pi\, n}{60}\right) T$$

$$(250 - 0.6 \times 20.7) \times 20.7 = \frac{2\pi\, n}{60} \times 40$$

or $$n = 1{,}174\ \text{rpm}$$

7.19 *A 15 kW, 250 V, 1,200 rpm shunt motor has 4 poles, 4 parallel armature paths, and 900 armature conductors; R_a = 0.2 Ω. At rated speed and rated output the armature current is 75 A and I_f = 1.5 A. Calculate: (a) the flux/pole, (b) the torque developed, (c) rotational losses, (d) η(e) the shaft load and (f) if the shaft load remains fixed, but the field flux is reduced to 70% of its value by field control, determine the new operating speed.*

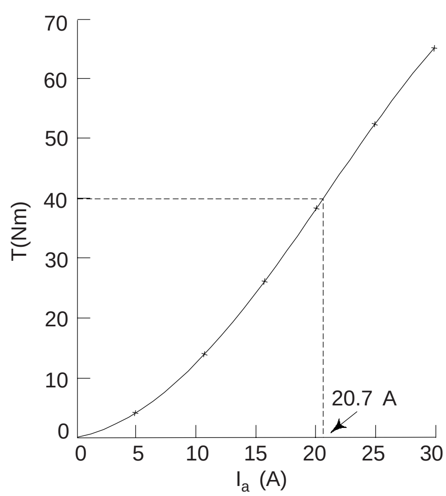


Fig. P7.19

Solution

(a) $E_a = \dfrac{\Phi\, nZ}{60}\left(\dfrac{P}{A}\right)$

$$250 - 75 \times 0.2 = \frac{\Phi \times 1{,}200\ 900}{60} \times \left(\frac{4}{4}\right)$$

or $$\Phi = 0.013\ \text{Wb}$$

(b) $T = \dfrac{1}{2\pi}\ \Phi Z I_a \left(\dfrac{P}{A}\right)$

$$= \frac{1}{2\pi} \times 0.013 \times 900 \times 75 \times 1 = 139.7\ \text{Nm}$$

(c), (d) and (e) Input = 250 × (75 + 1.5) = 19.125 kW

$$\text{Mechanical power developed} = 139.7 \times \frac{2\pi \times 1{,}200}{60} = 17.56 \text{ kW}$$

$$\text{Net mechanical output} = 15 \text{ kW}$$

$$\text{Rotational loss} = 17.56 - 15 = 2.56 \text{ kW}$$

$$\eta = \frac{15}{19.125} = 78.4\%$$

(f) $\Phi = 0.7 \times 0.01 = 0.007$ Wb

The torque load remains the same

$$\therefore \quad I_a = 75 \times \frac{0.01}{0.007} = 107.14 \text{ A}$$

$$E_a = 250 - 107.14 \times 0.2 = 228.57 \text{ V}$$

$$228.57 = \frac{0.007 \times n \times 900}{60} \times \left(\frac{4}{4}\right)$$

or $\quad n = 2{,}177$ rpm

7.20 *A 115 kW, 600 V dc series-wound railway track motor has a combined field and armature resistance (including brushes) of 0.155 Ω. The full-load current at rated voltage and speed is 216 A. The magnetization curve at 500 rpm is as follows:*

emf(V)	*375*	*400*	*425*	*450*	*475*
I_f*(A)*	*188*	*216*	*250*	*290*	*333*

(a) Neglecting armature reaction, calculate the speed in rpm at the rated current and voltage.
(b) Calculate the full-load internal (developed) torque.
(c) If the starting current is to be restricted to 290 A, calculate the external resistance to be added and the starting torque.

Solution

$$I_a = 216 \text{ A} = I_f$$

$$E_a = 600 - 216 \times 0.155 = 566.5 \text{ V}$$

(a) $E_a(500 \text{ rpm}) = 400$ V at $I_f = 216$ A

$$n = \frac{500}{400} \times 566.5 = 708.1 \text{ rpm}$$

(b) $$T = \frac{E_a I_a}{\omega_m} = \frac{566.6 \times 216}{(2\pi \times 708.1)/60} = 1650 \text{ Nm}$$

(c) $$R_a + R_{ext} = \frac{600}{290} = 2.07\ \Omega$$

$$\therefore \quad R_{ext} = 1.91\ \Omega$$

$$E_a = k_a \Phi \omega_m$$

$$\therefore \quad k_a \Phi = \frac{E_a}{\omega_m} = \frac{E_a}{(2\pi \times 500)/60}$$

At $I_f = I_a = 290$ A, $\quad E_a = 450$ V at 500 rpm

$$k_a \Phi = \frac{450}{(2\pi \times 500)/60} = 8.59 \text{ at } I_f = 290 \text{ A}$$

$$T = k_a \ \Phi I_a$$
$$= 8.59 \times 290 = 2{,}492 \text{ Nm}$$

7.21 *A 100 kW, 600 V, 600 rpm dc series-wound railway motor has a combined field and armature resistance (including brushes) of 0.155 Ω. The full-load current at rated voltage and speed is 206 A. The magnetization curve at 400 rpm is as follows:*

I_f(V)	*188*	*206*	*216*	*250*	*290*	*333*
emf (V)	*375*	*390*	*400*	*425*	*450*	*475*

(a) Determine the armature reaction in equivalent demagnetizing field current at 206 A.
(b) Calculate the internal (developed) torque at the full-load current.
(c) Assuming demagnetizing armature reaction mmf proportionate to I_a^2, determine the internal starting torque at the starting current of 350 A.

Solution

(a) $E_a = 600 - 206 \times 0.155 = 568.07$ at 600 rpm
or $= 378.71$ at 400 rpm
From the magnetizing curve of Fig. 7.7, the corresponding $I_f = I_a = 193$ A

Actual $I_f = I_a = 206$ A

Demagnetizing effect of armature reaction = 206 – 193 = 13 A of field current

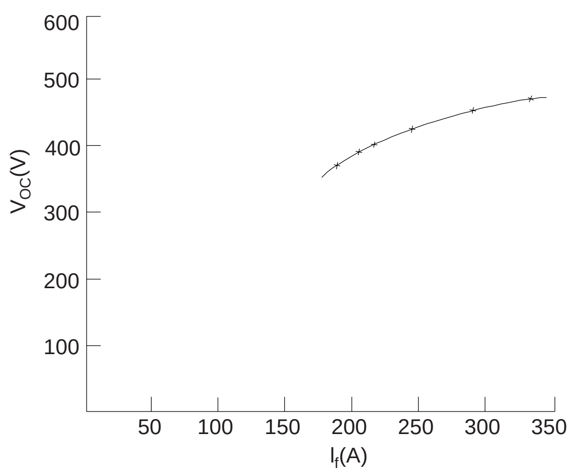


Fig. P7.21

(b) $T = \dfrac{E_a I_a}{\omega_m} = \dfrac{568.07 \times 206}{(2\pi \times 600)/60} = 1{,}862$ Nm

(c) I_a(start) = 350 A

Demagnetizing effect of armature reaction $= 13 \times \left(\dfrac{350}{200}\right)^2 = 37.5$ A

Effective $I_f = 350 - 37.5 = 312.5$ A
If $I_f = 312.5$ A, $\quad E_a = 46.2$ V at 400 rpm
$E_a = k_a \ \phi\omega_m$

or $$k_a\ \Phi = \frac{462}{(2\pi \times 400)/60} = 11.1 \text{ at } I_f = 312.5 \text{ A}$$

$\therefore$ $$T_s = k_a \Phi I_a \text{ (start)} = 11.1 \times 350 = 3{,}885 \text{ Nm}$$

7.22 *A 3kW series motor runs normally at 800 rpm on a 240 V supply, taking 16 A; the field coils are all connected in series. Estimate the speed and the current taken by the motor, if the coils are reconnected in two parallel groups of two in series. The load torque increases as the square of the speed. Assume that the flux is directly proportional to the current and ignore losses.*

Solution

$I_a(R_a + R_{se})$ voltage is assumed to be negligible. Hence

$$V \approx E_a = K_e n I_a \qquad (\phi \propto I_a) \tag{i}$$

$$T = K_T\, I_a^2 = k_L n^2 \tag{ii}$$

All coils in series:

$$240 = K_e \times 800 \times 16 \tag{iii}$$

$$K_T \times (16)^2 = K_L \times (800)^2. \qquad \text{or } 16\ \sqrt{K_T} = 800\ \sqrt{K_L} \tag{iv}$$

Two parallel groups of two in series

$$\text{Coil current} = \frac{I_a}{2}$$

$$240 = K_e \times n \times \frac{I_a}{2} \tag{v}$$

$$K_T I_a \times \frac{I_a}{2} = K_L n^2$$

or $$\sqrt{K_T}\, I_a = \sqrt{2}\, \sqrt{K_L}\ n \tag{vi}$$

From (iii) and (v) we get

$$nI_a = 32 \times 800 \tag{vii}$$

From (iv) and (vi) we get

$$\frac{I_a}{16} = \frac{\sqrt{2}n}{800} \tag{viii}$$

From (vii) and (viii) we get

$$n = 951 \text{ rpm}$$

$$I_a = 26.9 \text{ A}$$

7.23 *A 20 kW, 500 V shunt motor has an efficiency of 90% at full-load. The armature copper loss is 40% of the full-load loss. The field resistance is 250 Ω. Calculate the resistance values of a 4-section starter suitable for this motor in the following two cases:*

Case 1: *Starting current* $\le 2I_{f1}$

Case 2: *Starting current (min) = 120%* I_{f1}

Solution

$$P_{out} = 20 \text{ kW}$$

$$P_{in} = \frac{20}{0.9} = 22.222 \text{ kW}$$

$$P_{\text{Loss}}(\text{fl}) = 2{,}222 \text{ W}$$
$$P_{\text{c}}(\text{fl}) = 2{,}222 \times 0.4 = 888 \text{ W}$$
$$I_{\text{L}} = \frac{20 \times 1{,}000}{500} = 40 \text{ A}$$
$$I_{\text{f}} = \frac{500}{250} = 2 \text{ A}$$
$$I_{\text{a}}(\text{fl}) = 40 - 2 = 38 \text{ A}$$

$\therefore$
$$R_{\text{a}} = \frac{888}{(38)^2} = 0.615 \ \Omega$$

Case 1

$$I_{\text{a}} = 2I_{\text{a}}(\text{fl}) = 76 \text{ A}$$
$$R_1 = \frac{V}{I_1} = 6.58 \ \Omega$$
$$k = 4 + 1 = 5$$
$$\gamma = \left(\frac{R_1}{R_{\text{a}}}\right)^{1/4} = \left(\frac{6.58}{0.015}\right)^{1/4} = 1.81$$

Now

$$R_2 = \frac{R_1}{\gamma} = \frac{6.58}{1.81} = 3.64 \ \Omega \qquad r_1 = 2.94 \ \Omega$$
$$R_3 = \frac{R_2}{\gamma} = 2.01 \ \Omega \qquad r_2 = 1.63 \ \Omega$$
$$R_4 = \frac{R_3}{\gamma} = 1.11\Omega \qquad r_3 = 0.90 \ \Omega$$
$$R_5 = \frac{R_4}{1.81} = 0.615 \ \Omega \qquad r_4 = 0.50 \ \Omega$$

Case 2

$$I_2 = 1.2 \times 38 = 45.6$$
$$\frac{R_1}{R_{\text{a}}} = \gamma^4 = \left(\frac{I_1}{I_2}\right)^{\gamma}$$
$$I_1 = \frac{V}{R_1} \qquad I_2 = 4.56 \text{ A}$$
$$\left(\frac{500}{45.6\, R_1}\right)^4 = \frac{R_1}{0.615}$$

$$R_1^5 = (0.615)\left(\frac{500}{45.6}\right)^4$$

$$R_1 = (0.615)^{1/5}\left(\frac{500}{45.6}\right)^{4/5} = 6.16\ \Omega$$

$$\gamma = \left(\frac{R_1}{R_a}\right)^{1/4} = \left(\frac{6.16}{0.615}\right)^{1/4} = 1.779$$

$$R_2 = \frac{R_1}{\gamma} = \frac{6.16}{1.779} = 3.46\ \Omega \qquad r_1 = 2.70\ \Omega$$

$$R_3 = \frac{3.46}{1.779} = 1.94\ \Omega \qquad r_2 = 1.52\ \Omega$$

$$R_4 = \frac{1.94}{1.779} = 1.09\ \Omega \qquad r_3 = 0.85\ \Omega$$

$$R_5 = \frac{1.09}{1.779} = 0.615\ \Omega \qquad r_4 = 0.48\ \Omega$$

7.24 *A starter is to be designed for a 10 kW, 250 V shunt motor. The armature resistance is 0.15 Ω. This motor is to be started with a resistance in the armature circuit so that during the starting period the armature current does not exceed 200% of the rated value or fall below the rated value. That is, the machine is to start with 200% of armature current and, as soon as the current falls to rated value, sufficient series resistance is to be cut out to restore current to 200% (or less in the last step). The process is to be repeated till all the resistance is cut out.*

(a) Calculate the total resistance of the starter.

(b) Also calculate the resistance to be cut out in each step in the starting operation.

Solution

Given
$$I_L = \frac{10 \times 10^{-3}}{250} = 40\ \text{A}$$
$$I_a \approx I_L = 40\ \text{A}$$
$$I_1 = 2I_{f1} = 80\ \text{A}$$
$$I_2 = I_{f1} = 40\ \text{A}$$
$$\gamma = \frac{I_1}{I_2} = 2$$

Now
$$R_1 = \frac{V}{I_1} = \frac{250}{80} = 3.125\ \Omega$$
$$R_{starter}(\text{total}) = 3.125 - 0.15$$
$$= 2.975\ \Omega$$

Also
$$\gamma^{k-1} = \frac{R_1}{R_a} \quad \text{or} \quad 2^{k-1} = \frac{3.125}{0.15}$$

which gives

$$k = 5 \Rightarrow 4 \text{ sections are required.}$$

Resistances of various sections are computed below:

$$R_1 = 3.125\ \Omega$$

$$R_2 = \frac{3.125}{2} = 156\ \Omega \qquad r_1 = 1.56\ \Omega$$

$$R_3 = \frac{1.56}{2} = 0.78\ \Omega \qquad r_2 = 0.78\ \Omega$$

$$R_4 = \frac{0.78}{2} = 0.39\ \Omega \qquad r_3 = 0.39\ \Omega$$

$$R_5 = \frac{0.39}{2} = 0.15\ \Omega \qquad r_4 = 0.15\ \Omega$$

7.25 *A dc motor drives a 100 kW generator having an efficiency of 87%*

(a) What should be the kW rating of the motor?

(b) If the overall efficiency of the motor generator set is 74%, what is the efficiency of the motor?

Solution

(a) kW rating (motor) = $\frac{100}{0.87} = 115$ kW

(b) $\eta = \eta_G \eta_M$

$0.74 = 0.87\ \eta_m$ or $\eta_M = 0.85$ or 85%

(c) P_0 (G) = 100 kW

P_i (G) = 115 kW

P_{Loss} (G) = 15 kW

P_i (M) = 135.3 kW

P_{Loss} (M) = 135.3 – 115 = 20.3 kW

7.26 *A 600 V dc motor drives a 60 kW load at 900 rpm. The shunt field resistance is 100 Ω and the armature resistance is 0.16 Ω. If the motor efficiency is 85%, determine;*

(a) the speed at no-load and the speed regulation.

(b) the rotational loss.

Solution

(a) $P_{in} = \frac{60}{0.85} = 70.59$ kW

$$I_L = \frac{70.59 \times 1000}{600} = 117.65\ \text{A}$$

$$I_f = \frac{600}{100} = 6\ \text{A}$$

$$I_a = 117.65 - 6 = 111.65\ \text{A}$$

$$E_a = 600 - 111.65 \times 0.16 = 582.14\ \text{V}$$

No load : $E_a \approx V = 600$

$$n_0 = 900 \times \frac{600}{582.14} = 927.6\ \text{rpm}$$

$$\text{speed regulation} = \frac{927.6 - 900}{900} = 3.1\ \%$$

(b) Total loss = 70.59 – 60 = 10.59 kW

$I_a^2\ R_a$ = $(111.65)^2 \times 0.16$ = 1.99 kW

$I_f^2\ R_f$ = $(6)^2 \times 100$ = 3.6 kW

Rotational loss = 10.59 – (1.99 + 3.6)
= 5 kW

7.27 *Calculate the efficiency of a self-excited dc shunt generator from the following data: Rating –10 kW, 250 V, 1000 rpm.*

Armature resistance = 0.35 Ω
Voltage drop at brushes = 2V
Windage and friction losses = 150 W
Iron loss at 250 V = 180 W

Open-circuit characteristic:

emf(V):	*11*	*140*	*227*	*285*	*300*	*312*
Field current (A):	*0*	*1.0*	*1.5*	*2.0*	*2.2*	*2.4*

Solution

$$I_L = \frac{10 \times 10^3}{250} = 40 \text{ A}$$

$$I_a \approx I_L = 40 \text{ A}$$

$$\therefore \quad E_a = 250 + 40 \times 0.35 + 2 = 266 \text{ V}$$

From OCC of Fig. P7.27, the corresponding field current is

$$I_f = 1.8 \text{ A}$$

$$\therefore \quad I_a(\text{corrected}) = I_L + I_f = 41.8 \text{ A}$$

$$P_L = I_a^2 R_a + V_f I_f + V_b I_a + P_{wf} + P_{io}$$

$$= (41.8)^2 \times 0.35 + 250 \times 1.8 + 2 \times 41.8$$

$$+ 150 + 180 \times \left(\frac{266}{250}\right)^2 = 15 \text{ kW}$$

(Iron loss is assumed to be proportional to the square of the flux density)

$$\therefore \quad \eta_G = \frac{10}{10+15} = 86.96\%$$

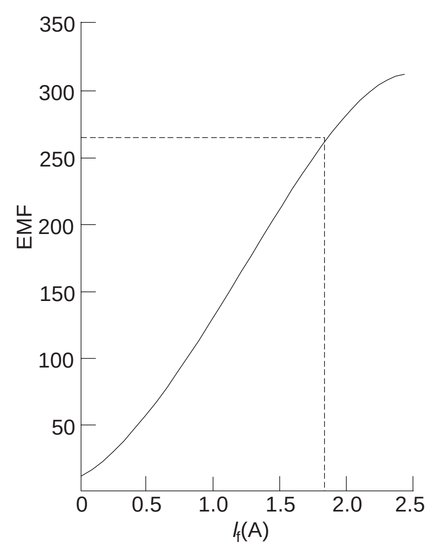


Fig. P7.27

Note Stray-load loss has been neglected.

7.28 *A 60 kW, 250 V shunt motor takes 16 A when running light at 1,440 rpm. The resistance of the armature and field are 0.2 Ω and 125 Ω respectively when hot. (a) Estimate the efficiency of the motor when taking 152 A. (b) Also estimate the efficiency if working as a generator and delivering a load current of 152 A at 250 V.*

Solution

$$I_f = \frac{250}{125} = 2\text{A}$$

$$I_{ao} = 16 - 2 = 14 \text{ A}$$

$$P_k = (P_{io} + P_{wf} + P_{sh})$$
$$= (250 \times 14 - (14)^2 \times 0.2) + 250 \times 2$$
$$= 3{,}960.8 \text{ W}$$

Assumption Stray-load loss will be ignored

(a) *Motoring*

$$I_a = 152 - 2 = 150 \text{ A}$$
$$P_L = I_a^2 R_a + P_k$$
$$= (150)^2 \times 0.2 + 3{,}9608 = 8.461 \text{ kW}$$
$$P_{in} = 250 \times 152 = 38 \text{ kW}$$
$$\therefore \quad \eta_M = \frac{38 - 8.461}{38} = 77.73\%$$

(b) *Generating*

$$I_a = 152 + 2 = 154 \text{ A}$$
$$P_L = I_a^2 R_a + P_k$$
$$= (154)^2 \times 0.2 + 3{,}960.8 = 8.704 \text{ kW}$$
$$P_{out} = 250 \times 152 = 38 \text{ kW}$$
$$\therefore \quad \eta_G = \frac{38}{38 + 8.704} = 81.36\%$$

7.29 *A 200 V shunt motor takes 10 A when running on no-load. At higher loads the brush drop is 2 V and at light loads it is negligible. The stray-load loss at a line current of 100 A is 50% of the no-load loss. Calculate the efficiency at a line current of 100 A if armature and field resistances are 0.2 and 100 Ω respectively.*

Solution

At no-load

No-load loss = 200 × 100 = 2,000 W

$$I_f = \frac{200}{100} = 2 \text{ A}$$
$$I_a = 10 - 2 = 8\text{A}$$
$$P_{io} + P_{wf} = 200 \times 8 - (8)^2 \times 0.2 = 1{,}587.2 \text{ W}$$

At load

$$I_a = 100 - 2 = 98 \text{ A}$$
$$I_a^2 R_a = (98)^2 \times 0.2 = 1{,}920.8 \text{ W}$$

Stray-load loss = 0.5 × 2,000 = 1,000 W

$$P_L = (I_a^2 R_a + V_b I_a + P_{st}) + (P_{io} + P_{wf} + P_{sh})$$
$$= (1{,}920.8 + 2 \times 98 + 1{,}000) + (1{,}587.2 + 200 \times 2) = 5{,}104 \text{ W}$$
$$\eta = \frac{P_{in} - P_L}{P_{in}} = \frac{200 \times 100 - 5{,}104}{200 \times 100} = 74.48\%$$

7.30 *The Hopkinson's test on two machines gave the following results for full-load: Line voltage 250 V; line current, excluding field current, 50A; motor armature current 38 A; field currents 5 A and 4.2 A. Calculate the efficiency of each machine. Armature resistance of each machine = 0.02 Ω. State the assumptions made.*

Solution

$$I_{ag} = I_{am} - I_L = 380 - 50 = 330 \text{ A}$$

Input to set = $VI_L = 250 \times 50 = 12.5$ kW

Armature copper loss $= (I_{am}^2 + I_{ag}^2)\ R_a$

$$= [(380)^2 + (330)^2]\ 0.02$$

$$= 5.066 \text{ kW}$$

Stray loss of each machine

$$W_s = \frac{1}{2}\ (12.5 - 5.066) = 3.72 \text{ kW}$$

Motor

$$I_{fm} = 4.2 \text{ A}$$

$$P_{in,m} = V(I_{am} + I_{fm})$$

$$= 250\ (380 + 4.2) = 96.05 \text{ kW}$$

$$P_{Lm} = P_{st} + I_{am}^2 R_a + Vl_{fm}$$

$$= 7.658 \text{ kW}$$

$$\eta_m = \left(\frac{96.05 - 7.658}{96.05}\right) \times 100 = 92.03\%$$

Generator

$$I_{fg} = 5 \text{ A}$$

$$P_{out,g} = 250 \times 330 = 82.5 \text{ kW}$$

$$P_{L,g} = P_{st} + I_{ag}^2 R_a + VI_{fg}$$

$$= 7.148 \text{ kW}$$

$$\eta_G = \left(\frac{82.5}{82.5 + 7.148}\right) \times 100 = 92.03\%$$

7.31 *Calculate the efficiency of a 500 V shunt motor, when taking 700 A, from the following data recorded when the motor was hot: Motor stationary; voltage drop in the armature winding 15 V, armature current 510 A, field current 9 A at normal voltage. Motor running at normal speed unloaded; armature current 22.5 A, applied voltage 550 V. Allow 2 V for brush contact drop and 1% of the rated output of 400 kW for stray-load losses.*

Solution

$$R_a = \frac{15}{510} = 0.029\ \Omega$$

$$R_f = \frac{500}{9} = 55.56\ \Omega$$

Using the data for unloaded motor:

Rotational loss = $(V - I_a R_a - V_b) I_a$ = (550 – 22.5 × 0.029 – 2) × 22.5 = 12.315 kW

$$\eta_M = \frac{P_{in} - I_a^2 R_a - V_f I_f - V_b I_a - \text{stray-load loss} - \text{rotational loss}}{P_{in}}$$

$$I_a = I_L - I_f = 700 - 9 = 691 \text{ A}$$

$$P_{in} = VI_L = 500 \times 700$$

$$= 350 \text{ kW}$$

$$\therefore \quad \eta_M = \frac{350{,}000 - (691)^2 \times 0.029 - 500 \times 9 - 2 \times 691 - 0.01 \times 400 \times 10^3 - 12{,}315}{350{,}000}$$

$$= 89.7\%$$

7.32 *A 480 V, 20 kW shunt motor took 2.5 A when running light. For an armature resistance to be 0.6 Ω, field resistance of 800 Ω and brush drop of 2 V, find the full-load efficiency.*

Solution

$$I_f = \frac{480}{800} = 0.6 \text{ A}$$

$$I_{i0} = 2.5 - 0.6 = 1.9 \text{ A}$$

$$P_{i0} + P_{mf} = (V - I_a R_a - V_b)\, I_a = (480 - 1.9 \times 0.6 - 2) \times 1.9 = 906 \text{ W}$$

$$P_{sh} = 480 \times 0.6 = 288 \text{ W}$$

$$P_k = 906 + 288 = 1{,}194 \text{ W}$$

At full-load,

$$P_{out} = 20 \text{ kW}$$

$$VI_L = 20{,}000 + (I_a^2 R_a + V_b I_a) + P_k \;(P_{st} \text{ ignored})$$

$$I_L = I_a + 0.6$$

$$\therefore \quad 480\,(I_a + 0.6) = 20{,}000 + 0.6 I_a^2 + 2I_a + 1{,}194$$

$$0.6 I_a^2 - 4.78 I_a + 20{,}906 = 0$$

Solving,

$$I_a = 46.42 \text{ A (taking the smaller of the two values)}$$

$$\therefore \quad P_{in} = 480\,(46.42 + 0.6) = 22{,}569 \text{ W}$$

$$\eta_M = \frac{20{,}000}{22{,}569} = 88.6\%$$

8.24 A 3-phase hydroelectric synchronous generator is read to be 110 MW, 0.8 pf lagging, 6-kV, Y-connected, 50 Hz, 100-rpm.
Determine:
(a) the number of poles
(b) the kVA rating
(c) the prime mover rating if the full-load generator efficiency is 97.1% (leave out field loss).
(d) the output torque of the prime-mover.

(a) $f = nP/120$, $P = (120 X 50)/ 100 = 60$

(b) $kVA = 110/0.8 = 137.5$ kVA

(a) kw (turbine) = $110/0.971 = 113.3$ kw

(d) Tpm(output) = $(113.3 X 1000 X 60) / 2 \pi X 100 = 10.82 X 10^3$ Nm

8.25 1000 kVA, 50 Hz, 2300 V, 3-phase synchronous generator gave the following test data:

Field current (A)	**40**	**80**	**100**	**120**	**140**	**170**	**240**
Voc (Line)(V)	**1000**	**1900**	**2200**	**2450**	**2600**	**2750**	**3000**
Isc(A)	**2000**						

(a) Find the field current required to deliver rated kVA at 0.8 lagging pf at rated terminal voltage.
(b) The OC voltage at this field current.
(c) The maximum kVAR that the machine can deliver as a synchronous condenser at rated voltage, if the rotor heating limits the field current to 240 A.

Refer fig 8.25
Xs(adjusted) = $(2300/\sqrt{3}) / 1800 = 0.738\ \Omega$

(a) $Ia = (1000 X 1000) / (\sqrt{3} X 2300) = 251$ A

$\varphi = \cos^{-1}0.8 = 36.9^\circ$

$Ef = 2300/\sqrt{3} \angle 0^\circ + j\ 0.738 X 251 \angle (-36.9^\circ)$

$= 1328 + 185 \angle (53.1^\circ)$
$= 1328 + 110 + j\ 148 = 1438 + j\ 148$

$Ef = 1445\sqrt{3} = 2503$ V

From Fig P – 8.2
If = 128 A

(b) Voc (If = 128A) = 2503 V

(c) At If = 240(A) (limit)
$E_f = 3000 / \sqrt{3} = 1732$

Ia = 404/0.737 = 5.48A
kVAR(out) = ($\sqrt{3}$ X 2300 X 548) / 1000 = 2183 lagging.

8.26 A 3-phase synchronous generator feeds into a 22kV grid. It has a synchronous reactance of 8 Ω / phase and is delivering 12 MW and 6 MVAR to the system.
Determine:
(a) the phase angle of the current
(b) the power angle
(c) the generated emf.

(a) S = 12 + j6
$= 13.42 \angle 26.56^\circ$
Phase angle of current = - 26.56°(reference grid voltage)

(b) Ia = (13.42 X 1000) / ($\sqrt{3}$ X 22) = 352.2 A

$E_f = 22/\sqrt{3} \times 1000 + j\,8 \times 352.2 \angle -26.56^\circ$
$= 12702 + 2818 \angle 63.4^\circ$
$= 12.7 + 1.26 + j\,2.52$
$= 13.96 + j\,2.52$

Ef = 14.18 kv

$\delta = 10.2^\circ$

(c) Generated emf = 14.18$\sqrt{3}$ = 24.56 kv

8.27 A 6.6 kV, 3-phase synchronous machine has as open circuit characteristic given by:

Field current (A)	**60**	**80**	**100**	**120**	**140**	**160**	**180**
Armature emf (kV) (line)	**5.3**	**6.2**	**6.8**	**7.2**	**7.5**	**7.7**	**7.9**

In short-circuit a field current of 80 A gave an armature current of 360 A. Determine the saturated synchronous reactance.

When developing 400 kW of mechanical power as a motor calculate the field current for pfs of 0.8 lagging, unity and 0.8 leading.

From Fig P-8.27

$Xs(sat) = (6.6 X 1000)/ (\sqrt{3} X 410) = 9.3 \Omega$

Neglecting armature loss

$Ia = (400 X 1000) / (\sqrt{3} X 6600 X 0.8) = 43.7 A$
$\Phi = \cos^{-1} 0.8 = 36.90$

<u>0.8 lagging pf</u>
$Ef = 6.6/\sqrt{3} + j (9.3 X 43.7) / 1000 \angle (-36.9^{\circ})$
$= 3.81 + 0.41 \angle 53.1^{\circ}$
$= 3.81 + 0.246 + j 4328$
$= 4.056 + j 0.328$
Or $Ef = 4.069\sqrt{3} = 7.05$ kv

From Fig P – 8.27
If = 115 A

<u>0.8 leading pf</u>

$Ef = 3.81 + j 0.41 \angle 36.8^{\circ}$
$= 3.81 + 0.41 \angle 126.8^{\circ}$
$= 3.81 - 0.246 + j 0.328$
$= 3.564 + j 0.328$
W $Ef = 3.58\sqrt{3} = 6.2$ kv

From fig P- 8.27
If = 76 A

8.28 A 200 kV A, 3.3 kV, 50 Hz three-phase synchronous generator is star-connected. The effective armature resistance is 5 Ω/phase and the synchronous reactance is 29.2 Ω/phase. At full-load calculate the voltage regulation for the following power factors:

(a) 0.707 leading (b) unity (c) 0.707 lagging

$Ia(fl) = (200 X 1000) / (\sqrt{3} X 3300) = 35 A$

1) PF 0.707 leading
$\varphi = \cos^{-1} 0.707 = + 45^{\circ}$
$Ef = 1.905 + (5 + j 29.2) X 35 \angle 45^{\circ}$
$= 1.905 + [(29.62 X 35) / 1000] \angle(45^{\circ} + 80.3^{\circ})$
$= 1.905 + 1.04 \angle 125.3^{\circ}$
$= 1.905 - 0.6 + j 0.849$
$= 1.305 + j 0849$

$Voc = \sqrt{3}E_f = 1.34\sqrt{3} = 2.32$ kV
$Reg = (2.32 - 3.3 X 100) / 3.3 = - 29.7 \%$

2) PF unity

$Ef = 1.905 + 1.04 \angle 80.3^\circ$
$= 1.905 + 0.175 + j\ 1.025$
$= 2.08 + j\ 1.025$

$Voc = \sqrt{3}\ Ef = 2.32\sqrt{3} = 4.016$ kV
$Reg = (4.016 – 3.3 X 100) / 3.3 = + 21.7\%$

3) PF 0.707 lagging

$Ef = 1.905 + 1.04 \angle(-45^\circ + 80.3^\circ)$
$= 1.905 + 1.04 \angle 35.3^\circ$
$= 1.905 + 0.849 + j\ 0.6$
$= 2.754 + j\ 0.6$

$Voc = \sqrt{3}Ef = 2.82\sqrt{3} = 4.88$ kv
$Reg = (4.88 – 3.3) / 3.3 X 100 = + 47.93$

8.29 A 3-phase, 4-pole star-connected synchronous motor has a resistance of 0.25 Ω/phase and a synchronous reactance of j 2.5 Ω/phase. The field is exited such that the open-circuit voltage of the machine is 25 kV. The motor is synchronized to 22kV mains. Calculate the maximum load on the motor (including rotational loss) before it could lose synchronism. What is the corresponding current and power factor?

$Zs = 0.25 + j\ 2.5$
$= 2.51 \angle 84.3^\circ$

$Pmech(max) = (25 X 22) / 2.51 = 219$ MW
$\delta = - 90^\circ$
$Ia = (22 – 25 \angle (-90^\circ)) / (\sqrt{3} X 2.51 \angle 84.3^\circ$

$= (22 + j25) / (4.347 \angle 84.3^\circ)$

$= (33.3 \angle 48.7^\circ) / (4.347 \angle 84.3^\circ)$

$= 7.66 \angle (-35.6^\circ)$

$Ia = 7.66$ kA
$Pf = \cos 35.6^\circ = 0.813$ lagging

8.30 A 6-pole, 3-phase, 50 Hz synchronous motor is supplied from 6.6 kV busbars. Its open-circuit voltage is 3.3kV/phase. The per phase resistance and synchronous reactance are 0.6 Ω and 4.8 Ω respectively. Calculate the current, power factor and torque developed, when the excitation emf lags the busbar voltage by 15˚, 25˚, and 35˚ (elect).

$Zs = 0.6 + j\,4.8$

$= 4.83 \angle 82.9^\circ\ \Omega$

$\underline{\delta = 15^\circ}$

$I0 = (3.81 - 3.3 \angle -15^\circ) / (4.83 \angle 82.9^\circ)$

$= (3.81 - 3.19 + j\,0.854) / (4.83 \angle 82.9^\circ)$

$= (0.62 + j\,0.854) / (4.83 \angle 82.9^\circ)$

$= (1.055 \angle 54^\circ) / (4.83 \angle 82.9^\circ)$

$= 218 \angle -28.9^\circ$

Ia = 218 A

Pf = cos 28.9 ° = 0.875 lagging

$Pmech = 3 \times 3.3 \times 218 \cos (28.9^\circ - 15^\circ)$

$= 3 \times 3.3 \times 218 \times 0.971$

= 2.1 Mw

$\underline{\delta = 25^\circ}$

$Ia = [3.81 - 3.3 \angle (-25^\circ)] / 4.83 \angle 82.9^\circ$

$= (1.613 \angle 59.9^\circ) / (4.83 \angle 82.9^\circ)$

$= 0.334 \angle (-23^\circ)$

$3.81 - (3.3 \angle -25^\circ) = 3.81 - 3 + j\,1.395$

$= 0.81 + j\,1.395$

$= 1.613 \angle 59.9^\circ$

Ia = 334 A

Pf = cos 23 ° = 0.92 lagging

$Pmech = 3 \times 3.3 \times 334 \cos 2^\circ = 3.3$ MW

$\underline{\delta = 35^\circ}$

$3.81 - 3.3 \angle (-35^\circ) = 3.81 - 2.7 + j\,1.893$

$= 1.11 + j\,1.893$

$= 2.19 \angle 59.6^\circ$

$Ia = (2.19 \angle 59.6^\circ) / (4.83 \angle 82.9^\circ) = 0.453 \angle (-23.3^\circ)$

Ia = 453 A, pf = cos 23.3 ° = 0.918 lagging

$Pmech = 3 \times 3.3 \times 453 \cos (25 - 23.3^\circ)$

= 4.48 MW

8.31 A 25 kVA, 400 V, 3-phase synchronous generator delivers rated kVA at rated voltage at 0.8 pf lagging. The per phase (star basis) armature resistance and synchronous reactance respectively are 0.66 Ω and 7.1 Ω. The field winding is supplied 10.6 A at 110 V. The friction and winding loss is estimated to be 480 W and iron loss as 580 W. Calculate:
(a) the full-load efficiency.
(b) the terminal voltage when the load is thrown off.

a) Input = 25 X 0.8 = 20kw
Rotational loss = 480 + 580 = 1060 w 1.06 kw
Mechanical input = 2v + 1.06 = 21.06 kw
Field loss = 110 X 10.6 = 1.166 kw
Total input = 21.06 + 1.166 = 22.226 kw

η = 20/22.226 = 90%

b) Full-load
Ia(μ) = (2s X 1000) / √3 X 400 = 36.1A
φ = - 36.9°

Ef = 231 + (0.66+ j 7.1) X 36.1 ∠ (-36.9°)
= 231 + 7.13 X 36.1 ∠84.7°-36.9°)
= 231 + 257.4 ∠47.8°
= 231 + 172.9 + j 190.7
= 403.9 + j 190.7
Ef = 446.6v

Voc = 446.6√3 = 773.5v

8.32 A 15 kW. 400 V, 3-phase, star-connected synchronous motor has a synchronous impedance of 0.4 + j4 Ω. Find the voltage to which the motor should be excited to give a full-load output at 0.866 leading pf. Assume an armature efficiency of 93%. Also calculate the mechanical power developed.

Full load output = 15 kw
Armature efficiency = 93%
Armature input = 15 / 0.93 = 16.13 kw

Ia = (16.12 X 1000) / (√3 X 400) = 23.3 A

Φ = cos-1 0.806 = -30 °

Ef = 231 – (0.4 + j4) X 23.3∠30 ° ; 0.4 + j 4 = 4.02 ∠ (84.3 °)
= 231 – 93.7 ∠ 114.3 ° = 231 + 38.6 – j 85.4
= 269.6 – j 85.4 = 282.8 ∠ (-17.6 °)

$\sqrt{3}Ef = \sqrt{3} X 282.8 = 490$ V
Mechanical power developed
$= 3 X 282.8 X 23.3 \cos(30^\circ - 17.6^\circ)$
$= 19.31$ kw

8.33 A 3-phase, star-connected synchronous generator is rated ar 1200 kVA, 11 kV. On short-circuit a field current of 55 A gives full-load current. The OC voltage waith the same excitation is 1580 V/phase. Calculate the voltage regulation at
(a) 0.8 lagging and
(b) 0.8 leading pf. Neglect armature resistance.

$Ia(\mu) = 1200 / (\sqrt{3} X 11) = 63 A = Isc$

$Voc = 1580 / \sqrt{3} = 912$ V
$Xs = 912 / 63 = 14.5\ \Omega$

<u>0.8 lagging pf</u>

$Ef = 6.35 + j\ 14.5 X 0.063 \angle (-36.9^\circ)$
$= 6.35 + 0.914 \angle (53.1^\circ)$
$= 6.35 + 0.549 + j\ 0.731$
$= 7.05 + j\ 0.731 = 7.09 \angle 5.9^\circ$

$Voc = \sqrt{3}Ef = \sqrt{3} X 7.09 = 12.28$ kv
Voltage Reg $= [(12.28 – 11) / 11] X 100$
$= 11.64\ \%$

<u>0.8 leading pf</u>

$Ef = 6.35 + 0.914 \angle 90^\circ + 36.9^\circ$
$= 6.35 + 0.914 \angle 126.9^\circ$
$= 6.35 – 0.549 + j\ 0.731$
$= 5.801 + j\ 0.731$
$= 5.847 \angle 7.2^\circ$

$Voc = \sqrt{3}Ef = \sqrt{3} X 5.817$
$= 10.12$kv

Voltage reg $= [(10.12 – 11) / 11] X 100$
$= - 7.94\ \%$

8.34 A 1500 kW. 3-phase, star-connected 2300 V, 50 Hz synchronous motor has a synchronous reactance of 2 Ω/phase. The motor is supplied from a 3-phase, star –

connected, 2300 V, 1750 kVA turbo generator whose synchronous reactance is 2.8 Ω/phase. When the motor is drawing full-load power, at upf, calculate:
(a) the induced emf o f the generator, and
(b) emf of the motor.
What maximum power can flow from the generator to the motor with machine excitation held fixed?

$I_a(\mu) = 1500 / (\sqrt{3} \times 2.3 \times 1) \angle 0^\circ = 376.5 \angle 0^\circ$ A
$V_t = (230 \angle 0^\circ) / \sqrt{3} = 1328 \angle 0^\circ$ V

(i) $E_{fg} = 1328 + j\,2.8 \times 376.5 \angle 0^\circ$
$= 1328 + j\,1054$
$= 1695 \angle 38.4^\circ$ V
$\sqrt{3}\,E_{fg} = \sqrt{3} \times 1695$
$= 2935$ V

(ii) $E_{fm} = 1328 - j\,2 \times 376.5 \angle 0^\circ$
$= 1328 - j753$
$= 1527 \angle (-29.6^\circ)$
$\sqrt{3}\,E_{fm} = \sqrt{3} \times 1527$
$= 2645$ V

$P_{max} = (2.935 \times 2.645) / (2.8 + 2)$
$= 1617$ kw

8.35 A 13.8 kV 1250 kVA 3-phase, star-connected synchronous generator has a resistance of 2.1 Ω/phase. Data for its OCC and ZPFC characteristics is given below:

Field current (A)	**40**	**50**	**110**
	140	**180**	
Open circuit	**7.28**	**8.78**	**15.68**
Volts (line)	**17.25**	**18.82**	
Zero pf volts	**0**	**1.88**	**10.66**
(line)	**13.17**	**15.68**	

Find the voltage regulation o f the generator for full-load 0.8 pf lagging.

$I_{fl} = 1250/(\sqrt{3} \times 13.8) = 52.3$ A

$V_t(\text{phase}) = 13.8 / \sqrt{3} = 7.97$ kv

From fig P- 8.35 (b)
$I_f = 116$ A
From OCC of Fig P– 8.35 (a)
$V_{oc}(\text{line}) = 16$kv
Voltage regulation = (16 – 13.8) / 13.8 = 15.9 %

8.36 A 440 V, 50 Hz, Y-connected salient-pole synchronous generator has a direct-axis reactance of 0.12 Ω and a quadrature-axis reactance of 0.075 Ω per phase, the armature resistance being negligible. The generator is supplying 1000 A at 0.8 lagging pf.

(a) Find the excitation emf, neglecting saliency and assuming $X_s = X_d$.

(b) Find the excitation emf accounting for saliency.
On equivalent star basis, per phase reactance are

Xd = 0.12/3 = 0.04Ω
Xq = 0.75/3 = 0.025Ω

(b) Ia = 1000A, φ = +36.9°(lagging)
Vt = 440/√3 = 254 V

Tan V = (Vt sin φ + Ia Xq) / (Vt cos φ + Iara)
= (254 X 0.6 + 1000 X 0.025) / (254 X 0.8)
= 177.4/203.2
= 0.873
V = 41.1 °

δ = V - φ = 41.1° - 36.9 ° = 4.2 °

Ef = Vt Cos δ + Id xd ; Id = Ia sin V
= 254 Cos 4.2 ° + 73.2 X 0.04 = 1000 X sin 4.2 ° = 73.2
= 253.3 + 3 = 256.3 or 440 v(line)

√3Ef = √3 X 256.3 = 444 V

(a) Saliency ignored, Xs = Xd
Ef = 254 ∠ 0° + j 0.04 X 1000∠ (-36.9 °)
= 254 + 40 ∠ 531 °
= 254 + 24 + j 32
√3 Ef = √3 X 279.8 = 484.6 V

8.37 Figure P: 8.36 shows two generators supplying in parallel a load o f 2.8 MW at 0.8 pf lagging:
(a) At what frequency is the system operating and what is the load supplied by each generator?

(b) If the load is now increased by 1 MW, what will be the new frequency and the load sharing?

(c) In part (b) which should be the set point o f G2 for the system frequency to be 50 Hz? What would be the load sharing now?

G2: $f2 = 51 - G2$ ---------- (i)

G1: $f1 = 51.8 - G1$ ------- (ii)

(a) $G1 + G2 = 2.8$ --------- (iii)

$f1 - f2 = f$

Adding (i) and (ii)

$$2f = 102.8 - (a1 + a2)$$
$$2f = 102.8 - 2.8 = 100$$
$$f = 50 \text{ Hz}$$

(b) $G1 + G2 = 3.8$

$$2f = 102.8 - 3.8 = 99$$
$$f = 49.5 \text{ Hz}$$

(c) $G1 + G2 = 3.8$

$$2 \times 50 = (51.8 + f02) - 3.8$$
$$f02 = 103.8 - 51.8$$
$$= 52 \text{ Hz}$$

8.38 A generating station comprises four 125 kWA, 22 kV, 0.84 pf lagging synchronous generators with a frequency drop of 5 Hz from no-load to full-load, at a frequency of 50 Hz, three generators supply a steady load of 75 MW each while the balance is shared by the fourth generator (called swing generator).

(a) For a total load o f260 MW at 50Hz, find the no-load frequency setting of the generators.

(b) With no change in governor setting as in part(a), find the system frequency if the system load rises to 310 MW.

(c) Find the no-load frequency of the swing generator for the system frequency to be restored to 50 Hz for the load in part (b).

(d) Find the system frequency if the swing generator trips off with load as in part (b), the governor setting remaining unchanged as in part (a).

(a) Generator rating = 125 X 0.84 = 105 MW
Frequency drop = 5/105 = 1/21 Hz/MW

G = G1 + G2 + G3 = 3 X 75
= 225 MW

Combined drop = 5/(105 X 3)
= 1/63 Hz/MW

G4 = 260 – 225 = 35 MW

f = f0 – 1/43G ---------- (i)
f4 = f04 – 1/21 G4 ----- (ii)

f = f4 = 50
50 = f0 – 1/63 X 225 or f0 = 53.6 Hz
50 = f04 – 1/21 X 35 or f04 = 51.7 Hz

(b) Load rise to 310MW = (3 X 75 + 85)
G4 = 85
f4 = f

Adding (i) and (ii)

2f = (53.6 + 51.7) – 1/63 X 225 – 1/21 X 85
Or f = 48.84 Hz

(c) f = f4 = 50
Adding (i) and (ii)

100 = 53.6 + f04 – 1/63 X 225 – 1/21 X 81
Or f04 = 53.83 Hz

(d) Load = 310 MW
f = 53.6 – 1/63 X 310 = 48.68 Hz

8.39 An 11 kW, 3-phase alternator has Xd and Xq of 0.6pu and 0.1pu respectively and negligible armature resistance. It is delivering rated kVA at 0.8 pf lagging. Determine its generated emf.

V_L = 11kW X_d = 0.6pu X_q = 0.1pu R_a = 0
Rated k VA at 0.8 pf Eg = ?

V = 1pu

Cos φ = 0.8 Sin φ = 0.6 φ = 36.9°

I_a = 1pu δ = 26.56

Tan φ = (I_a X_q Cos φ)/(V+ I_a X_q Sin φ)
= (1 X 0.1 X 0.8)/(1+0.1 X 0.6)
= 0.08/1.06

$I_d = I_a$ Sin ($\varphi + \delta$)
$= 1$ X Sin (4.31 + 36.9)
$= 0.65$A

Eg = V Cos $\delta + I_d$ X I_d
$= 1$ X 0.997 + 0.65 X 0.6
= **1.387**

8.40 Compute the distribution factor and coil span factor for a 3 phase winding with 4 slots per pole per phase and with coil span of 10 slot pitch.

m = 4   No of slots =   4 X 4 X 3 = 48

$\beta = (180^\circ \text{ X } 4) / 48 = 15^\circ$

Kd = Sin(4 X (15˚/2)) / 4 sin(15˚/2)

= 0.5/ 0.522

Kd = 0.956

Kc = cos θsp/2

Full pitch = 4 X 3 = 12 slot pitches

Coil pitch = 5 slot pitch

Therefore, $\theta_{sp} = 15^\circ$

Therefore, Kc = cos(15˚/2) = **0.991**

8.41 A 3.3kV, 3 phase star connected synchronous generator has full load current of 100A. Under short circuit condition, it takes 5 A field current to produce full load short circuit current. The open circuit voltage is 900 V (line to line). Determine synchronous reactance per phase and voltage regulation for 0.8 pf lagging. Assume armature resistance os 0.9 ohms/phase.

Given Data:

3.3kV   3 φ   Y - connected

I_a = 100A, I_f = 5A → short circuit current
E_o =900 (L-L)
p.f = 0.8 pf lag
R_a = 0.9 Ω/phase

Z_s = (O.C Voltage/ Phase) / (S.C Current / Phase)
= (900 / $\sqrt{}$ 3) / 100
= 5.196 Ω

$X_s = (\sqrt{} Z_s^{2} - R_a^{2})$
= ($\sqrt{}$ $5.19^2 - 0.9^2$
X_s = 5.12 Ω

At 0.8 pf lagging

$E_o = \sqrt{}(V \cos φ + I_a R_a)^2 + (V \sin φ + Ia X_S)^2$

V_t / Phase = 3300 / √3 = 1905.25 V

E_o = 1905.25 X 0.8 + 100

I_a = 1905.25 X 0.8 + 100 X 0.9)^2 + (1905.25 X 0.6 + 100 X 5.12)^2
= √(2605656.848 + 2739521.523
= 2311.96 volts

% Reg^n = (2311.96 – 1905.25) / 1905.25
Reg^n = 21.34 %

8.42 Two similar 6.6 kV synchronous generators supply a total load of 1000 kW at 0.8 pf lagging so that power supplied by each machine is same. Determine the load current of second synchronous generator and power factor of each machine, when excitation of first synchronous generator is decreased so that load current reduces to 100A.

V_1 = 6.6 kV, V_2= 6.6kV P_{L1} = 200kW P_{L2} = 100 kW 0.8pf lag

V_{PL} = 3810 V

I_{total} = 100k/ (√3 X 6.6k X 0.8) = 21.86

Assume PL = 1000 kW

I_{total} = 109.3A

If I_{total} = 100 A
P_L = √3 X 6.6k X cos θ X 100 = 2000kW

I = 100 (0.8 –j0.6)
I_{total} = 80 – j60
Cos φ = 2000kW / (√3 X 6.6k X 100) = 0.87

$I2 = 54.65$ A
$I1 = 100-54.65 = 45.35$

$\cos\theta 1 = 500 / (\sqrt{3} \text{ X } 6.6\text{k X}45.35) = 0.96$

$\cos\theta 2 = 500 / (\sqrt{3} \text{ X } 6.6\text{k X}54.65) = 0.8$

8.43 A 3-phase, 2.5 MVA, 6.6 kV synchronous generator gave the following test results,

OCC

I_f(A)	16	20	25	32	45
V_{oc} (line)(V)	4400	5500	6600	7700	8800

SC test
$I_f = 18$A for rated armature current
ZPF test
$I_f = 20$ A for rated armature current at rated voltage.
Determine the field current and voltage regulation when the generator is supplying rated current at 0.8 pf lagging and rated voltage, $R_a = 0$
Use the following methods and compare the results and draw conclusions:

(a) Unsaturated synchronous reactance.
(b) Saturated synchronous reactance.
(c) mmf method
(d) ASA method

Rated $I_a = 2.5\text{M} / 6.6\text{k} = 378.8$A

Vr = 6.6kV => Vph = 3811 V

By mmf metthos:
From the Graph
$I_{f1} = 25$ A
$I_{f2} = 18$ A

$I_f = \sqrt{(25^2 + 18^2 - 2 \text{ X}25 \text{ X } 18 \cos(90 + 36.86) + 540)}$

$= 38.6$ A

(4800 – 3811) / 3811 = 26 %

$E = \sqrt{((V \cos\varphi + I R_a)^2 + (V \sin\varphi + I X_L)^2)}$
$= \sqrt{((3811 \text{ X}0.8)^2 + (3811 \text{ X } 0.6 + 300)^2)}$
6690493.56
$= 3998$ V

$I_{f1} = 21$ A

$I_{f2} = 18$ A

$I_f = \sqrt{}\,(21^2 + 18^2 - 2 \text{ X}25 \text{ X } 18 \cos 126.86)$
$= \sqrt{}\ 1218.6$
$= 453.6$
$I_f = 34.9$ A + 6
$= 40.9$ A ≈ 41 A

VR = (4890 – 3811) / 3811 = 28.3 %

CHAPTER 8: SYNCHRONOUS MACHINES

8.1 *The open and short circuit tests data on a 3-phase, 1 MVA, 3.6 kV, star connected synchronous generator is given below.*

I_f(A):	*60*	*70*	*80*	*90*	*100*	*110*
Voc (line) (V):	*2560*	*3000*	*3360*	*3600*	*3800*	*3960*
I_{SC}(A):	*180*					

Find:

(a) The unsaturated synchronous reactance

(b) The adjusted synchronous reactance

(c) The short circuit ratio

(d) The excitation voltage needed to give rated voltage at full load, 0.8 lagging pf. Use adjusted synchronous reactance.

(e) Voltage regulation for the load specified in part (d).

Solution

The OCC and SCC as per the data are drawn in Fig. P8.1(a)

(a) For rated voltage, and air-gap line, the short circuit current is 255A, for constant I_f

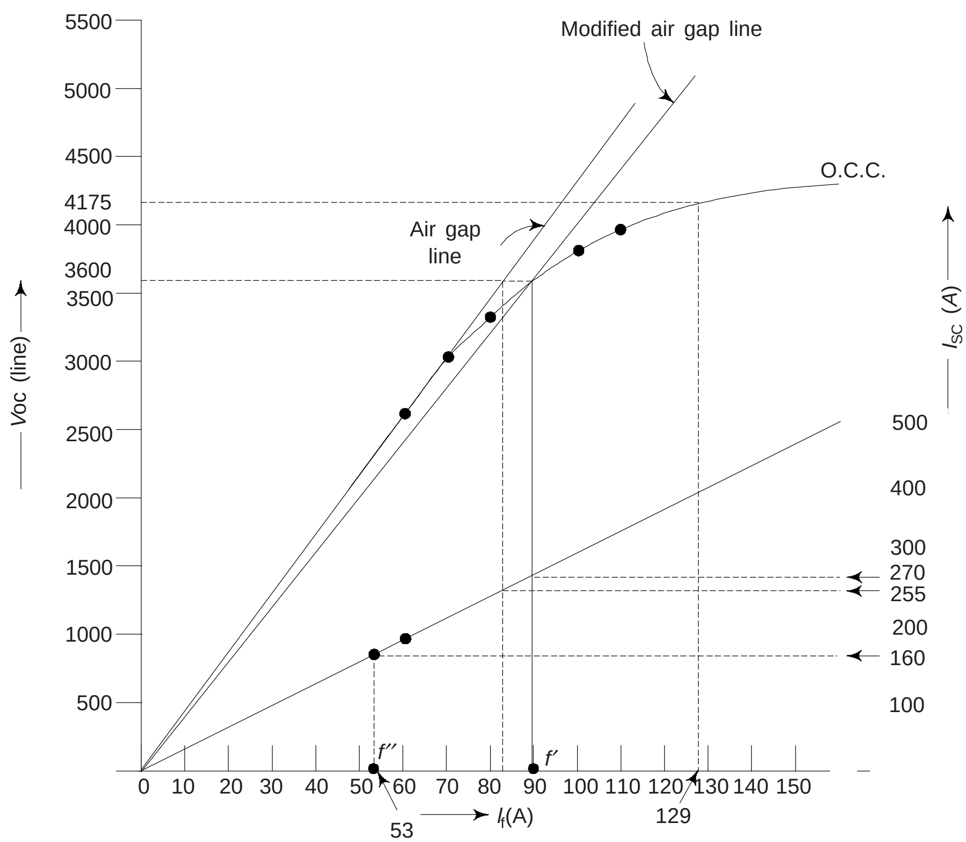


Fig. P8.1(a)

$\therefore \quad x_s \text{ (unsaturated)} = \dfrac{3600/\sqrt{3}}{255} = 8.15\ \Omega$

(b) $\quad x_s \text{ (adjusted)} = \dfrac{3600/\sqrt{3}}{270} = 7.7\ \Omega$

(c) Refer to the circuit diagram of Fig. P8.1(b)

$$I_a \text{ (rated)} = \frac{1 \times 10^6}{\sqrt{3} \times 3.6 \times 10^3} = 160.4 \text{ A}$$

$$\text{SCR} = \frac{Of'}{Of''} = \frac{90}{53} = 1.698$$

(d)
$$I_a = 160.4\ (0.8 - j\ 0.6)$$
$$E_f = 2078 + j\ 7.7 \times 160.4\ (0.8 - j\ 0.6)$$
$$= 2078 + 741 + j\ 988$$
$$= 2819 + j\ 988$$
$$= 2987 \text{ or } 5173 \text{ V (line)}$$

(e) The new field current for 5173 V (line) is obtained from modified air gap line. On linear basis we get,

$$I_f = \frac{90}{3600} \times 5173 = 129 \text{ A}$$

For this value of field current when the machine is open circuited.

$$V_{OC} \text{ (line)} = 4175 \text{ V.}$$

$$\therefore \quad \text{Voltage regulation} = \frac{4175 - 3600}{3600}$$
$$= 15.97\%$$

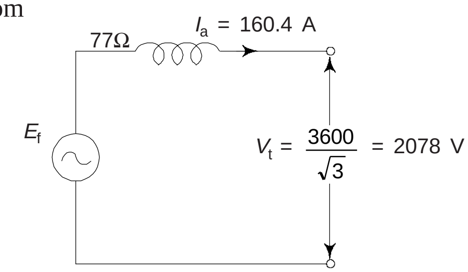


Fig. **P8.1(b)**

8.2 *A 3-phase, 2.5 MVA, 6.6 kV synchronous generator gave the following data for OCC at synchronous speed:*

I_f(A)	*16*	*20*	*25*	*32*	*45*
V_{OC}(line) (V)	*4400*	*5500*	*6600*	*7700*	*8800*

With the armature short-circuited and full-load current flowing, the field current is 18 A. When the machine is applying full-load current at zero pf at rated voltage, the field current is 45 A.

Determine the leakage reactance in Ω per phase and the full-load armature reaction in terms of equivalent field amperes. Find also the field current and voltage regulation when the machine is supplying full-load at 0.8 pf lagging at rated voltage. Neglect armature resistance.

Solution

$$I_a(\text{rated}) = \frac{2.5 \times 1,000}{\sqrt{3} \times 6.6} = 218.7 \text{ A}$$

From Potier's triangle of Fig. P8.2(a)

$$\sqrt{3} I_a(\text{rated})\ x_l = 800 \text{ V}$$

$$\therefore \qquad x_l = \frac{800}{\sqrt{3} \times 218.7} = 2.11\ \Omega$$

Full-load armature reaction, $I_f^{ar} = 15$ A

$$V_t = \frac{6,600}{\sqrt{3}} = 3,811 \text{ V}$$

$$\cos\phi = 0.8 \text{ lag} \qquad \phi = 36.9°$$

From the phasor diagram of Fig. P8.2(b)

$$E_r = 4,130 \text{ V or } 7,153 \text{ V (line)} \xrightarrow{\text{OCC}} 28.3 \text{ A}$$

$$= I_f^r$$

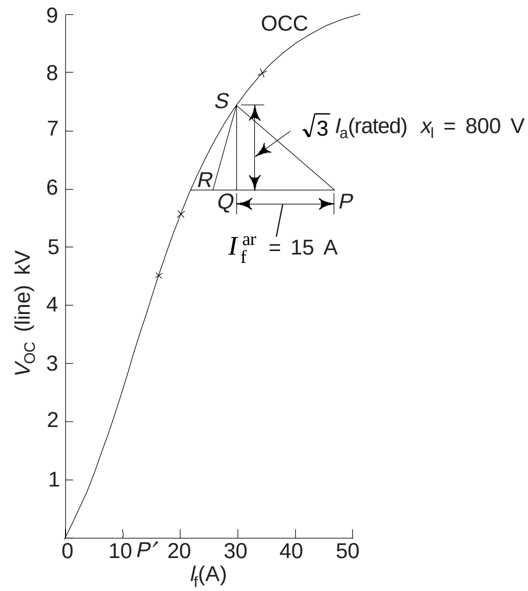


Fig. P8.2(a)

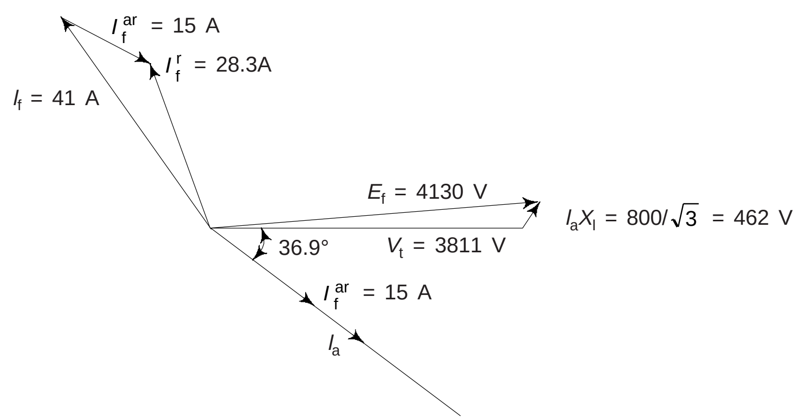


Fig. P8.2(b)

From OCC (Fig. P8.2 (a)) at $I_f = 41$ A,

$$E_f = 8,600 \text{ V (line)}$$

$$\text{Voltage regulation} = \frac{8,600 - 6,600}{6,600} = 30.3\%$$

8.3 *In Prob. 8.1, the armature leakage reactance is estimated to be 0.15 pu, Solve part (d) by using the mmf phasor diagram.*

Solution

MMF diagram in drawn in Fig. P8.3.

(d) $\bar{E}_r = \bar{V}_t + j\bar{I}_a x_l$

$$= \frac{3600}{\sqrt{3}} \angle 0° + j160.4 \angle -36.86°$$

$$\times 0.15 \times \frac{3.6^2}{1}$$

$$= 2265.44 + j248.97 = 2278.72 \angle 6.27°$$

E_r = 2278.72 V (phase); 3946.87 V (line)

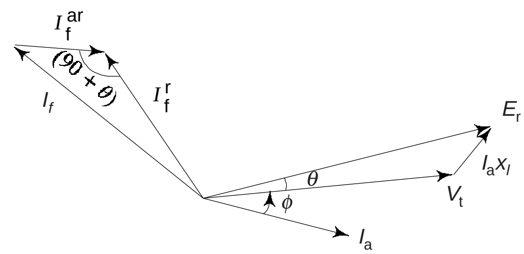


Fig. P8.3 *MMF Phasor Diagram*

From Fig. P8.1(a)

$$I_f^r = 113 \text{ A} \quad \text{and} \quad I_f^{ar} = 53.3 \text{ A.}$$

From Fig. P8.3.

$$I_f = ((I_f^r)^2 + (I_f^{ar})^2 - 2\, I_f^r\, I_f^{ar} \cos(90 + \theta))^{1/2}$$

$$= (113^2 + 53.3^2 - 2 \times 113 \times 53.3 \cos 96.27°)^{0.5}$$

$$= 130.1 \text{ A}$$

Again refer to Fig. P8.1(a)

$$E_f = 4175 \text{ V (line)}$$

8.4 *A 1 MVA, 11 kV, 3-phase star-connected synchronous machine has the following OCC test data:*

I_f(A)	*50*	*110*	*140*	*180*
V_{OC}(line) (V)	*7000*	*12500*	*13750*	*15000*

The short-circuit test yielded full-load current at a field current of 40 A. The ZPF test yielded full-load current at rated terminal voltage for a field current of 150 A. The armature resistance is negligible.

Calculate the field current needed for the machine to draw full-load 0.8 pf leading current when operated as a motor connected to an 11 kV supply.

Solution

$$I_a(\text{rated}) = \frac{1 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 52.5 \text{ A}$$

From the Potier triangle of Fig. 8.4(a)

$$\sqrt{3}\, I_a(\text{rated}) x_1 = 2{,}060 \text{ V}$$

$$\therefore \quad x_1 = \frac{2{,}060}{\sqrt{3} \times 52.5} = 22.65 \ \Omega$$

and $I_a(\text{rated}) x_1 = 1{,}189$ V

Full-load armature reaction, $I_f^{ar} = 27.5$ A

$$V_t = \frac{11{,}000}{\sqrt{3}} = 6{,}351 \text{ V}$$

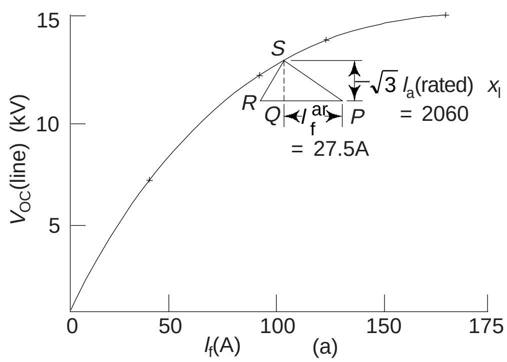


Fig. **P8.4(a)**

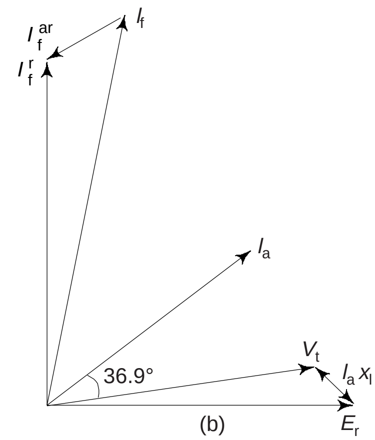


Fig. **P8.4(b)**

$$\cos\ \phi = 0.8 \text{ lead} \qquad \phi = 36.9°$$

From the phasor diagram of Fig. P8.4(b)

$$E_r = 7{,}100 \text{ V} \quad \text{or} \quad 12.3 \text{ kV(line)} \xrightarrow{\text{OCC}} I_f^r = 105 \text{ A}$$

$$I_f = 122.5 \text{ A}$$

8.7 *The full-load torque angle of a synchronous motor at rated voltage and frequency is 30° elect. The stator resistance is negligible. How would the torque angle be affected by the following changes?*

(a) The load torque and terminal voltage remaining constant, the excitation and frequency are raised by 10%.

(b) The load power and terminal voltage remaining constant, the excitation and frequency are reduced by 10%.

(c) The load torque and excitation remaining constant, the terminal voltage and frequency are raised by 10%.

(d) The load power and excitation remaining constant, the terminal voltage and frequency are reduced by 10%.

Solution

$$P_m = \frac{V_t E_f}{x_d} \sin 30°$$

$$T_m = \frac{V_t E_f}{\omega_s x_d} \sin 30°$$

(a) $$T_m = \frac{V_t \times 1.1\, E_f}{1.1\omega_s \times 1.1 x_d} \sin\delta = \frac{V_t E_f}{\omega_s x_d} \sin 30°$$

$\therefore$ $$\sin\ \delta = 1.1 \sin 30° = 0.55$$

$$\delta = 33.36°$$

(b) $$P_m = \frac{V_t \times 0.9 E_f}{0.9 x_d} \sin\ \delta = \frac{V_t E_f}{x_d} \sin 30°$$

$$\sin\ \delta = \sin 30°$$

$$\delta = 30°$$

(c) $T_m = \dfrac{1.1V_t E_f}{1.1\omega_s \times 1.1x_d} \sin\delta = \dfrac{V_t E_f}{\omega_s x_d} \sin 30°$

$$\sin \delta = 1.1 \sin 30°$$

or $\delta = 33.36°$

(d) $P_m = \dfrac{0.9V_t E_f}{0.9x_d} \sin\delta = \dfrac{V_t E_f}{x_d} \sin 30°$

$$\sin\delta = \sin 30°$$

or $\delta = 30°$

8.8 *A 1000 kVA, 3-phase, 11 kV, star-connected synchronous motor has negligible resistance and a synchronous reactance of 35 Ω per phase.*

(a) What is the excitation emf of the motor if the power angle is 10° and the motor takes rated current at: (i) lagging power factor, and (ii) leading power factor.

(b) What is the mechanical power developed and the power factor in part (a)?

(c) At what power angle will this motor operate if it develops an output of 500 kW at the rated line voltage and with an excitation emf of 10 kV (line)? What is the corresponding power factor?

(d) What is the minimum excitation at which the motor can deliver 500 kW at the rated line voltage without losing synchronism?

Solution

$$x_s = 35\ \Omega$$

$$V_t = \frac{11}{\sqrt{3}} = 6.35 \text{ kV}$$

$$I_a(\text{rated}) = \frac{1{,}000}{\sqrt{3} \times 11} = 52.5 \text{ A}$$

(a) Power angle $\delta = 10°$

$$I_a(\text{rated})x_s = 52.5 \times 35 = 1.84 \text{ kV}$$

V_t is drawn in Fig. P8.8. The locus of $jI_a x_s$ is a circle as the power factor of I_a varies.

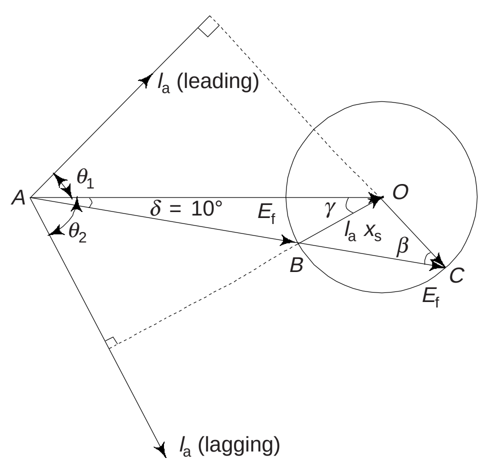


Fig. P8.8

Corresponding to a value of power angle δ, as is clear from the diagram, we have two points C and B which give values of E_f for leading and lagging currents correspondingly. From Fig. 8.8

$$V_t^2 + E_f^2 - 2V_tE_f \cos 10° = (I_a x_s)^2$$

or
$$E_f^2 - 2 \times 6.35 \times \cos 10° \; E_f - (1.84)^2 + (6.35)^2 = 0$$

$$E_f^2 - 12.51 \; E_f + 36.93 = 0$$

(i) Lagging pf: $E_f = 4.77$ kV

(ii) Leading pf: $E_f = 7.74$ kV

(b) (i) I_a lagging

Mechanical power developed

$$P_m(\text{out}) = 3 \times \frac{V_t E_f}{x_s} \sin \delta$$

$$= 3 \times \frac{6.35 \times 4.77}{35} \times \sin 10°$$

$$= 451 \text{ kW}$$

From triangle OAB

$$\frac{I_a x_s}{\sin \delta} = \frac{E_f}{\sin \gamma}$$

$$\frac{1.84}{\sin 10°} = \frac{4.77}{\sin \gamma}$$

or
$$\gamma = 26.80°$$

pf = cos θ_2 = cos (90° – 26.8°) = 0.451 lagging

(ii) I_a leading

Mechanical power developed

$$P_m(\text{out}) = 3 \times \frac{V_t E_f}{x_s} \sin \delta$$

$$= 3 \; \frac{6.35 \times 7.74}{35} \sin 10°$$

$$= 7315 \text{ kW}$$

From triangle OAC

$$\frac{I_a x_s}{\sin \delta} = \frac{V_t}{\sin \beta}$$

$$\frac{1.84}{\sin 10°} = \frac{6.35}{\sin \beta}; \quad \therefore \; \beta = 36.8°$$

pf = cos θ_1 = cos (90° – 36.8° – 10°) = 0.73 leading

(c) $P_m(\text{out}) = \dfrac{V_t E_f}{x_s} \sin \delta$

$$E_f = \frac{10}{\sqrt{3}} = 5.77 \text{ kV}$$

$$0.5 = 3 \times \frac{6.35 \times 5.77}{35} \times \sin \delta$$

$$\sin \delta = 0.159$$

$$\therefore \qquad \delta = 9.2°$$

Since $E_f < V_t$ only lagging pf solution is possible

$$\frac{1.84}{\sin 9.2°} = \frac{5.77}{\sin \gamma}$$

$$\gamma = 30.1°$$

$$\cos \theta_2 = \cos (90 - 30.1°) = 0.5 \text{ lagging}$$

(d) Without losing synchronism minimum excitation corresponds to power angle $\delta = 90°$

$$P_m(\text{out}) = \frac{V_t E_f}{x_s} \sin 90°$$

$$0.5 = 3 \times \frac{6.35 \times E_f}{35}$$

$$E_f = 0.919 \text{ kV or } 1.59 \text{ kV (line)}$$

8.9 *A 1000 kVA, 6.6 kV, 3-phase star-connected synchronous generator has a synchronous reactance of 25 Ω per phase. It supplies full-load current at 0.8 lagging pf and a rated terminal voltage. Compute the terminal voltage for the same excitation when the generator supplies full-load current at 0.8 leading pf.*

Solution

$$I_a(\text{rated}) = \frac{1,000}{\sqrt{3} \times 6.6} = 87.5 \text{ A}$$

$$V_t = \frac{6,600}{\sqrt{3}} = 3,810.6 \text{ V}$$

Operation at 0.8 lagging pf rated terminal voltage

$$\bar{E}_f = V_t + jI_a x_s$$
$$= 3,810.6 + j\, 87.5(0.8 - j\, 0.6)\, 25$$
$$= 5,123 + j\, 1,750 \qquad \therefore \qquad E_f = 5,413.7 \text{ V}$$

Operation at 0.8 leading pf excitation remaining unchanged. From the phasor diagram of Fig. P8.9

$$\frac{5,413.7}{\sin 53.1°} = \frac{2,187.5}{\sin \delta}$$

or $$\sin \delta = 0.323$$

$$\delta = 18.8°$$

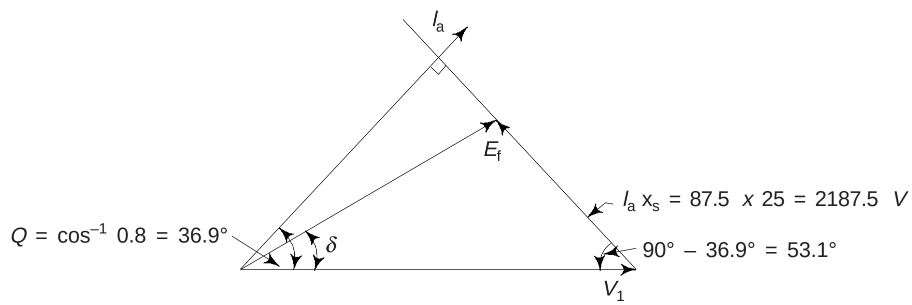


Fig. **P8.9**

$$180° - 18.8° - 53.1° = 108.1°$$

$$\frac{V_t}{\sin 108.1°} = \frac{2,187.5}{0.323}$$

or $\quad V_t = 6,437.3$ V or 11.15 kV (line)

8.10 *A 750 kW, 11 kV, 3-phase, star-connected synchronous motor has a synchronous reactance of 35 Ω/phase and negligible resistance. Determine the excitation emf per phase when the motor is operating on full-load at 0.8 pf leading. Its efficiency under this condition is 93%.*

Solution

$$\eta = 0.93$$

$$\text{Input} = \frac{750}{0.93} = 806.45 \text{ kW}$$

$$I_a = \frac{806.45}{\sqrt{3} \times 11 \times 0.8} = 52.9 \text{ A, 0.8 pf leading}$$

$$V_t = \frac{11,000}{\sqrt{3}} = 6,351 \text{ V}$$

$$E_f = V_t - j\, I_a x_s$$

$$E_f = 6.351 - j\ 52.9(0.8 + j\ 0.6) \times 35$$

$$= 7,462 - j\ 1,481$$

or $\quad E_f = 7,607.5$ V or 13.18 kV (line)

8.11 *A synchronous generator having synchronous reactance of 1.0 pu is connected to infinite busbars of 1.0 pu voltage through two parallel lines each of 0.5 pu reactance.*

(a) Calculate the generator excitation, terminal voltage and power output when it delivers rated current (1.0 pu) at unity power factor at its terminals. What active and reactive power are delivered to the infinite busbars?

(b) Calculate the generator excitation and terminal voltage when the generator is delivering zero active power and 0.5 pu lagging reactive power to the infinite busbars.

(c) With one line disconnected, can the generator deliver the same active power to the infinite busbars at the same excitation as in part (a)? Explain.

Solution

(a) The circuit model of the system is drawn in Fig. P8.11(a) and its phasor diagram (upf) in Fig. P8.11(b)

$$V_t = \sqrt{1-(0.25)^2} = 0.968 \text{ pu}$$

$$E_f = V\sqrt{V_t^2 + (1)^2} = 1.39 \text{ pu}$$

$$\delta_1 = \sin^{-1} 0.25 = 14.48°$$

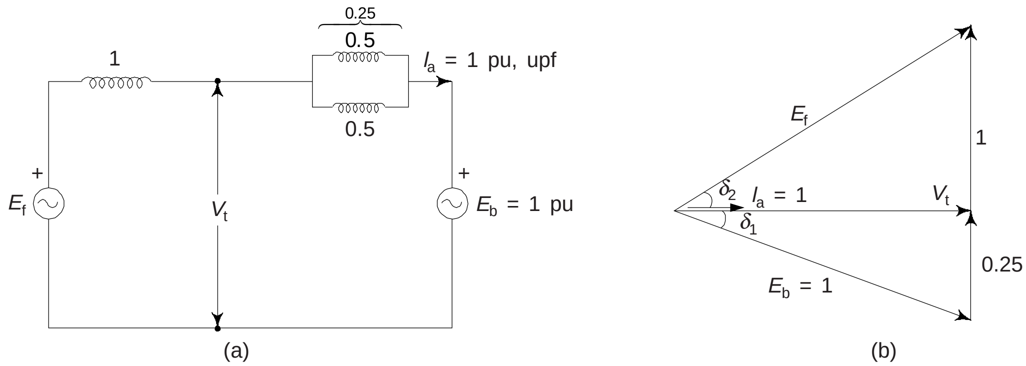


Fig. P8.11

Generator power output = 0.968 × 1 = 0.968 pu

Active power delivered to busbars = 1 × 1 × cos δ_1 = 0.968 pu

Reactive power delivered to busbars = –1 × 1 × sin δ_1

= – 0.25 pu (negative because current lags voltage)

(b) The phasor diagram is drawn in Fig. P8.11(c) from which it follows that

$$E_f = 1.625 \text{ V}, \quad V_t = 1.125 \text{ pu}$$

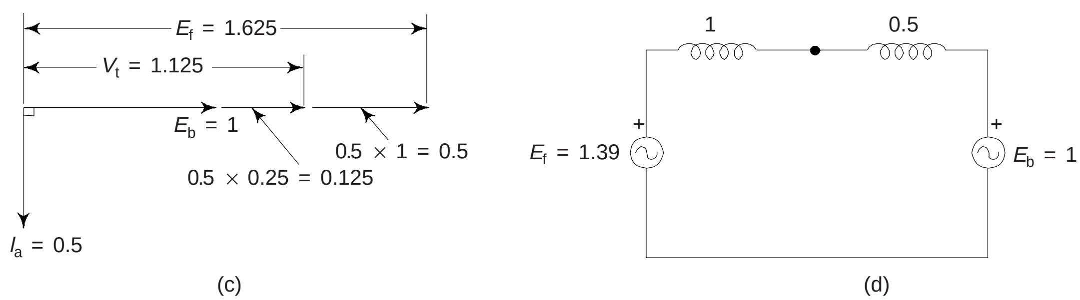


Fig. P8.11

(c) The circuit model now modifies to that drawn in Fig. P8.11(d).

Maximum power that can be delivered = (1 × 1.39)/1.5 = 0.926 pu. The power (0.968) as in part (a) cannot be delivered as this is more than the maximum power that can be delivered (0.926).

Therefore, the answer is *no*.

8.12 *Consider a synchronous generator-motor set whose data is given below.*

Generator: 1200 kVA, 3-phase, 3.3 kV, 2-pole, 50 Hz star-connected, $x_s = 4.55\ \Omega/\text{ph}$.
Motor: 1000 kW, 3-phase, 3.3 kV, 24-pole, 50 Hz star-connected, $x_s = 3.24\ \Omega/\text{ph}$.

(a) That set is operating at rated terminal voltage and frequency with the motor drawing 800 kW at upf. Compute the excitation emfs of both the machines. With the excitation emfs held fixed at these values, what maximum torque can the motor supply? Also determine the armature current, terminal voltage and power factor under this condition.

(b) The motor shaft load is now gradually increased while the field currents of both the generator and motor are continuously adjusted so as to maintain the rated terminal voltage and upf operation. What maximum torque can the motor now deliver without losing synchronism?

Solution

(a) The circuit model of the set is drawn in Fig. P8.12(a).

$$V_t = \frac{3,300}{\sqrt{3}} = 1,905 \text{ V}$$

$$I_a = \frac{800}{\sqrt{3} \times 3.3 \times 1} = 140 \text{ A upf}$$

or

$$\bar{E}_{fg} = 1,905 + j\,140 \times 4.55 = 1,905 + j\,637$$

$$E_{fg} = 2,008.7 \text{ V} \quad \text{or} \quad 3,479 \text{ V (line)}$$

$$\bar{E}_{fm} = 1,905 - j\,140 \times 3.24 = 1,905 - j\,453.6$$

$$E_{fm} = 1,958.3 \text{ V} \quad \text{or} \quad 3,392 \text{ V (line)}$$

$$P_m(\max) = 3 \times \frac{2,008.7 \times 1,958.3}{(4.55 + 3.24)} = 1,514.9 \text{ kW}$$

$$n_{sm} = \frac{120 \times 50}{24} = 250 \text{ rpm}$$

$$\omega_{sm} = \frac{2\pi \times 250}{60} = 26.18 \text{ rad/s}$$

$\therefore$

$$T_m(\max) = \frac{1,514.9 \times 1,000}{26.18} = 57.86 \times 10^3 \text{ Nm}$$

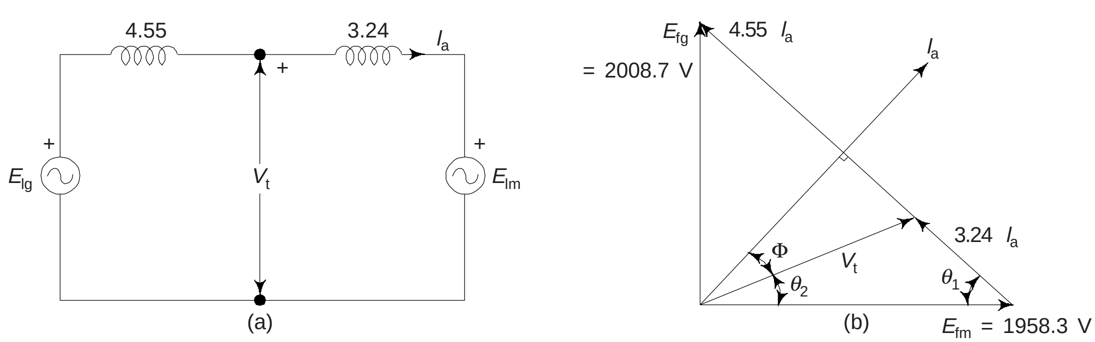


Fig. P8.12

From the phasor diagram of Fig. P8.10(b)

$$(4.55 + 3.24)\ I_a = \sqrt{(2,008.7)^2 + (1,9583)^2}$$

$$I_a = 360 \text{ A}$$

$$\theta_1 = \tan^{-1}\frac{2,008.7}{1,958.3} = 45.73°$$

$$V_t = [(1,958.3)^2 + (3.24 \times 360)^2 - 2 \times 1,958.3 \times (3.24 \times 360) \cos 45.73°]^{0.5}$$
$$= 1,416.5 \text{ V}$$

$$\frac{3.24 \times 360}{\sin\theta_2} = \frac{1,416.5}{\sin 45.73°}$$

$$\sin\theta_2 = 0.5896$$

$$\theta_2 = 36.13°$$

$$\phi = 90° - 45.73° - 36.13 = 8.14°$$

$$\cos\phi = 0.99 \text{ leading}$$

(b) Under these conditions the phasor diagram for maximum torque (power) is drawn in Fig. P8.12(c) where

$$\theta_1 + \theta_2 = 90°$$

$$\tan^{-1}\frac{3.34 I_a}{1,905} + \tan^{-1}\frac{4.55 I_a}{1,905} = 90° \quad (\text{Find } I_a)$$

$$I_a = 500 \text{ (It gives } \theta_1 + \theta_2 = 90.44°)$$

$$P_m(\max) = 3 \times 1,905 \times 500 = 2857.5 \text{ kW}$$

$$T_m(\max) = \frac{2,857.5 \times 1,000}{26.18} = 109.15 \times 10^3 \text{ Nm}$$

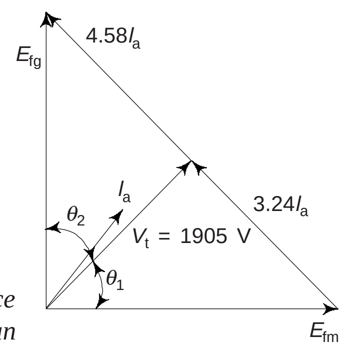


Fig. P8.12(c)

8.13 *A 2500 V, 3-phase, star-connected motor has a synchronous reactance of 5 Ω per phase. The motor input is 1000 kW at rated voltage and an excitation emf of 3600 V (line). Calculate the line current and power factor.*

Solution

$$V_t = \frac{2,500}{\sqrt{3}} = 1,443.4 \text{ V}$$

$$E_f = \frac{3,600}{\sqrt{3}} = 2,078.5 \text{ V}$$

$$P_e = \frac{1,000}{\sqrt{3}} = 333.3 \text{ kW}$$

$$\frac{P_e x_s}{V_t} = \frac{333.3 \times 1,000 \times 5}{1,443.4} = 1154.6$$

With reference to the geometry of the phasor diagram of Fig. 8.11

$$OQ = \sqrt{(2,078.5)^2 - (1,154.6)^2} = 1,728.3$$

$$MQ = 1{,}728.3 - 1{,}443.4 = 284.9$$

$$PM = \sqrt{(1{,}154.6)^2 + (284.9)^2} = 1{,}189.2 = I_a x_s = 5I_a$$

or $$I_a = 237.8 \text{ A}$$

$$\theta = \tan^{-1} \frac{1{,}154.6}{284.9} = 76.14°$$

$$\phi = 90° - \theta = 13.86°$$

$$\cos \phi = 0.97 \text{ leading}$$

8.14 *Repeat Prob. 8.13 considering a motor resistance per phase of 0.1 Ω.*

Solution

$$V_t = 1{,}443.4 \text{ V}$$

$$E_f = 2{,}078.5 \text{ V}$$

$$z_s = 0.1 + j\,5 = 5.00 \angle 88.85°$$

$$\alpha = 90° - 88.85 = 1.15°$$

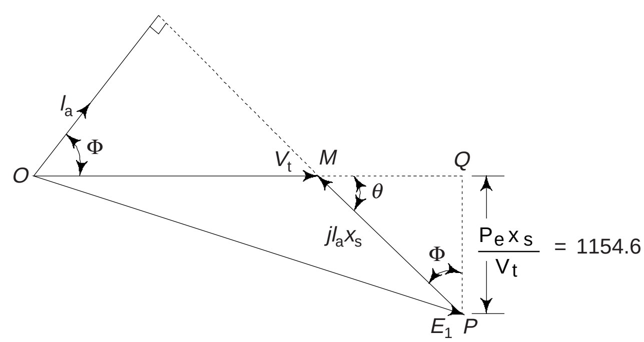


Fig. P8.14

For motoring operation

$$P_e(\text{in}) = \frac{V_t^2 r_a}{z_s^2} + \frac{V_t E_f}{z_s} \sin(\delta - \alpha)$$

$$\frac{1{,}000 \times 1{,}000}{3} = \frac{(1{,}443.4)^2 \times 0.1}{(5)^2} + \frac{1{,}443.4 \times 2{,}078.5}{5} \sin(\delta - 1.15°)$$

$$\frac{1}{3} = 0.00833 + 0.6 \sin(\delta - 1.15°)$$

$$\delta = 33.95°$$

$$Q_e(\text{in}) = \frac{V_t^2 x_s}{z_s^2} - \frac{V_t E_f}{z_s} \cos(\delta - \alpha)$$

$$= \frac{(1{,}443.4)^2 \times 5}{(5)^2} + \frac{(1{,}443.4 \times 2{,}078.5)}{5} \cos 32.8°$$

$$= -87.68 \text{ kVAR}$$

$$\cos\,\phi = \cos\,\tan^{-1}\,\frac{87.68}{333.3} = 0.967 \text{ leading}$$

$$S_e = \sqrt{P_e^2 + Q_e^2} = 344.6 \text{ kVA}$$

$$3 \times 344.6 = \sqrt{3} \times 2.5 \times I_a$$

or $$I_a = 238.7 \text{ A}$$

Remark Notice that consideration of the resistance has hardly made any difference to the result. Resistance can therefore be neglected except for calculation of efficiency.

8.15 *A 20 MVA, 11 kV, 3-phase, delta-connected synchronous motor has a synchronous impedance of 15 Ω/phase. Windage, friction and iron losses amount to 1200 kW.*

(a) *Find the value of the unity power factor current drawn by the motor at a shaft load of 15 MW. What is the excitation emf under this condition?*

(b) *If the excitation emf is adjusted to 15.5 kV (line) and the shaft load is adjusted so that the motor draws upf current, find the motor output (net).*

Solution

(a) $V_t = \dfrac{11}{\sqrt{3}} = 6.35$ kV

$$x_s(\text{eqv. star}) = \frac{15}{3} = 5\ \Omega$$

$$P_m = 15 + 1.2 = 16.2 \text{ MW(3-phase)}$$

$$P_e(\text{in}) = P_m(\text{out}) = 16.2 \text{ MW(3-phase)}$$

$$I_a = \frac{16.2}{\sqrt{3} \times 11 \times 1} = 0.85 \text{ kA (upf)}$$

The phasor diagram is drawn in Fig. P8.15.

$$\bar{E}_f = 6.35 - j\,5 \times 0.85$$

$$= 6.35 - j\,4.25$$

$$E_f = 7.64 \text{ kV} \quad \text{or} \quad 13.23 \text{ kV (line)}$$

(b) $E_f = \dfrac{15.5}{\sqrt{3}} = 8.95$ kV

$$I_a x_s = \sqrt{(8.95)^2 - (6.35)^2} = 6.31 \text{ kV}$$

$$I_a = \frac{6.31}{5} = 126 \text{ kA}$$

$$P_e(\text{in}) = P_m(\text{out})\Big|_{\text{gross}}$$

$$= \sqrt{3} \times 11 \times 126 = 24 \text{ MW (3-phase)}$$

$$P_m(\text{out})\Big|_{\text{net}} = 24 - 1.2 = 22.8 \text{ MW.}$$

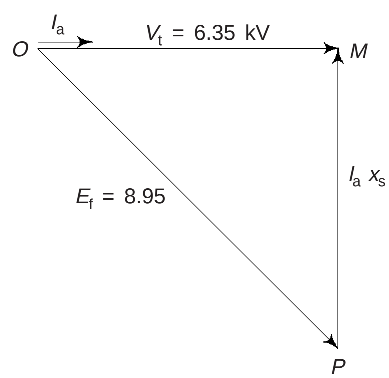


Fig. P8.15

8.16 *A 600 V, 6-pole, 3-phase, 50 Hz, star-connected synchronous motor has a resistance and synchronous reactance of 0.4 Ω and 7 Ω respectively. It takes a current of 15 A at upf when operating with a certain field current. With the field current remaining constant, the load torque is increased until the motor draws a current of 50 A. Find the torque (gross) developed and the new power factor.*

Solution

$$V_t = \frac{600}{\sqrt{3}} = 346.4 \text{ V}$$

$$z_s = 0.4 + j\,7 = 7.011\ \angle 86.7°$$

$$I_a = 15\ \angle 0°\ \text{A}$$

$$\bar{E}_f = 364.4 - 15\,(0.4 + j\,7)$$

$$= 340.4 - j\,105$$

$$E_f = 356.2 \text{ V} \quad \text{or} \quad 617 \text{ V (line)}$$

Now

$$I_a = 50 \text{ A}$$

$$I_a z_s = 50 \times 7.011 = 350.6 \text{ V}$$

The phasor diagram for these conditions is drawn in Fig. P8.16 from which it follows that:

$$(346.4)^2 + (350.6)^2 - 2 \times 346.4 \times 350.6 \cos\ \beta = (356.2)^2$$

$$\beta = 61.46°$$

$$\phi = 90° - 61.46° - 3.3° = 25.24°$$

$$P_e(\text{in}) = \sqrt{3} \times 600 \times 50 \cos 25.24°$$

$$= 47 \text{ kW (3-phase)}$$

$$3 I_a^2 r_a = 3 \times (50)^2 \times 0.4 = 3 \text{ kW}$$

$$P_m(\text{out})\Big|_{\text{gross}} = 47 - 3 = 44 \text{ kW}$$

$$n_s = 1{,}000 \text{ rpm}$$

$$\omega_s = \frac{2\pi \times 1{,}000}{60} = 104.72 \text{ rad/s}$$

$$T_m(\text{developed}) = \frac{44 \times 1{,}000}{104.72} = 420.2 \text{ Nm}$$

$$\text{pf} = \cos 25.24° = 0.9 \text{ lagging}$$

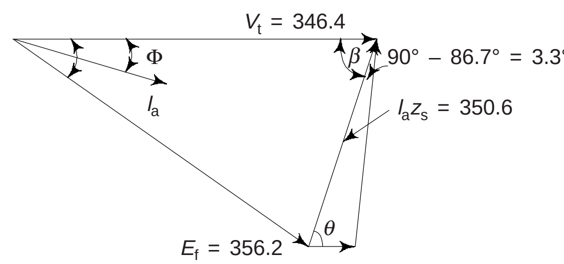


Fig. P8.16

8.17 *A 500 V, 3-phase, mesh-connected motor has an excitation emf of 600 V. The motor synchronous impedance is (0.4 + j 5) Ω while the windage, friction and iron losses are 1200 W. What is the maximum power output that it can deliver? What is the corresponding line current, pf and motor efficiency?*

Solution

$$V_t = \frac{500}{\sqrt{3}} = 288.7 \text{ V}$$

$$E_f = \frac{600}{\sqrt{3}} = 346.4 \text{ V}$$

$$z_s(\text{eqv. star}) = \frac{1}{3}\,(0.4 + j\,5) = 0.133 + j\,1.67 = 1.675\ \angle 85.44°$$

From Eq. (8.19)

$$P_m(\text{out, gross})\Big|_{\max} = -\frac{E_f^2 r_a}{z_s^2} + \frac{E_f V_t}{z_s}$$

$$= -\frac{(346.4)^2 \times 0.133}{(1.673)^2} + \frac{346.4 \times 288.7}{1.675}$$

$$= 54 \text{ kW (per phase)}$$

$$P_m(\text{out, net})\Big|_{\max} = 54 \times 3 - 1.2$$

$$= 160.8 \text{ kW (3-phase)}$$

For max power output

$$\delta = \theta = 85.44°$$

$$I_a = \frac{V_t \angle 0 - E_f \angle -\delta}{z_s \angle\theta}$$

$$= \frac{288.7 - 346.4\angle -85.44°}{1.675\angle 85.44°}$$

$$= 172.4 \angle -85.44° - 206.8 \angle -170.9°$$

$$= 217.9 - j\,139.14$$

or

$$I_a = 258.5 \angle -32.56°$$

$$I_a = 258.5 \text{ A}$$

$$\text{pf} = \cos 32.56° = 0.842 \text{ lagging}$$

$$P_e(\text{in}) = \sqrt{3} \times 500 \times 258.5 \times 0.842$$

$$= 188.49 \text{ kW}$$

$$\eta = \frac{160.8}{188.49} = 85.3\%$$

8.18 *A 3-phase synchronous generator has a direct-axis synchronous reactance of 0.8 pu and a quadrature-axis synchronous reactance of 0.5 pu. The generator is supplying full-load at 0.8 lagging pf at 1.0 pu terminal voltage. Calculate the power angle and the no-load voltage if the excitation remains unchanged.*

Solution

$$V_t = 1.0 \text{ pu}$$

$$I_a = 1.0 \text{ pu, } 0.8 \text{ pf lagging}$$

$$\phi = \cos^{-1} 0.8 = 36.9°$$

$$x_d = 0.8 \text{ pu} \qquad x_q = 0.5 \text{ pu}$$

From Eq. (8.21)

$$\tan\psi = \frac{V_t \sin\phi + I_a x_q}{V_t \cos\phi + I_a x_a}$$

$$= \frac{1 \times 0.6 + 1 \times 0.5}{1 \times 0.8 + 0} = 1.375$$

or $\psi = 54°$

Power angle, $\delta = \psi - \phi = 54° - 36.9°$

$= 17.1°$

No-load voltage (Eq. (8.23))

$$E_f = V_t \cos \delta + I_d x_d$$
$$= V_t \cos \delta + (I_a \sin \psi) x_d$$
$$= 1 \cos 17.1° + (1 \times \sin 54°) \times 0.8$$
$$= 1.6 \text{ pu}$$

8.19 *A 3.5 MVA, slow-speed, 3-phase synchronous generator rated at 6.6 kV has 32 poles. Its direct-and quadrature-axis synchronous reactances as measured by the slip test are 9.6 and 6 Ω respectively. Neglecting armature resistance, determine the regulation and the excitation emf needed to maintain 6.6 kV at the terminals when supplying a load of 2.5 MW at 0.8 pf lagging. What maximum power can the generator supply at the rated terminal voltage, if the field becomes open-circuited.*

Solution

$$x_d = 9.6 \ \Omega \qquad x_q = 6 \ \Omega$$

$$V_t = \frac{6.6}{\sqrt{3}} = 3.81 \text{ kV}$$

$$I_a(\text{rated}) = \frac{3.5 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 306.17 \text{ A}$$

I_a at 2.5 MW at 0.8 pf is

$$= \frac{2.5 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3 \times 0.8} = 273.4 \text{ A}$$

$$\phi = \cos^{-1}(0.8) = 36.9° \text{ lag}$$

From Eq. 8.21,

$$\tan \psi = \frac{V_t \sin\phi + I_a x_q}{V_t \cos\phi + I_a r_a}$$

$$= \frac{3{,}810 \times 0.6 + 273.4 \times 6}{3{,}810 \times 0.8}$$

$$= 129$$

or $\psi = 52.2°, \quad \delta = \psi - \phi = 52.2° - 36.9° = 15.3°$

From Eq. 8.23,

$$E_f = V_t \cos \delta + I_d x_d$$
$$= 3{,}810 \cos 15.3° + (I_a \sin \psi)\, x_d$$
$$= 3{,}810 \cos 15.3° + 273.4 \times \sin 52.2 \times 9.6$$
$$= 5{,}749 \text{ V or } 9.96 \text{ kV (line)}$$

$$\text{Regulation} = \frac{5{,}749 - 3{,}810}{3{,}810} = 50.9\%$$

From Eq. 8.24,

$$P_e = \frac{E_f V_t}{x_d} \sin \delta + V_t^2 \left(\frac{x_d - x_q}{2x_d x_q} \right) \sin 2\delta$$

Given $E_f = 0$

$\therefore$ $$P_e = V_t^2 \left(\frac{x_d - x_q}{2x_d x_q} \right) \sin 2\delta$$

$$P_{e,max} = V_b^2 \left(\frac{x_d - x_q}{2x_d x_q} \right)$$

$$= (3.81)^2 \times \left(\frac{9.6 - 6}{2 \times 9.6 \times 6} \right)$$

$$= 0.454 \text{ MW}$$

8.20 *A salient-pole synchronous motor has x_d = 0.85 pu. It is connected to busbars of 1.0 pu voltage, while its excitation is adjusted to 1.2 pu. Calculate the maximum power output the motor can supply without loss of synchronism. Compute the minimum pu excitation that is necessary for the machine to stay in synchronism while supplying the full-load torque, (i.e., 1.0 pu power).*

Solution

$$P_m = \frac{E_f V_t}{x_d} \sin \delta + V_t^2 \frac{x_d \; x_q}{2x_d x_q} \sin 2\delta$$

$$P_m = \frac{1.2}{0.85} \sin \delta + 1 \times \frac{0.3}{2 \times 0.85 \times 0.55} \sin 2\delta$$

$$P_m = 1.41 \sin \delta + 0.32 \sin 2\delta$$

$$\frac{dP_m}{d\delta} = 1.41 \cos \delta + 0.64 \cos 2\delta = 0$$

By trial

$$\delta = 70°$$

$$P_m(\text{max}) = 1.41 \sin 70° + 0.32 \sin 140°$$

$$= 1.53 \text{ pu}$$

When excitation is variable

$$P_m = \frac{E_f}{0.85} \sin \delta + \frac{0.3}{2 \times 0.85 \times 0.55} \sin 2\delta$$

$$P_m = 1.176 \; E_f \sin \delta + 0.32 \sin 2\delta$$

$$1.0 = 1.176 \; E_f \sin \delta + 0.32 \sin 2\delta \qquad \text{(i)}$$

For maximum power output

$$1.176 \; E_f \cos \delta + 0.64 \cos 2\delta = 0 \qquad \text{(ii)}$$

Solution of Eqs (i) and (ii) gives

$$\delta = 63°$$

$$E_f(\text{min}) = 0.705 \text{ pu}$$

8.21 *From the phasor diagram of the salient-pole synchronous machine given in Fig. P8.73, prove that*

$$\tan\ \delta = \frac{I_a x_q \cos\phi - I_a r_a \sin\phi}{V_1 - I_a x_q \sin\phi - I_a r_a \cos\phi}$$

Solution

$$OG = V_t - I_a x_q \sin\ \phi - I_a r_a \cos\ \phi$$
$$GC = I_a x_q \cos\ \phi\ - I_a r_a \sin\ \phi$$

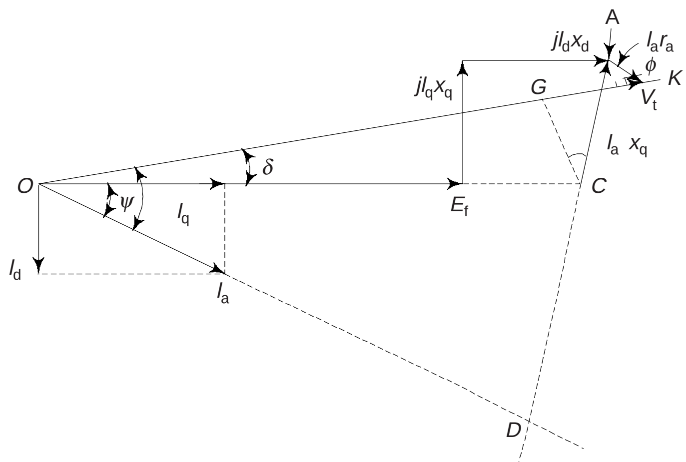


Fig. P8.21

Hence

$$\tan\ \delta = \frac{GC}{OG} = \frac{I_a x_q \cos\phi - I_a r_a \sin\phi}{V_t - I_a x_q \sin\phi - I_a r_a \cos\phi};\ \phi \text{ lagging}$$

For the generating case

$$\tan\ \delta = \frac{I_a x_q \cos\phi - I_a r_a \sin\phi}{V_t - I_a x_q \sin\phi - I_a r_a \cos\phi};\ \phi \text{ lagging}$$

8.22 *Two star-connected generators are connected in parallel and supply a balanced load of 1500 kVA at 11 kV line voltage and 0.8 lagging power factor. The synchronous reactances of the two machines respectively are 35 Ω and 40 Ω. The prime-mover governors of the two machines are adjusted so as to equally share the power load. The phase current in one machine is 43 A, at a lagging power factor. Calculate:*

(a) the phase current in the second machine

(b) the induced emf of each machine

(c) the power factor at which each machine operates.

Solution

The circuit model of the system is drawn in Fig. P8.22.

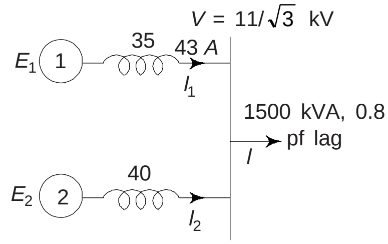


Fig. P8.22

$$P = 1{,}500 \times 0.8 = 1{,}200 \text{ kW}$$

$$P_1 = P_2 = 600 \text{ kW}$$

$$\sqrt{3} \times 11 \times 43 \cos \phi_1 = 600$$

$$\cos \phi_1 = 0.73 \text{ lag}; \qquad \phi_1 = 42.9°$$

$$\bar{I}_1 = 31.5 - j\,29.3 \text{ A}$$

$$I = \frac{1,500}{\sqrt{3} \times 11} = 78.7 \text{ A}$$

$$\cos \phi = 0.8 \text{ lag}, \qquad \phi = 36.9°$$

$$\bar{I} = 63 - j\,47.2$$

$$\therefore \qquad \bar{I}_2 + \bar{I} - \bar{I}_1 = (63 - j\,47.2) - (31 - 5 - j\,29.3)$$

$$= 31.5 - j\,17.9 = 36.2 \angle -29.6°$$

$$I_2 = 36.2 \text{ A} \qquad \cos \phi_2 = 0.87 \text{ lag}$$

$$\bar{E}_1 = \frac{11,000}{\sqrt{3}} + j\,35(31.5 - j\,29.3)$$

$$= 4,351 + 1,025 + j\,1,102 = 7,376 + j\,1,102$$

$$E_1 = 7,458 \text{ V} \qquad \text{or} \qquad 12.92 \text{ kV (line)}$$

$$\bar{E}_2 = \frac{11,000}{\sqrt{3}} + j\,40\,(31.5 - j\,17.9)$$

$$= 6,351 + 716 + j\,1,260 = 7,067 + j\,1,260$$

$$E_2 = 7,178 \text{ V} \qquad \text{or} \qquad 12.43 \text{ kV (line)}$$

8.23 *Calculate the synchronizing coefficient (in kW and Nm per mechanical degree) at full-load for a 1000 kVA, 0.8-pf(lag), 6.6 kV, 8-pole, star-connected cylindrical rotor generator of negligible resistance and synchronous reactance of 0.8 pu.*

Solution

Refer to the circuit diagram drawn in Fig. P8.23.

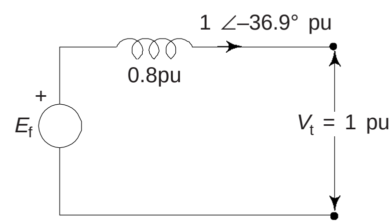


Fig. P8.23

$$\bar{E}_f = 1 + j\,0.8(0.8 - j\,0.6)$$

$$= 1.48 + j\,0.64 = 1.61 \angle 23.4°$$

$$E_f = 1.61 \text{ pu}$$

$$\delta_0 = 23.4°$$

$$P_e = \frac{V_t E_f}{x_s} \sin \delta$$

$$\left(\frac{\partial P_e}{\partial \delta}\right)_0 = \frac{V_t E_f}{x_s} \cos \delta_0$$

$$= \frac{1 \times 1.61}{0.8} \cos 23.4°$$

$$= 1.847 \text{ pu per elect rad}$$

$$1 \text{ pu} = 1,000 \text{ kW}$$

$$\text{Stiffness} = 1,847 \times \frac{\pi}{180} \text{ kW/elect deg.} = 32.55 \text{ kW/elect deg.}$$

$$1 \text{ elect degree} = \frac{8}{2} \text{ mech deg.}$$

$$\therefore \quad \text{Stiffness} = 32.25 \times 4 \text{ kW/mech deg.}$$

$$= 129 \text{ kW/mech deg.}$$

$$W_s = \frac{120 \times 50}{8} \times \frac{2\pi}{60} = 78.5 \text{ rad/s}$$

$$\text{Stiffness} = \frac{129 \times 10^3}{78.5}$$

$$= 1.64 \times 10^3 \text{ Nm/mech deg.}$$

CHAPTER 9: INDUCTION MACHINE

9.1 *A 4-pole wound-rotor induction motor is used as a frequency changer. The starter is connected to a 50 Hz, 3-phase supply. The load is connected to the rotor slip rings. What are the possible speeds at which the rotor can supply power to this load at 25 Hz? What would be the ratio of voltages at load terminals at these speeds? Assume the rotor impedance to be negligible.*

Solution

$$sf = 25$$

$$\therefore \quad s = \frac{25}{50} = 0.5$$

We can get 25 Hz voltage at slip = ± 0.5

$$\text{or} \quad \frac{n_s - n}{n_s} = \pm 0.5$$

$$\text{where} \quad n_s = \frac{120 \times 50}{4} = 1{,}500 \text{ rpm}$$

$$\therefore \quad n = 750 \text{ rpm for } s = +0.5$$

$$= 2{,}250 \text{ rpm for } s = -0.5$$

The rotor voltage at any slip s is

$$= s \times \text{rotor voltage at standstill}$$

Therefore the rotor voltage is the same for both 0.5 and –0.5 slip. Therefore,

$$\text{ratio of voltages} = 1$$

9.2 *A 6-pole, 50 Hz, 3-phase induction motor running on full-load develops a useful torque of 160 Nm and the rotor emf is absorbed to make 120 cycles/min. Calculate the net mechanical power developed. If the torque loss in windage and friction is 12 Nm, find the copper loss in the rotor windings, the input to the motor and efficiency. Given: starter losses = 200 W (inclusive of core loss).*

Solution

$$n_s = 1{,}000 \text{ rpm}, \qquad \omega_s = 104.7 \text{ rad/s}$$

$$f_2 = sf$$

$$\frac{120}{60} = s \times 50 \quad \text{or} \quad s = \frac{120}{60 \times 50} = 0.04$$

$$\omega = (1 - s)\,\omega_s = (1 - 0.04) \times 104.7 = 100.5 \text{ rad/s}$$

Mechanical power developed (net) = 160 × 100.5 = 16.08 kW

Mechanical power developed (gross) = (160 + 12) × 100.5

= 17.29 kW

$$3 I_2'^2 r_2' \left(\frac{1}{s} - 1\right) = P_m$$

$$\text{Rotor copper loss} = 17{,}290 \times \left(\frac{0.04}{1 - 0.04}\right) = 720 \text{ W}$$

Motor input = 17.29 + 0.72 + 0.8 = 18.81 kW

$$\eta = \frac{16.08}{18.81} = 85.5\%$$

9.3 *A 12-pole, 3-phase, 50 Hz induction motor draws 2.80 A and 110 kW under the blocked-rotor test. Find the starting torque when switched on direct to rated voltage and frequency supply. Assume the starter and rotor copper losses to be equal under the blocked-rotor test. What would be the starting torque if the motor is started by connecting the phase windings in star. (Try this part after studying Sec. 9.8)*

Solution

$$3 \times (280)^2\ (r_1 + r_2') = 110 \times 10^3$$

$$r_1 + r_2' = 0.468\ \Omega$$

$$r_1 = r_2' = 0.234\ \Omega \text{ (equivalent rotor)}$$

$$\omega_s = \frac{120 \times 50}{12} \times \frac{2\pi}{60} = 52.36 \text{ rad/s}$$

$$T_s = \frac{3}{\omega_s}\ (I_2'^2 r_2')$$

$$= \frac{3}{52.36}\ (280)^2 \times 0.234$$

$$= 1{,}051 \text{ Nm}$$

Phase windings connected in star

$$I_2' = \frac{280}{3}$$

$$r_2' = 3 \times 0.234$$

$$\therefore \quad T_s = \frac{3}{52.36} \times \left(\frac{280}{3}\right)^2 \times 3 \times 0.234 = \frac{1{,}051}{3} = 350 \text{ Nm}$$

9.4 *A 3.3 kV, 20-pole, 50 Hz, 3-phase, star-connected induction motor has a slip ring rotor of resistance 0.025 Ω and standstill reactance of 0.28 Ω per phase. The motor has a speed of 294 rpm when full-load torque is applied. Compute: (a) slip at maximum torque, and (b) ratio of maximum to full-load torque. Neglect stator impedance.*

Solution

(a) $S_{\max, T} = \dfrac{0.025}{0.28} = 0.09$

(b) $$T_{\max} = \frac{3}{\omega_s} \frac{0.5 V^2}{x_2'} \qquad \text{(i)}$$

$$T_{fl} = \frac{3}{\omega_s} \frac{V^2 (r_2'/s_{fl})}{(r_2'/s_{fl})^2 + x_2'^2} \qquad \text{(ii)}$$

$$n_s = \frac{120 \times 50}{20} = 300 \text{ rpm}; \qquad n = 294 \text{ rpm}$$

$$s_{fl} = \frac{300 - 294}{300} = 0.02$$

Dividing Eq. (i) by (ii)

$$\frac{T_{\max}}{T_{\mathrm{fl}}} = \frac{0.5[(r_2'/s_{\mathrm{fl}})^2 + x_2'^2]}{x_2'(r_2'/s_{\mathrm{fl}})}$$

$$= \frac{0.5[(0.025/0.02)^2 + (0.28)^2]}{0.28 \times (0.025/0.02)}$$

$$= 2.34$$

9.5 *An 8-pole, 3-phase, 50 Hz induction motor runs at a speed of 710 rpm with an input power of 35 kW. The starter copper loss at this operating condition is 1200 W while the rotational losses are 600 W. Find: (a) rotor copper loss, (b) gross torque developed, (c) gross mechanical power developed, and (d) net torque and mechanical power output.*

Solution

$$n_s = \frac{120 \times 50}{80} = 750 \text{ rpm} \qquad \omega_s = \frac{2\pi \times 750}{60} = 78.54 \text{ rad/s}$$

$$n = 710 \text{ rpm}$$

$$s = \frac{750 - 710}{750} = 0.053 \qquad \omega = (1 - 0.053) \times 78.54 = 74.38 \text{ rad/s}$$

Electrical power input = 35 kW

Stator copper loss = 12 kW

∴ Power across air-gap, $P_G = 35 - 1.2 = 33.8$ kW

(a) Rotor copper loss = $sP_G = 0.053 \times 33.8 = 1.79$ kW

(b) Torque (gross) in synchronous watts = 33.8×10^3 W

$$T(\text{gross}) = \frac{33.8 \times 10^3}{78.54} = 430.4 \text{ Nm}$$

(c) Mechanical power developed (gross) = $(1 - s)P_G$

$$= (1 - 0.053) \times 33.8 = 32 \text{ kW}$$

(d) Mechanical power output (net) = 32 – 0.6 = 31.4 kW

$$\text{Torque (net)} = \frac{31.4 \times 1{,}000}{74.38} = 422 \text{ Nm}$$

9.6 *A 7.5 kW, 440 V, 3-phase, star-connected, 50 Hz, 4-pole squirrel-cage induction motor develops full-load torque at a slip of 5% when operated at rated voltage and frequency. Rotational losses (core, windage and friction) are to be neglected. Motor impedance data is as follows:*

$$r_1 = 1.32\ \Omega$$

$$x_1 = x_2' = 1.46\ \Omega$$

$$x_m = 22.7\ \Omega$$

Determine the maximum motor torque at rated voltage and the slip at which it will occur. Also calculate the starting torque.

Solution

Taking the Thevenin equivalent in Fig. P9.6.

$$V_{TH} = \frac{254 \times 22.7}{|1.32 + j\,(1.46 + 22.7)|} = 238.3 \text{ V}$$

$$\bar{Z}_{TH} = \frac{j\,22.7(1.32 + j\,1.46)}{24.19\,\angle 86.87°} = 1.85\,\angle 51°$$

$$= 1.16 + j\,1.44 = R_1 + jX_1$$

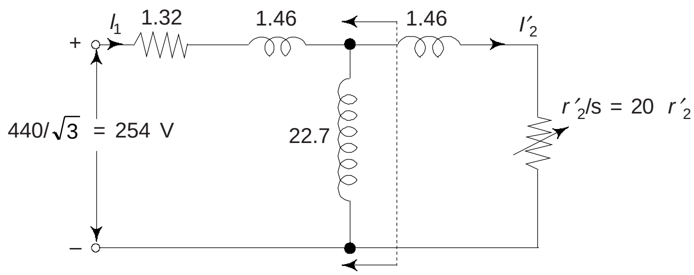


Fig. P9.6

Full-load output = 7.5 kW

$$\omega_s = \frac{1{,}500 \times 2\pi}{60} = 157.1 \text{ rad/s} \qquad \omega = 157.1\,(1 - 0.05) = 149.2 \text{ rad/s}$$

$$T(\text{full-load}) = \frac{7{,}500}{149.2} = 50.27 \text{ Nm}$$

Now,

$$T = \frac{3}{\omega_s} \frac{V_{TH}^2\,(r_2'/s)}{(R_1 + r_2'/s)^2 + (X_1 + x_2')^2} \tag{9.17}$$

Substituting the values corresponding to full-load

$$50.27 = \frac{3}{157.1} \frac{(238.3)^2 \times 20 \times r_2'}{(1.16 + 20r_2')^2 + 8.41}$$

$$(1.16 + 20r_2')^2 + 8.41 = 431.4r_2'$$

$$400r_2' + 46.4r_2' + 1.35 + 8.41 = 431.4r_2'$$

$$400r_2'^2 - 385r_2' + 9.76 = 0$$

Solving

$$r_2' = 0.026\ \Omega \quad \text{or} \quad 0.936\ \Omega$$

$$r_2' = 0.936\ \Omega \text{ (larger value is selected)}$$

Now,

$$T_{max} = \frac{3}{\omega_s} \frac{0.5\,V_{TH}^2}{R_1 + \sqrt{R_1^2 + (X_1 + x_2')^2}} \tag{9.18}$$

$$= \frac{3}{157.1} \frac{0.5 \times (238.3)^2}{1.16 + \sqrt{(1.16)^2 + (2.9)^2}}$$

$$= 126.6 \text{ Nm}$$

$$S_{\max,T} = \frac{r_2'}{\sqrt{R_1^2 + (X_1 + x_2')^2}} \tag{9.19}$$

$$= \frac{0.936}{\sqrt{(1.16)^2 + (2.9)^2}} = 0.3 \quad \text{or} \quad 30\%$$

$$T_{\text{start}} = \frac{3}{\omega_s} \frac{V_{TH}^2 r_2'}{(R_1 + r_2')^2 + (X_1 + x_2')^2} \tag{9.20}$$

$$= \frac{3}{157.1} \frac{(238.3)^2 \times 0.936}{(1.16 + 0.936)^2 + (1.44 + 1.46)^2}$$

$$= 79.3 \text{ Nm}$$

9.7 *The motor of Prob 9.6 is fed through a feeder from 440 V, 50 Hz mains. The feeder has an impedance of (1.8 + j 1.2) Ω/phase. Find the maximum torque that the motor can deliver and corresponding slip, stator current and terminal voltage.*

Solution

The per phase circuit model of the motor and feeder is drawn in Fig. P9.7. Taking the Thevenin equivalent,

$$V_{TH} = \frac{254 \times 22.7}{|(1.8 + 1.32) + j\,(1.2 + 1.46 + 22.7)|}$$

$$= 225.67 \text{ V}$$

$$Z_{TH} = \frac{j\,22.7\,(3.12 + j\,2.66)}{28.55\,\angle 83^\circ}$$

$$= 3.64\ \angle 47.4^\circ = 2.46 + j\,2.67 = R_1 + jX_1$$

$$S_{\max,\,T} = \frac{r_2'}{\sqrt{R_1^2 + (X_1 + x_2')^2}} = \frac{0.936}{\sqrt{(2.46)^2 + (2.67 + 1.46)^2}} = 0.195$$

$$T_{\max} = \frac{3}{\omega_s} \frac{0.5 V_{TH}^2}{R_1 + \sqrt{R_1^2 + (X_1 + x_2')^2}} = \frac{3}{157.1} \frac{0.5 \times (225.67)^2}{2.46 + \sqrt{(2.46)^2 + (2.67 + 1.46)^2}}$$

$$= 66.9 \text{ Nm}$$

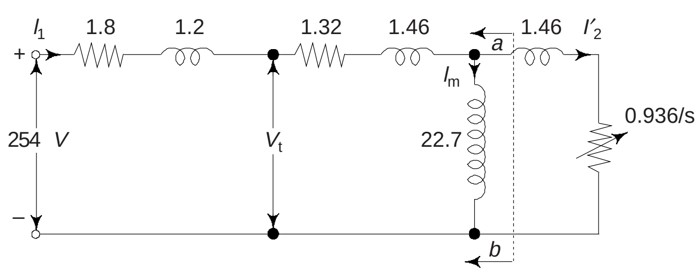


Fig. P9.7

Notice the reduction in T_{max}.

For the remaining part it is convenient to proceed by finding Z_f.

$$\frac{r_2'}{s} = \frac{0.936}{0.195} = 4.8$$

$$Z_f = \frac{j\,22.7\,(4.8 + j\,1.46)}{4.8 + j\,(1.46 + 22.7)}$$

$$= 4.63\ \angle 28.1° = 4.08 + j\ 2.18$$

$$Z(\text{total}) = (3.12 + 4.08) + j\ (2.66 + 2.18) = 8.68\ \angle 33.9°$$

$$I_1 = \frac{254\,\angle 0}{8.68\,\angle 33.9°} = 29.26\ \angle{-33.9}°\text{A}$$

$$\bar{V}_t = 254\ \angle 0° - 29.26\ \angle{-\,33.9}° \times (1.8 + j\ 1.2)$$

$$= 254 - 63.3\ \angle{-0.2}$$

$$= 254 - 63.3 + j\ 0.22$$

or $$V_t = 190.7\text{ V} \qquad \text{or} \qquad 330.3\text{ V (line)}$$

Notice that the voltage of the stator terminals is considerably reduced because of the voltage drop in feeder impedance.

9.8 *A 400 V, 3-phase, star-connected induction motor gave the following test results:*

No-load	*400 V*	*8.5 A*	*1,100 W*
Blocked-rotor	*180 V*	*45 A*	*5,700 W*

Determine the ohmic values of the components in the circuit model and calculate the line current and power factor when the motor is operating at 5% slip. The stator resistance per phase is 1.5 Ω and the standstill leakage reactance of the rotor winding referred to the stator is equal to that of the stator winding.

Solution

No-load test

$$y_0 = \frac{8.5}{400/\sqrt{3}} = 0.0368\ \mho$$

$$g_i = \frac{[1{,}100 - 3 \times (8.5)^2 \times 0.5]}{(400)^2} = 0.0062\ \mho$$

$$b_m = \sqrt{y_0^2 - g_i^2} = 0.0363\ \mho$$

Hence

$$x_m = 27.55\ \Omega$$

$$r_i = 161.3\ \Omega$$

Note: r_i here accounts for rotational losses.

Blocked-rotor test

$$Z = \frac{180/\sqrt{3}}{45} = 2.31\ \Omega$$

$$R = \frac{5{,}700/3}{(45)^2} = 0.938\ \Omega$$

$$X = \sqrt{(2.31)^2 - (0.94)^2} = 2.11\ \Omega$$

Now

$$R = r_1 + r_2'$$

$$0.938 = 0.5 + r_2'$$

$\therefore$ $$r_2' = 0.438$$

also,

$$x_1 = x_2' = \frac{2.11}{2} = 1.055\ \Omega$$

Performance calculations

$$s = 0.05$$

$$\frac{0.438}{S} = \frac{0.438}{0.05} = 8.76\ \Omega$$

In Fig. P9.8,

$$\frac{1}{\bar{Z}_F} = (0.0062 - j\ 0.0363) + \frac{1}{8.76 + j\,1.055}$$

$$= (0.0062 - j\ 0.0363) + (0.112 - j\ 0.0135)$$

$$= (0.118 - j\ 0.05) = 0.128\ \angle{-23°}$$

or $$\bar{Z}_f = 7.8\ \angle 23° = 7.18 + j\ 3.05$$

$$\bar{Z}\,(\text{total}) = (0.5 + j\ 1.055) + (7.18 + j\ 3.05)$$

$$= (7.68 + j\ 4.105) = 8.71\ \angle 28.1°$$

$$I_1 = \frac{231}{8.71} = 26.5\ \text{A}$$

$$\text{pf} = \cos 28.1° = 0.882 \text{ lagging}$$

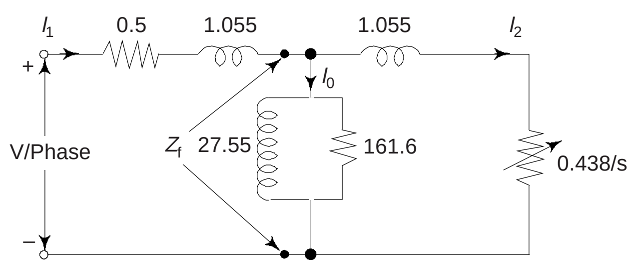


Fig. P9.8 *Circuit Model*

9.9 *A 15 kW, 415 V, 4-pole, 50 Hz delta-connected motor gave the following results on test (voltages and currents are in line values):*

No-load test	*415 V*	*10.5 A*	*1,510 W*
Blocked-rotor test	*105 V*	*28 A*	*2,040 W*

Using the approximate circuit model, determine:

(a) the line current and power factor for rated output,

(b) the maximum torque, and

(c) the starting torque and line current if the motor is started with the stator star-connected. Assume that the stator and rotor copper losses are equal at standstill.

Hint *Part (a) is best attempted by means of a circle diagram. For proceeding computationally from the circuit model, we have to compute the complete output-slip curve and then read the slip for rated output.*

Solution

$$OP_0 = 10.5 \text{ A}$$

$$\cos \phi_0 = \frac{1{,}510}{\sqrt{3} \times 415 \times 10.5} = 0.2$$

or
$$\phi_0 = 78.5°$$

$$OP_{SC} = \frac{28 \times 415}{105} = 110.7 \text{ A (at 415 V)}$$

$$\cos \phi_{SC} = \frac{2{,}040}{\sqrt{3} \times 105 \times 28} = 0.4$$

$$\phi_{SC} = 66.4°$$

$$\text{Rated output} = 15 \text{ kW} = \sqrt{3}\ V(I \cos \phi)_{\text{rated}}$$

$$(I \cos \phi)_{\text{rated}} = \frac{15{,}000}{\sqrt{3} \times 415} = 20.87 \text{ A}$$

(a) The circle diagram is drawn in Fig. P9.9. PP' is drawn parallel to the output line P_0P_{SC} at a vertical distance 20.87A above P_0P_{SC}. The point P pertains to this point (P' is the second but unacceptable solution—too large a current):
Then,

$$I_1 = OP = 30.5 \text{ A}$$
$$\text{pf} = \cos \phi = \cos 30° = 0.866 \text{ lagging}$$

(b) Torque line P_0F is drawn by locating F as the midpoint of $P_{SC}G$ (equal stator and rotor, losses). The maximum torque point is located by drawing a tangent to the circle parallel to P_0F. The maximum torque is given as

$$RS = 44 \text{ A}$$

$$T_{\max} = 3 \times \frac{415}{\sqrt{3}} \times 44 = 31{,}626 \text{ syn watts}$$

$$n_s = 1{,}500 \text{ rpm}, \qquad \omega_s = 157.1 \text{ rad/s}$$

$$T_{\max} = \frac{31{,}626}{157.1} = 201.3 \text{ Nm}$$

(c) If the motor is started to delta (this is the connection for which test data are given)

$$I_s(\text{delta}) = OP_{SC} = 110.7 \text{ A}$$
$$P_{SC}F = 22 \text{ A}$$

$$T_s(\text{delta}) = \frac{(\sqrt{3} \times 415 \times 22)}{157.1} = 100.7 \text{ Nm}$$

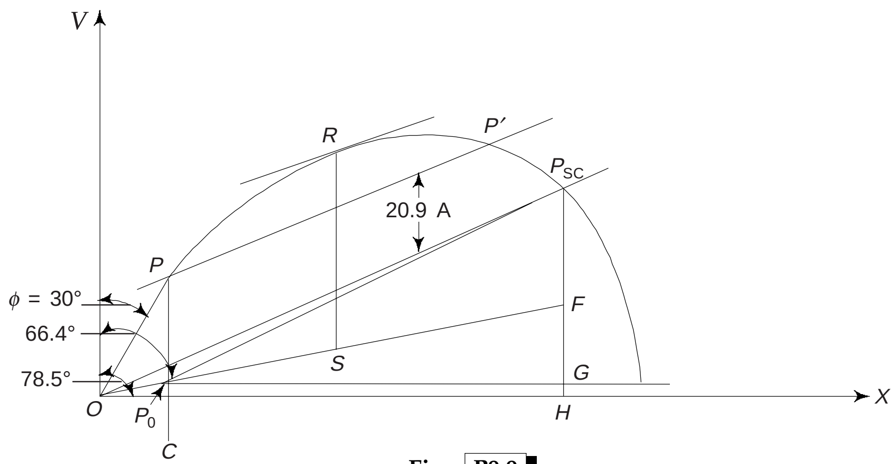


Fig. P9.9

Star connection

$$I_s(\text{star}) = \frac{110.7}{3} = 36.9 \text{ A}$$

$$T_s(\text{star}) = \frac{100.7}{3} = 33.57 \text{ Nm}$$

9.10 *A 400 V, 3-phase, 6-pole, 50 Hz induction motor gave the following test results:*

No-load	*400 V*	*8 V*	*0.16 power factor*
Blocked-rotor	*200 V*	*39 A*	*0.36 power factor*

Determine the mechanical output, torque and slip when the motor draws a current of 30 A from the mains. Assume the stator and rotor copper losses to be equal.

Solution

$$\cos\phi_0 = 0.16; \qquad \phi_0 = 80.8°$$
$$\cos\phi_{SC} = 0.36; \qquad \phi_{SC} = 69°$$

The circle diagram is drawn in Fig. P9.10. From the circle diagram we get the following results.

$$P_m = \sqrt{3} \times 400 \times PB(= 10.75 \text{ A})$$
$$= 7.45 \text{ kW}$$

$$\text{Slip } s = \frac{BC}{PC} = \frac{3.5 \text{ A}}{13.5 \text{ A}} = 0.26$$

$$\text{Torque } T = \frac{\sqrt{3} \times 400 \times PC\ (= 13.5 \text{ A})}{\omega_s}$$

and
$$\omega_s = \frac{120 \times 50}{6} \frac{2\pi}{60} = 104.7 \text{ rad/s}$$

∴
$$T = \frac{\sqrt{3} \times 400 \times 13.5}{104.7} = 89.33 \text{ Nm}$$

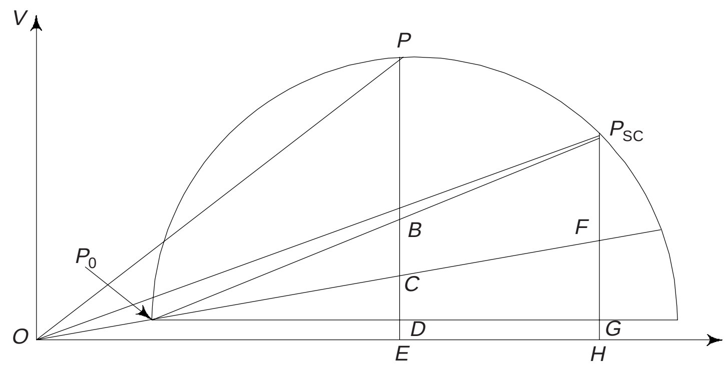


Fig. P9.10

9.11 *A 4-pole, 3-phase, 400 V, 50 Hz induction motor has the following parameters for its circuit model (referred to the stator side on equivalent-star basis)*

$$r_1 = 1.2 \ \Omega \qquad x_1 = 1.16 \ \Omega$$

$$r_2' = 0.4 \ \Omega \qquad x_2' = 1.16 \ \Omega$$

$$x_m = 35 \ \Omega$$

Rotational losses are 800 W.

(a) For a speed of 1,440 rpm, calculate the input current, power factor, net mechanical power and torque and efficiency.

(b) Calculate the maximum torque and the slip at which it occurs.

Solution

(a) $n_s = 1{,}500$ rpm $\quad n = 1{,}440$ rpm $\quad$ or $\quad \omega = \dfrac{2\pi \times 1{,}440}{60} = 150.8$ rad/s

$$s = \frac{60}{1{,}500} = 0.04$$

$$\frac{r_2'}{s} = \frac{0.4}{0.04} = 10 \ \Omega$$

In this part it is convenient to proceed by finding Z_f (Fig. P9.11), in order that the identity of the input current is preserved.

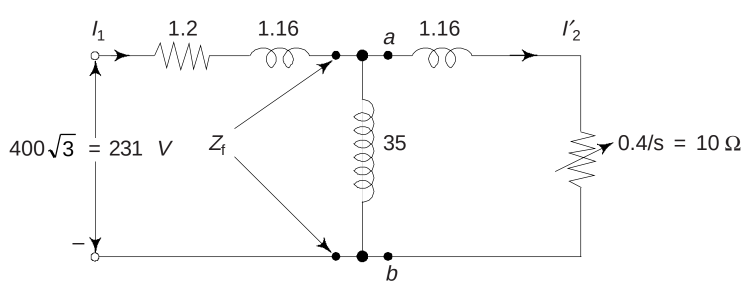


Fig. P9.11

$$\bar{Z}_f = \frac{j\,35\,(10 + j\,1.16)}{10 + j\,(35 + 1.16)}$$

$$= \frac{352.45\,\angle 96.6°}{37.51\,\angle 74.5°} = 9.4\ \angle 22.1° = 8.71 + j\,3.54$$

Total impedance,

$$\bar{Z}_i = 9.91 + j\,4.70$$

$$= 10.97\ \angle 25.4°$$

Stator current,

$$I_1 = \frac{231}{10.97} = 21.1\ \text{A}$$

$$\text{pf} = \cos 25.4° = 0.9 \text{ lagging}$$

Mechanical power output (gross) $= 3I_1^2\ R_{fi}\ (1 - s)$

$$= 3 \times (21.1)^2 \times 9.91\ (1 - 0.04) = 12.71\ \text{kW}$$

Rotational losses = 0.8 kW

Mechanical power output (net) = 12.71 – 0.8 = 11.91 kW

$$n = 1{,}440 \text{ rpm} \quad \text{or} \quad 150.8 \text{ rad/s}$$

$$\text{Torque (net)} = \frac{11.91 \times 1{,}000}{150.8} = 78.98\ \text{Nm}$$

$$\text{Power input} = \sqrt{3} \times 400 \times 21.1 \times 0.9 = 13.16\ \text{kW}$$

$$\eta = \frac{11.91}{13.16} = 90.5\%$$

(b) It will be necessary here to use Thevenin theorem. Finding the Thevenin equivalent to the left of *ab* in Fig. P9.9, we get

$$V_{TH} = \frac{231 \times 35}{|1.2 + j\,(35 + 1.16)|} = 223.5\ \text{V}$$

$$\bar{Z}_{TH} = \frac{j\,35\,(1.2 + j\,1.16)}{1.2 + j\,(35 + 1.16)} = 1.62\ \angle 45.9°$$

$$= 1.13 + j\,1.16 = R_1 + jX_1$$

$$S_{max,\ T} = \frac{r_2'}{\sqrt{R_1^2 + (X_1 + x_2')^2}}$$

$$= \frac{0.4}{\sqrt{(1.13)^2 + (1.16 + 1.16)^2}} = 0.155$$

$$n_s = 1{,}500 \text{ rpm}$$

$$\omega_s = 157.1 \text{ rad/s}$$

According to Eq. 9.10,

$$T_{max} = \frac{3}{\omega_s} \frac{0.5V_{TH}^2}{R_1 + \sqrt{R_1^2 + (X_1 + x_2')^2}}$$

$$= \frac{3}{157.1} \times \frac{0.5 \times (223.5)^2}{1.13 + \sqrt{(1.13)^2 + (1.16 + 1.16)^2}}$$

$$= 128.5 \text{ Nm}$$

9.12 *A 3-phase, 3.3 kV, 50 Hz, 10-pole, star-connected induction motor has a no-load magnetizing current of 45 A and a core loss of 3.5 kW. The stator and referred rotor standstill leakage impedances are respectively (0.2 + j 1.8) Ω/phase. The motor is supplied from 3.3 kV mains through a line of reactance 0.5 Ω/phase. Use the approximate circuit model:*

(a) The motor is running at 0.03 slip. Estimate the gross torque, stator current and power factor. Assume voltage at motor terminals to be 3.3 kV.

(b) Calculate the starting torque and current when the motor is switched on direct to line with voltage at the far end of the line being 3.3 kV.

Solution

(a)
$$\bar{Z}(s) = (0.2 + j\,1.8) + (0.45/0.03 + j\,1.8)$$
$$= 15.2 + j\,3.6 = 15.62\ \angle 13.3°\ \Omega$$
$$I_m = 45 \text{ A}$$
$$I_i = \frac{35}{3 \times 3.3/\sqrt{3}} = 6 \text{ A}$$
$$\bar{I}_0 = 6 - j\,45$$
$$\bar{I}_2' = \frac{(3.3/\sqrt{3}) \times 1{,}000}{15.62\ \angle 13.3°} = 122\ \angle -13.3° \text{ A}$$
$$= 118.7 - j\,28$$
$$\bar{I}_1 = (118.7 - j\,28) + (6 - j\,45)$$
$$= 124.7 - j\,73$$
$$I_1 = 144.4 \text{ A}$$
$$\text{pf} = 0.863 \text{ lagging}$$
$$n_s = \frac{120 \times 50}{10} = 600 \text{ rpm} \qquad \omega_s = \frac{2\pi \times 600}{60} = 62.83 \text{ rad/s}$$
$$\text{Torque (gross)} = \frac{3}{62.83} (122)^2 \times \frac{0.45}{0.03}$$
$$= 10{,}660 \text{ Nm}$$

(b) We shall neglect magnetizing current.

$$\bar{Z}(\text{total})|_{s=1} = j\,0.5 + (0.2 + j\,1.8) + (0.45 + j\,1.8)$$
$$= 0.65 + j\,4.1 = 4.15\ \angle 81°$$
$$I_s = \frac{(3.3/\sqrt{3}) \times 1{,}000}{4.15} = 459 \text{ A}$$
$$T_s = \frac{3}{62.83} (459)^2 \times 0.45 = 4{,}527 \text{ Nm}$$

9.13 *A 6-pole, 440 V, 3-phase, 50 Hz induction motor has the following parameters of its circuit model (referred to the stator or equivalent star basis):*

r_1 = 0.0 Ω (stator copper loss negligible); x_1 = 0.7 Ω

r_2' = 0.3 Ω x_2' = 0.7 Ω

x_m = 35 Ω

Rotational loss = 750 W

Calculate the net mechanical power output, stator current and power factor when the motor runs at a speed of 950 rpm.

Solution

The circuit model of the motor is drawn in Fig. P9.13(a) and its thevenin equivalent is given in Fig. P9.10(b).

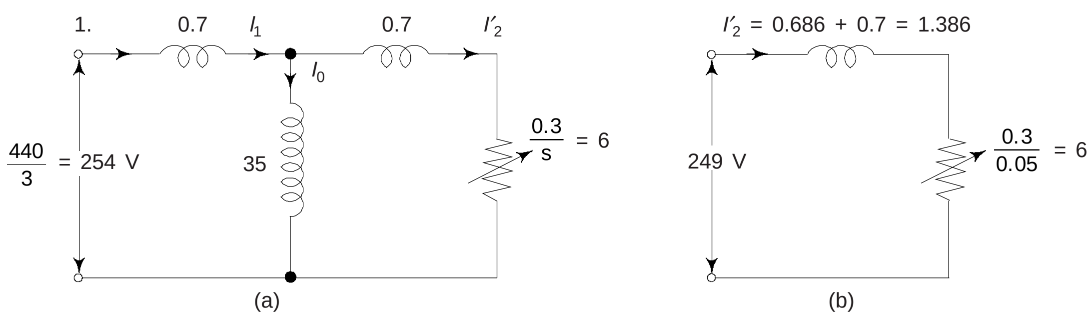


Fig. P9.13

$$s = \frac{1{,}000 - 950}{1{,}000} = 0.05$$

$$V_{TH} = \frac{254 \times 35}{35 + 0.7} = 249 \text{ V}$$

$$x_{TH} = \frac{35 \times 0.7}{35 + 0.7} = 0.686 \ \Omega$$

$$I_2' = \frac{249}{\sqrt{(1.386)^2 + (6)^2}} = 40.44 \text{ A}$$

$$P_m(\text{gross}) = \left(\frac{1}{s} - 1\right) \times 3 I_2'^2 r_2'$$

$$= \left(\frac{1}{0.05} - 1\right) \times 3 \times (40.44)^2 \times 0.3 = 27.96 \text{ kW}$$

$$P_m(\text{net}) = 27.96 - 0.75 = 27.21 \text{ kW}$$

Input current

$$\bar{Z}_f = \frac{j\,35\,(6 + j\,0.7)}{6 + j\,35.7}$$

$$\bar{Z}_f = \frac{j\,35\,(6 + j\,0.7)}{6 + j\,35.7}$$

$$= 5.84 \angle 16.2° = 5.61 + j\ 163$$

$$\bar{Z}\,(\text{total}) = 5.61 + j(0.7 + 1.63) = 5.61 + j\ 2.33$$

$$= 6.07 \angle 22.6°$$

$$I_1 = \frac{254}{6.07} = 41.8\ \text{A} \qquad \text{pf} = \cos 22.6° = 0.923 \text{ lagging}$$

9.14 *A 75 kW, 440 V, 3-phase, 6-pole, 50 Hz, wound-rotor induction motor has a full-load slip of 0.04 and a slip at maximum torque of 0.2 when operating at rated voltage and frequency with rotor winding short-circuited at the slip-rings. Assume the stator resistance and rotational losses to be negligible. Find:*

(a) Maximum torque
(b) Starting torque
(c) Full-load rotor copper loss.

The rotor resistance is now doubled by adding an external series resistance.
Determine:

(d) Slip at full-load
(e) Full-load torque
(f) Slip at maximum torque.

Solution

If the stator resistance and rotational losses are neglected, the circuit model on the Thevenin basis is as in Fig. P9.14.

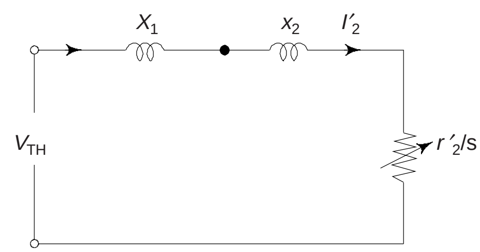


Fig. P9.14

$$S_{\max,\,T} = \frac{r_2'}{X_1 + x_2'} \qquad \text{(i)}$$

$$T_{fl} = \frac{3}{\omega_s} \frac{V_{TH}^2\,(r_2'/s_{fl})}{(r_2'/s_{fl})^2 + (X_1 + x_2')^2} \qquad \text{(ii)}$$

$$T_{\max} = \frac{3}{\omega_s} \frac{0.5 V_{TH}^2}{(X_1 + x_2')} \qquad \text{(iii)}$$

Dividing Eq. (iii) by Eq. (ii)

$$\frac{T_{\max}}{T_{fl}} = 0.5 \times \frac{(r_2'/s_{fl})^2\ (X_1 + x_2')}{r_2'\ (X_1 + x_2')}\ s_{fl}$$

$$= 0.5 \times \frac{s_{\max}^2\ T + s_{fl}^2}{s_{\max}\ T\ s_{fl}} \qquad \text{(iv)}$$

$$s_{fl} = 0.04$$

$$\omega_s = \frac{2\pi \times 1{,}000}{60} = 104.7 \text{ rad/s}$$

$$\omega = (1 - 0.04) \times 104.7 = 100.5 \text{ rad/s}$$

$$100.5 \times T_{fl} = 75{,}000 \text{ (losses negligible)}$$

$$\therefore \qquad T_{fl} = 746.2 \text{ Nm}$$

(a) $S_{max,T} = 0.2$

Substituting in Eq. (iv)

$$\frac{T_{max}}{T_{fl}} = 0.5 \times \frac{(0.2)^2 + (0.04)^2}{0.2 \times 0.04} = 2.6$$

$\therefore$ $T_{max} = 2.6 \times 746.2 = 1{,}940$ Nm

(b) $$T_s = \frac{3}{\omega_s} \frac{V_{TH}^2 r_2'}{r_2'^2 + (X_1 + x_2')^2} \qquad \text{(v)}$$

Dividing Eq. (v) by Eq. (ii)

$$\frac{T_s}{T_{fl}} = \frac{(r_2'/s_{fl})^2 + (X_1 + x_2')^2}{r_2'^2 \ (X_1 + x_2')^2} \ s_{fl}$$

$$= \frac{s_{max,T}^2 + s_{fl}^2}{s_{max,T}^2 + 1} \left(\frac{1}{s_{fl}}\right)$$

$$= \frac{(0.2)^2 + (0.04)^2}{(0.2)^2 + 1} \times \frac{1}{0.04} = 1$$

$\therefore$ $T_s = 1 \times 746.2 = 746.2$ Nm

(c) $P_m = 75{,}000 = 3I_2'^2 r_2' \left(\frac{1}{0.04} - 1\right)$

or $3I_2'^2 r_2'$ (fl) = 3.125 kW = full-load rotor copper loss

(d) $S_{max,\,T} = 2 \times 0.2 = 0.4$

$\therefore$ $T_{max} = 1{,}940.1$ Nm (remains unchanged)

Substituting in Eq. (iv)

$$\frac{1{,}940.1}{T_{fl}'} = 0.5 \times \frac{(0.4)^2 + s_{fl}'^2}{0.4 s_{fl}'} \qquad \text{(vi)}$$

Also

$$104.7\ (1 - s_{fl}')\ T_{fl}' = 75{,}000 \qquad \text{(vii)}$$

Multiplying Eq. (vi) by Eq. (vii)

$$104.7 \times 1{,}940.1\ (1 - s_{fl}') = 75{,}000 \times 0.5 \times \frac{(0.4)^2 + s_{fl}'^2}{0.4 s_{fl}}$$

$$2.17\ (1 - s_{fl}')\ s_{fl}' = 0.16 + s_{fl}'^2$$

$$3.17 s_{fl}'^2 - 2.17 s_{fl}' + 0.16 = 0$$

$$s_{fl}'^2 - 0.685 s_{fl}' + 0.05 = 0$$

$$s_{fl}' = 0.083^*, 0.6$$

(e) $104.7\ (1 - 0.083)\ T'_{fl} = 75{,}000$

$$T'_{fl} = 781.2 \text{ Nm}$$

(f) $s'_{max,T} = 2 \times 0.2 = 0.4$

9.15 *A 3-phase induction motor has a 4-pole, star-connected stator winding and runs on 50 Hz with 400 V between lines. The rotor resistance and standstill reactance per phase are 0.4 Ω and 3.6 Ω respectively. The effective ratio of rotor to stator turns is 0.67. Calculate: (a) the gross torque at 4% slip, (b) the gross mechanical power at 4% slip, (c) maximum torque, (d) speed at maximum torque, and (e) maximum mechanical power (gross). Neglect stator impedance.*

Solution

Rotor impedance (standstill) referred to stator

$$= \frac{1}{(0.67)^2}\ (0.4 + j\ 3.6)$$

or
$$r'_2 + j\ x'_2 = 0.89 + j\ 8.02$$

$$\text{slip} = 0.04;\ \frac{r'_2}{s} = \frac{0.89}{0.04} = 22.25\ \Omega$$

(a) $T(\text{gross}) = \dfrac{2}{\omega_s}\dfrac{V^2\ (r'_2/s)}{(r'_2/s)^2 + x'^2_2}$

$$\omega_s = \frac{1{,}500 \times 2\pi}{60} = 157.1 \text{ rad/s}$$

$$T = \frac{3}{157.1} \times \frac{(400/\sqrt{3})^2 \times 22.25}{(22.25)^2 + (8.02)^2}$$

$$= 40.51 \text{ Nm}$$

(b) $P_m(\text{gross}) = 40.51 \times 157.1\ (1 - 0.04)$

$= 6.11$ kW

(c) $T_{max} = \dfrac{3}{\omega_s}\left(\dfrac{0.5V^2}{x'_2}\right)$

$$= \frac{3}{157.1}\ \frac{0.5 \times (400/\sqrt{3})^2}{8.02}$$

$= 63.49$ Nm

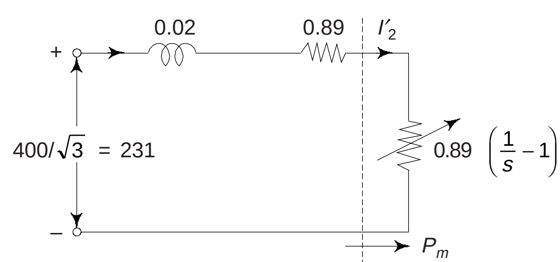


Fig. P9.15

(d) $s_{max,T} = \dfrac{r'_2}{x'_2} = \dfrac{0.89}{8.02} = 0.11$

$\omega_{max,T} = 1{,}500\ (1 - 0.11) = 1{,}335$ rpm

(e) Refer to Fig. P9.15

For $P_m(\text{max})$

$$\sqrt{(0.89)^2 + (8.02)^2} = 0.89\ \left(\frac{1}{s} - 1\right)$$

or
$$s_{P_m(\max)} = 0.1$$

$$I_2' = \frac{231}{|(0.89/0.1 + j\,8.02)|} = 19.28 \text{ A}$$

$$P_m(\max) = 3 \times (19.28)^2 \times 0.89 \left(\frac{1}{0.1} - 1\right)$$

$$= 8.934 \text{ kW}$$

9.16 *A 30 kW, 440 V, 50 Hz, 3-phase, 10-pole, delta-connected squirrel-cage induction motor has the following parameters referred to a stator phase:*

$$r_1 = 0.54\ \Omega \qquad r_2' = 0.81\ \Omega$$

$$x_1 + x_2' = 6.48\ \Omega$$

$$r_i = 414\ \Omega \qquad x_m = 48.6\ \Omega$$

Calculate the machine performance (input current, power, power factor, mechanical output (gross) and torque developed (gross) for the following conditions:

(a) as a motor at a slip of 0.025.
(b) as a generator at a slip of –0.025.
(c) as a break at a slip of 2.0.

Solution

On equivalent-star basis

$$V = \frac{440}{\sqrt{3}} = 254 \text{ V}$$

$$r_1 = 0.18\ \Omega \qquad r_2' = 0.27\ \Omega$$

$$x_1 + x_2' = 2.16\ \Omega$$

$$r_i = 138\ \Omega \qquad x_m = 16.2\ \Omega$$

$$\omega_s = \frac{120 \times 50}{10} \times \frac{2\pi}{60} = 62.83 \text{ rad/s}$$

$$\bar{I}_0 = \frac{254}{138} - j\frac{254}{16.2} = (1.84 - j\ 15.67) \text{ A}$$

The equivalent circuit is drawn in Fig. P9.16.

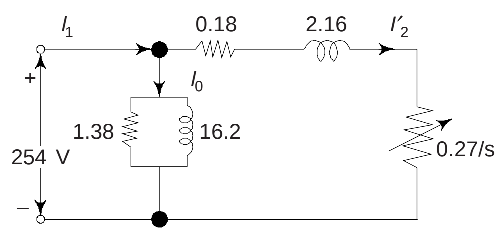


Fig. P9.16

(a) Motoring

$$s = 0.025$$

$$\frac{r_2'}{s} = \frac{0.27}{0.025} = 10.8\ \Omega$$

$$\bar{z}\,(\text{total}) = (10.8 + 0.18) + j\ 2.16$$
$$= 10.98 + j\ 2.16 = 11.9\ \angle 11.1°$$

$$\bar{I}_2' = \frac{254}{11.19}\ \angle{-11.1°} = 22.7\ \angle{-11.1°}$$
$$= 22.27 - j\ 4.37$$

$$\bar{I}_1 = (22.27 - j\ 4.37) + (184 - j\ 15.67)$$
$$= 24.11 - j\ 20.04 = 31.35\ \angle{-39.7°}$$

$$I_1 = 31.35 \text{ A}; \qquad \text{pf} = 0.77 \text{ lagging}$$

$$\text{Input power} = \sqrt{3} \times 440 \times 31.35 \times 0.77 = 18.4 \text{ kW}$$

$$P_m = \left(\frac{1}{s} - 1\right) \times 3I_2'^2 r_2'$$

$$= \left(\frac{1}{0.025} - 1\right) \times 3 \times (22.7)^2 \times 0.27$$

$$= 16.28 \text{ kW}$$

$$T = \frac{3}{62.83} \times \frac{(22.7)^2 \times 0.27}{0.025}$$

$$= 265.7 \text{ Nm}$$

(b) Generating

$$s = -0.025$$

$$\frac{r_2'}{s} = \frac{-0.27}{0.025} = -10.8 \ \Omega$$

$$z(\text{total}) = (-10.8 + 1.8) + j\ 2.16$$
$$= -9 + j\ 2.16 = 9.26 \ \angle 166.5°$$

$$\bar{I}_2' = \frac{254}{9.26} \angle -166.5° = 27.43 \ \angle -166.5°$$
$$= (-26.67 - j\ 6.4)$$

$$\bar{I}_1 = (-26.67 - j\ 6.4) + (1.84 - j\ 15.67)$$
$$= -24.83 - j\ 22.07 = 33.22 \ \angle -138.4°$$

$$\bar{I}_1 \text{ (out)} = 33.22 \ \angle 416\ °$$

$I_1 = 33.22$ pf = 0.747 lagging power output = 18.91 kW

$$P_m = \left(\frac{1}{-0.025} - 1\right) \times 3 \times (27.43)^2 \times 0.27$$

$$= -25 \text{ kW}$$

$$P_m(\text{in}) = 25 \text{ kW}$$

$$T = \frac{3}{62.88} \times \frac{(27.43)^2 \times 0.27}{-0.025}$$

$= -338$ Nm (in direction opposite to which the rotor is running)

(c) Braking

$$s = 2.0$$

$$\frac{r_2'}{s} = \frac{0.27}{s} = 0.135 \ \Omega$$

$$\bar{z}(\text{total}) = (10.135 + 0.18) + j\ 2.16 = 0.315 + j\ 2.16$$
$$= 2.18 \ \angle 81.7°$$

$$\bar{I}_2' = \frac{254}{2.18} \angle -81.7° = 116.5 \ \angle -817°$$
$$= 16.82 - j\ 115.28$$

$$\bar{I}_1 = (16.8 - j\ 115.28) + (184 - j\ 15.67)$$
$$= 18.64 - j\ 130.95 = 132.27\ \angle{-81.9^\circ}\ \text{A}$$
$$I_1 = 132.27\ \text{A}; \qquad \text{pf} = 0.141\ \text{lagging}; \qquad \text{power input} = 14.21\ \text{kW}$$
$$P_m = \left(\frac{1}{2} - 1\right) \times 3 \times (116.5)^2 \times 0.27$$
$$= -5.5\ \text{kW}$$
$$P_m(\text{in}) = 5.5\ \text{kW}$$
$$T = \frac{3}{62.83} \times \frac{(116.5)^2 \times 0.27}{2}$$
$$= 87.49\ \text{Nm}$$ (in the direction of field but opposite to the direction in which the rotor is rotating)

Torque is developed in the same direction in which the rotor is running. The field rotates in an opposite direction to the rotor (s = 2). The total electrical and mechanical power input (14.21 + 5.5 = 19.71 kW) is consumed in stator and rotor copper loss and in core loss. Braking, therefore, can only be a short-time operation.

9.17 *The following test results were obtained on a 7.5 kW, 400 V, 4-pole, 50 Hz, delta-connected induction motor with a stator resistance of 2.1 Ω/phase:*

No-load	*400 V*	*5.5 A*	*410 W*
Rotor-blocked	*140 V*	*20 A*	*1,550 W*

Obtain the approximate equivalent circuit modal.
Estimate the braking torque developed when the motor, running with a slip of 0.05 has two of its supply terminals suddenly interchanged.

Solution

No-load test

$$\left(\frac{400}{\sqrt{3}}\right)^2 g_i = \frac{410}{3}$$
$$g_i = 2.56 \times 10^{-3}\ \mho$$
$$r_i = 390\ \Omega \text{ (includes windage and friction loss)}$$
$$\left(\frac{400}{\sqrt{3}}\right) y_0 = 5.5$$
$$y_0 = 0.0238\ \mho$$
$$b_m = \sqrt{(0.0238)^2 - (2.56 \times 10^{-3})^2}$$
$$= 0.0237\ \mho$$
$$x_m = 42.2\ \Omega$$

Rotor-blocked test

$$Z(\text{total}) = \frac{140/\sqrt{3}}{20}\ 40.04\ \Omega$$
$$(20)^2 R = \frac{1,550}{3}$$

$$R = 1.29\ \Omega$$
$$X = 3.83\ \Omega$$
$$r_1 = \frac{2.1}{3} = 0.7\ \Omega \text{ (eqv. star)}$$
$$r_2' = 1.29 - 0.7 = 0.59$$

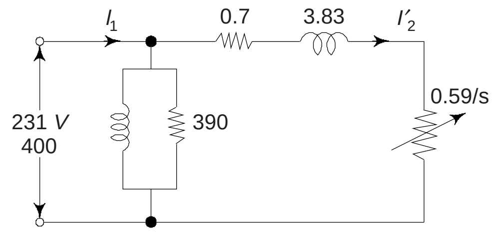


Fig. P9.17

Circuit model (approximate) (Fig. P9.17)
Braking torque

$$s = 1.95$$
$$\frac{r_2'}{s} = \frac{0.59}{1.95} = 0.3\ \Omega$$
$$Z\text{(total)} = (0.7 + 0.3) + j\,3.83 = 1 + j\,3.83$$
$$= 3.96\ \angle 75.4^\circ\ \Omega$$
$$I_2' = \frac{231}{3.96} = 58.3\ \text{A}$$
$$\omega_s = 157\ \text{rad/s} \qquad T_{\text{breaking}} = \frac{3}{157}\,(58.3)^2\,\frac{0.59}{1.95} = 19.65\ \text{Nm}$$

9.18 *A 3-phase wound-rotor induction motor has star-connected rotor winding with a rotor resistance of 0.12 Ω/phase. With the slip rings shorted the motor develops rated torque at a slip of 0.04 and a line current of 100 A. What external resistance must be inserted in each rotor phase to limit the starting current to 100 A? What pu torque will be developed with rotor resistance starting ?*

Solution

We must assume here that the magnetizing current can be neglected.

$$\frac{V'}{(0.1/0.04)^2 + x_2^2} = \frac{V'}{(0.1 + R_{\text{ext}})^2 + x_2^2} = 100\ \text{A}$$
$$R_{\text{ext}} = 2.4\ \Omega$$
$$T_{\text{fl}} = \frac{3}{\omega_s} \times \frac{(100)^2 \times 0.1}{0.04}$$
$$T_s = \frac{3}{\omega_s} \times \frac{(100)^2 \times 2.5}{1}$$
$$\frac{T_s}{T_{\text{fl}}} = \frac{2.5 \times 0.04}{0.1} = 1$$

or $$T_s = 1\ \text{pu}$$

9.19 *In Prob 9.18 what external resistance must be inserted per rotor phase to develop full load torque at 3/4th synchronous speed with a line current of 100 A?*

Solution

$$s = 0.25$$
$$\frac{3}{\omega_s}\,\frac{(100)^2 \times (0.1 + R_{\text{ext}})}{0.25} = \frac{3}{\omega_s}\,\frac{(100)^2 \times 0.1}{0.04}$$
$$0.1 + R_{\text{ext}} = \frac{0.25 \times 0.1}{0.04}$$
$$R_{\text{ext}} = 0.525\ \Omega$$

9.20 *A 4-pole, 50 Hz, 3-phase induction motor has a rotor resistance of 4.5 Ω/phase and a standstill reactance of 8.5 Ω/phase. With no external resistance in the rotor circuit, the starting torque of the motor is 85 Nm.*

(a) What is the rotor voltage at standstill ?

(b) What would be the starting torque if a 3 Ω resistance were added in each rotor phase ?

(c) Neglecting stator voltage drop, what would be the induced rotor voltage and the torque at a slip of 0.03?

Solution

$$r_2 = 4.5\ \Omega \qquad x_2 = 8.5\ \Omega$$

(a) $$T_{start} = \frac{3}{\omega_s}\frac{V^2 r_2'}{r_2'^2 + x_2'^2} \qquad \text{[Eq. (9.16) with } R'_{ext} = 0]$$

$$= \frac{3}{\omega_s}\frac{V'^2 r_2}{r_2^2 + x_2^2}$$

But $$\omega_s = \frac{120 \times 50}{4} \times \frac{2\pi}{60} = 157.1 \text{ rad/s}$$

∴ $$85 = \frac{3}{157.1}\frac{V'^2 \times 4.5}{(4.5)^2 + (8.5)^2}$$

Standstill rotor voltage,

$$V' = 302.5 \text{ V} \qquad \text{or} \qquad 523.9 \text{ V (line)}$$

(b) With external resistance $R_{ext} = 3\ \Omega$

$$T_{start} = \frac{3}{\omega_s}\frac{V'^2 (r_2 + R_{ext})}{(r_2 + R_{ext})^2 + x_2^2}$$

$$= \frac{3}{157.1}\frac{(302.5)^2\ (4.5 + 3)}{(4.5 + 3)^2 + (8.5)^2}$$

$$= 102 \text{ Nm}$$

(c) Induced rotor voltage at slip s = $s \times$ standstill voltage

$$= 0.03 \times 302.5$$

$$= 9.1 \text{ V} \qquad \text{or} \qquad 15.7 \text{ V (line)}$$

$$T = \frac{3}{\omega_s}\frac{V^2\ (r_2'/s)}{(r_2'/s)^2 + x_2'^2}$$

or

$$T = \frac{3}{\omega_s}\frac{V'^2\ (r_2/s)}{(r_2/s)^2 + x_2^2}$$

$$= \frac{3}{157.1}\frac{(302.5)^2\ (4.5/0.03)}{(4.5/0.03)^2 + (8.5)^2}$$

$$= 11.6 \text{ Nm}$$

9.21 *Calculate the ratio of transformation of an auto transformer starter for a 25 kW, 400 V, 3-phase induction motor if the starting torque is to be 75% of full-load torque. Assume the slip at full-load to be 3.5 % and the short-circuit current to be six times full-load current. Ignore the magnetizing current of the transformer and of the motor.*

Solution

$$T_{\text{start}} = x^2 \left(\frac{I_{\text{SC}}}{I_{\text{fl}}}\right)^2 s_{\text{fl}}$$

or $$0.75 = x^2(6)^2\ 0.035$$

or $$x = 0.77$$

9.22 *With reference to the circuit model of Fig. P9.13 (as reduced by the Thevenin theorem) show that*

$$\frac{T}{T_{\max}} = \frac{1+\sqrt{K^2+1}}{1+1/2\sqrt{K^2+1}\,(s/s_{\max,\text{T}} + s_{\max,\text{T}}/s)}$$

where

$$K = \frac{X_1 + x_2'}{R_{\text{I}}}$$

Solution

For the circuit model of Fig. 9.3 we have the following relationships

$$T = \frac{3}{\omega_{\text{s}}} \frac{V_{\text{TH}}^2\ (r_2'/s)}{(R_1 + r_2'/s)^2 + (X_1 + x_2')^2} \tag{i}$$

$$T_{\max} = \frac{3}{\omega_{\text{s}}} \frac{0.5V_{\text{TH}}^2}{R_1 + \sqrt{R_1^2 + (X_1 + x_2')^2}} \tag{ii}$$

$$s_{\max,\text{ T}} = \frac{r_2'}{\sqrt{R_1^2 + (X_1 + x_2')^2}} \tag{iii}$$

Dividing (i) by (ii), we have

$$\frac{T}{T_{\max}} = \frac{R_1 + \sqrt{R_1^2 + (X_1 + x_2')^2}\left(r_2'/s\right)}{\frac{1}{2}[(R_1 + r_2'/s)^2 + (X_1 + x_2')^2]}$$

$$= \frac{R_1\left[1+\sqrt{1+((X_1 + x_2')/R_1)^2}\right] r_2'/s}{R_1/r_2'/s + \frac{1}{2}[(r_2'/s)^2 + R_1^2 + (X_1 + x_2')^2]}$$

$$= \frac{1+\sqrt{1+((X_1 + x_2')/R_1)^2}}{1+1/2\,[(r_2'/s)^2 + R_1^2 + (X_1 + x_2')^2/\,R_1 r_2'/s]}$$

$$= \frac{1+\sqrt{1+K^2}}{1+(1/2)\,1/R_1\,\sqrt{R_1^2 + (X_1 + x_2')^2}\,[(r_2'/s)/(R_1^2 + (X_1 + x_2')^2) + (R_1^2 + (X_1 + x_2')^2)/(r_2'/s)]}$$

$$= \frac{1+\sqrt{1+K^2}}{1+1/2\sqrt{1+((X_1+x_2')/R_1)^2}\,[s_{\max,T}/s+s/s_{\max,T}]}$$

$$= \frac{1+\sqrt{1+K^2}}{1+1/2\sqrt{1+K^2}\,[s_{\max,T}/s+s/s_{\max,T}]}$$

9.23 *A 3-phase, 50 Hz, 75 kW induction motor develops its rated power at a rotor slip of 2%. The maximum torque is 250% of rated torque (that is, the torque developed at rated power). The motor has a K-ratio (defined in Prob 9.22) of K = 4.33. Find:*

(a) $s_{max,T}$ *at maximum torque*

(b) rotor current referred to the stator at maximum torque,

(c) starting torque, and

(d) starting current.

The answers to parts (b), (c), and (d) should be expressed in terms of current and torque at full-load speed.

Solution

$$K = \frac{X_1+x_2'}{R_1} = 4.33 \qquad s_{\text{fl}} = 0.02$$

(a) We use the result of Prob 9.22.

$$\frac{T_{\text{fl}}}{T_{\max}} = \frac{1}{2.5} = \frac{1+\sqrt{K^2+1}}{1+1/2\sqrt{K^2+1}\,[s_{\max,T}/s_{\text{fl}}+s_{\text{fl}}/s_{\max,T}]}$$

or
$$\frac{1}{2.5} = \frac{1+\sqrt{4.33^2+1}}{1+1/2\sqrt{4.33^2+1}\,[(s^2_{\max,T}/s^2_{\text{fl}})/s_{\text{fl}}\,s_{\max,T}]}$$

or
$$s^2_{\max,T}+s^2_{\text{fl}} = 4.22 \qquad s_{\text{fl}}\,s_{\max,T}$$

or
$$s^2_{\max,T} - 0.09\ s_{\max,T} + 4\times 10^{-4} = 0$$

or
$$s_{\max,T} = 0.09,\ 2.79\times 10^{-4}$$

Here $s_{\max,T} = 2.79\times 10^{-4}$ means the machine has an unrealistically small resistance and therefore the answer is

$$s_{\max,T} = 0.09$$

(b) $I_2'^2 = \dfrac{V_{\text{TH}}^2}{(R_1+r_2'/s)^2+(X_1+x_2')^2}$; follows from Thevenin equivalent of Fig. 9.3

$$\therefore \qquad \left(\frac{I_2'|_{\max,T}}{I_2'|_{\text{fl}}}\right)^2 = \frac{(R_1+r_2'/s_{\text{fl}})^2+(X_1+x_2')^2}{(R_1+r_2'/s_{\max,T})^2+(X_1+x_2')^2}$$

$$= \frac{(1+r_2'/R_1\ s_{\text{fl}})^2+((X_1+x_2')/R_1)^2}{(1+r_2'/R_1\ s_{\max,T})^2+((X_1+x_2')/R_1)^2} \qquad \text{(i)}$$

We know that

$$s_{\text{max, T}} = \frac{r_2'}{\sqrt{R_1^2 + (X_1 + x_2')^2}} \tag{9.9}$$

$$\frac{r_2'}{R_1} = s_{\text{max, T}} \sqrt{1 + ((X_1 + x_2')/R_1)^2}$$

$$= s_{\text{max, T}} \sqrt{1 + K^2}$$

$$= 0.09 \sqrt{1 + (4.33)^2} = 0.4 \tag{ii}$$

Using (ii), (i) can be rewritten as

$$\left(\frac{I_2'|_{\text{max,T}}}{I_2'|_{\text{fl}}}\right)^2 = \frac{(1 + 0.4/0.02)^2 + 4.33^2}{(1 + 0.4/0.09)^2 + 4.33^2} = \frac{459.75}{48.38}$$

or $\quad I'_{2\,\text{max, T}} = 3.08 I_2'|_{\text{fl}}$

The reader should note that the actual value of referred rotor current cannot be computed.

(c) $$T = \frac{3}{\omega_s} \frac{V_{TH}^2 (r_2'/s)}{(R_1 + r_2'/s)^2 + (X_1 + x_2')^2} \tag{9.8}$$

$$\therefore \quad \frac{T_s}{T_{\text{fl}}} = \left[\frac{(R_1 + r_2'/s_{\text{fl}})^2 + (X_1 + x_2')^2}{(R_1 + r_2')^2 + (X_1 + x_2')^2}\right] s_{\text{fl}}$$

$$= \left(\frac{(1 + 0.4/0.02)^2 + 4.33^2}{(1 + 0.4)^2 + 4.33^2}\right) \times 0.02 = \left[\frac{(1 + r_2'/R_1 s_{\text{fl}})^2 + (X_1 + x_2'/R_1)^2}{(1 + r_2'/R_1)^2 + (X_1 + x_2'/R_1)^2}\right] s_{\text{fl}}$$

$$= 0.02 \; \frac{459.75}{20.096}$$

$$\therefore \quad T_s = 0.458 \; T_{\text{fl}}$$

(d) Similar to part (b), we have

$$\left(\frac{I_2'/s}{I'_{2\text{fl}}}\right) = \frac{(R_1 + r_2'/s_{\text{fl}})^2 + (X_1 + x_2')^2}{(R_1 + r_2')^2 + (X_1 + x_2')^2}$$

$$= \frac{(1 + r_2'/R_1 s_{\text{fl}})^2 + ((X_1 + x_2'/R_1)^2}{(1 + r_2'/R_1)^2 + ((X_1 + x_2'/R_1)^2}$$

$$= \frac{(1 + 0.4/0.02)^2 + 4.33^2}{(1 + 0.4)^2 + 4.33^2}$$

$$\therefore \quad I_2'|_{s=1} = 22.9 \; I'_{2\text{fl}}$$

9.24 *A 3-phase induction motor is wound for P poles. If the modulation poles are P_M, obtain the general condition to suppress $P_2 = (P + P_M)$ poles. Under this condition show that the angle between the phase axes for $P_1 = (P - P_M)$ poles is $r(2\pi/3)$, where r = integer non-multiple of 3. If P = 10, find P_M and P_1.*

Solution

To suppress $P_2 = (P + P_M)$ poles,

$$\left(\frac{P + P_M}{P}\right) r \left(\frac{2\pi}{3}\right) = n(2\pi)$$

where r = integer non-multiple of 3

n = integer

This gives

$$\frac{P_M}{P} = \left(\frac{3n}{r} - 1\right)$$

The angle between the phase axes of $P_1 = (P - P_M)$ poles is

$$\left(1 - \frac{P_M}{P}\right) r \left(\frac{2\pi}{3}\right) = \left(1 - \frac{3n}{r} + 1\right) r \left(\frac{2\pi}{3}\right)$$

$$= \left(2 - \frac{3n}{r}\right) r \left(\frac{2\pi}{3}\right)$$

$$= 2r \left(\frac{2\pi}{3}\right) - n(2\pi)$$

$$= 2r \left(\frac{2\pi}{3}\right)$$

$$\frac{P_M}{P} = \left(\frac{3n}{r} - 1\right)$$

$$P_M = 10 \left(\frac{3n}{r} - 1\right)$$

$$n = 2, \qquad r = 5$$

$$P_M = 10 \left(\frac{6}{5} - 1\right) = 2$$

Hence

$$P_1 = 10 - 2 = 8 \text{ poles}$$

9.25 *The two cages of a 3-phase, 50 Hz, 4-pole, delta-connected induction motor have respective standstill leakage impedances of (2 + j 8) and (9 + j2) Ω/phase. Estimate the gross torque developed;*

(i) at standstill, the effective rotor voltage being 230 V/phase.

(ii) at 1450 rpm when the effective rotor voltage is 400 V/phase. The rotor quantities given are all referred to the stator; the stator impedances are negligible.

What is the gross starting torque if a star-delta starter is used ?

Solution

$$\omega_s = \frac{1,500 \times 2\pi}{60} = 157 \text{ rad/s}$$

(i) $$T_1 = \frac{3}{157} \frac{(230)^2 \times 2}{(2)^2 + (8)^2} = 29.7 \text{ Nm}$$

$$T_2 = \frac{3}{157} \frac{(230)^2 \times 9}{(9)^2 + (2)^2} = 107 \text{ Nm}$$

$$T(\text{total}) = 29.8 + 107 = 136.8 \text{ Nm}$$

(ii) $s = \dfrac{1{,}500 - 1{,}450}{1{,}500} = \dfrac{1}{30}$

$$T_1 = \frac{3}{157}\frac{(400)^2 \times (2\times 30)}{(2\times 30)^2 + (8)^2} = 50.1 \text{ Nm}$$

$$T_2 = \frac{3}{157}\frac{(400)^2 \times (9\times 30)}{(9\times 30)^2 + (2)^2} = 11.32 \text{ Nm}$$

$$T(\text{total}) = 50.1 + 11.3 = 61.4 \text{ Nm}$$

Star-delta starting:
In star-delta starting, starting voltage/phase is

$$\frac{400}{\sqrt{3}} = 230 \text{ V}$$

Hence,
Starting torque = 136.7 Nm

9.26 A 3-phase, 50 Hz, 4-pole, 400 V, wound rotor induction motor has a Δ-connected stator winding and Y-connected rotor winding. There are 80% as many rotor conductors as stator conductors. For speed of 1425 rpm, calculate
(a) the slip,
(b) the rotor induced emf between the two slip rings, and
(c) the rotor frequency

(a) ns = (120 X 50) / 4 = 1500 rpm
n = 1425
s = [(1500 – 1425) / 1500] X 100 = 5%

(b) Turn ratio rotor phase (V)/rotor phase Δ = 0.8
Rotor phase voltage = 400 X 0.3 = 320 V
Voltage between two slip rings = √3 X 320 = 554 V

(c) f2 = 0.05 X 50 = 2.5 Hz

9.27 A squirrel-cage induction motor is rated 25 kW, 440 V, 3-phase, 50 Hz. On full-load it draws 28.7kW with line current 50 A and runs at 720 rpm. Calculate
(a) the slip,
(b) the power factor, and
(c) the efficiency.

f = nb/120
50 = (720 X p) / 120 or p = 8
ns = 750 rpm

a) n= 720 rpm
ship, s = (750 – 720) / 750 = 0.04

b) 28.7 = (√3 X 440 X 50 X cosφ) / 1000

pf = cos φ = 0.753 lagging

c) η = (25 / 28.7) X 100 = 87.1 %

9.28 A 3-phase, 400V, 6-pole, 50 Hz induction motor develops mechanical power of 20 kW at 985 rpm. Calculate:
(a) the rotor copper loss.
(b) the total input power, and
(c) rotor frequency.

The stator losses are equal to 1800 W. Neglect mechanical loss.

(i) $3[(I'^2 X r'_2)/s – I'^2 X r_2] = Pm$

S = (1000-985) / 1000 X 100 = 1.5 %

$3((1/0.015) – 1)\ I'^2 r'_2 = 200w$

Rotor copper loss $= 3I'^2 r'_2 = 20000 \text{ X } 0.015/0.985$
$= 304.6$ w

(ii) Pc(pome asass air-gap) = 304.6/0.015 = 20307 w
Stator loss = 1300 w
Input power = 20307 + 1800 = 2.211 kw

(iii) $f_2 = 0.015 \text{ X } 50 = 0.75$ Hz

9.29 A 400 V, 5kW, 50 Hz induction motor runs at 1445 rpm at full-load. The rotational losses are 285 W. If the maximum torque occurs at 900 rpm, Calculate its value.

Mechanical output = 5000w
Rotational loss = 285 w
Mechanical power developed = 5285 w
Slip = (1500 – 1445) / 1500 = 0.0367
Stator impedance will be ignored

$I'2 = 400\sqrt{3} /(\sqrt{(r'_2/s)^2} + x'^2_2)$

Then in terms of mechanical power developed

$3(1/s - 1)\ I'^2_2\ r'_2 = Pm = 5285$

$3(1/0.0367 – 1)\ [\ (400)^2\ (1/3) \text{ X } r'_2\ ]\ /\ [\ (r'_2/0.0367)^2 + x'^2_2] = 5285$

$(0.9633 / 0.0367) \text{ X } [(400)^2 \text{ X } (0.0367)^2\ r'_2]\ /\ r'^2_2 + 0.00135 \text{ X } x'^2_2 = 5285$

$W = \quad r'_2/\ (r'^2_2 + 0.00135 \text{ X } x'^2_2) = 5285\ /\ ((400)^2 \text{ X } 0.0367 \text{ X } 0.9633)$

$W \quad r'_2/\ (r'^2_2 + 0.00135 \text{ X } x'^2_2) = 0.934$ ---------- (i)

At max torque
$r'_2 = x'_2$ $\qquad$ s = (1500 – 900) / 1500 = 0.4

or $r'_2 = 0.4 \text{ X } x'_2$

Substituting (ii) in (i)

$0.4\ x'_2\ /\ (0.16\ x'^2_2 + 0.00135 \text{ X } x'^2_2) = 0.934$
Or $(0.4/\ 0.161135 \text{ X } x'_2) = 0.934$

Or $x'_2 = 2.65\ \Omega$

Then $r'_2 = 0.4 \text{ X } 2.65 = 1.06\ \Omega$

$T_{max} = (3/\omega s)\ (0.5\ V2)/x'_2$ ----------------- (iii)

$\omega_s = (2\Pi \text{ X } 1500) / 60 = 50\ \Pi$ rad/s

Therefore $T_{max} = (3 / 50\ \Pi)\ (0.5 \text{ X } (400 / \sqrt{3})^2 / 265)$
$= 192.2$ Nm

9.30 The rotor of a 6-pole, 50 Hz slip ring induction motor has a resistance of 0.25Ω / phase and runs at 960 rpm. Calculate the external resistance/phase to be added to lower the speed to 800 rpm with load torque reducing to 3/4th of the previous value.

Assumption : Stator impedence is negligible. For the range of ships considered

$R'_2/s >> x'_2$

Now $T = (3/\omega_s) \text{ X } (sV^2/r'_2) = K \text{ X } s/r_2$

At 960 rpm $s = (1000\text{-}960) / 1000 = 0.04$
At 800 rpm $s = (1000 - 800) / 1000 = 0.2$

$T = K\ (0.04/0.15)$ -------- (i)

$\frac{3}{4}\ T = K\ (0.2/(0.25 + Rext)$ --------- (ii)

Dividing (i) by (ii)

$4/3 = (0.25 + Rext)/0.2 \text{ X } (0.04/0.25)$

$0.25 + Rext = 4/3 \text{ X } 0.2 \text{ X } 0.25/0.04 = 1.67$
$Rext = 1.42\ \Omega$

9.32 A 5kW, 400 V, 50 Hz, 4-pole induction motor gave the following lest data:

No-load test:
$V_0 = 400V$, $P_0 = 350$ W, $I_0 = 3.1$ A

Blocked rotor test:
$V_{sc} = 52$ V, $P_{sc} = 440W$, $I_{sc} = 7.6$ A,

24V, dc when applied between the two stator terminals causes a current of 7.6 A to flow.
Calculate the motor efficiency at rated voltage at a slip of 4%.

No load test:
$F0 = 3.1 / 400 / \sqrt{3} = 0.0134\ \Omega$

Gi = 350 / (400)2 = 0.0022 w

Ri = 457 Ω (accounts for rotational loss)

$Bm = \sqrt{(0.0134)^2 - (0.0022)^2}$
$= 0.0132\ \Omega$

Xm = 75.65 Ω

Blocked rotor test:
$Z = (52 / \sqrt{3}) / 7.6$
$= 3.95\ \Omega$

$R = (440/3) / (7.6)^2$
$= 2.54\ \Omega$

$X = \sqrt{((3.95)^2 - (2.54)^2)}$
$= 3.025\ \Omega$
$= x_1 + x'_2$

$R_1 = 24 / (2 \text{ X } 7.6) = 1.58\ \Omega$

$R'_2 = 2.54 - 1.58 = 0.96\ \Omega$

At slip of 4 %
$r'_2/s = (0.96/ 0.04) = 24\ \Omega$

$I'_2 = 231 / (125.58 + j\ 3.025) = 9$ A

Pm (next) = $3 \text{ X } ((1/0.04) - 1) \text{ X } (9^2 \text{ X } 0.96)$
= 5.6 kw

Pi = $5.506 + 3 \text{ X } (9^2 \text{ X}(2.54 / 1000))+ 0.35$
= 5.506 + 0.607 + 0.35
= 6.463 kw

$\eta = 5.506/6.463 = 85.2$ %

9.33 A 3-phase, 20kW, 600 V, 50 Hz, 6-pole, Y-connected squirrel-cage induction motor has the following parameters/phase referred to the stator:

$R_1 = 0.937$ W $\quad R'_2 = 0.7\ \Omega$
X (equivalent) = 3.42 Ω
$X_m = 72.9\ \Omega$

The rotational and core losses equal 545 W.
For a slip of 3.5% find:

(a) the line current and the power factor
(b) the mechanical output and shaft torque
(c) the efficiency.

$r'_2 / s = (0.7 / 0.035) = 20\ \Omega$

$r_1 + r'_2/s = 20.937\ \Omega$

a) $I'_2 = (600\ \sqrt{3}) / (20.937 + j\ 3.42)$

$= 346.4 / (21.21 \angle 93^\circ)$
$= 16.33 \angle 9.3^\circ$
$= 16.12 - j\ 2.64$

$I0 = 346.4 / j\ 72.9 = -\ j\ 4.75$

I1 bar $= -\ j\ 4.75 + 16.12 - j\ 2.64$
$= 16.12 - j\ 7.39$
$= 17.73 \angle (-24.6^\circ)$

I1 = 17.73 A
Pf = cos24.6 °
= 0.909 logging

b) Pm = 3 X ((1/0.035) – 1) X (16.33)2 X 0.7
= 15.44 kw

Mechanical output = 15.44 – 0.545 = 14.895 kw

N = (1 – 0.035) X 1000 = 965 rpm, ω = (2 Π X 965)/60 = 101.05 rad's

Shaft torque = 14895 / 101.05
= 147.4 Nm

c) Pn = √3 X 600 X 17.73 X 0.909
= 16.748kw
η =14.895 / 16.748 = 88.9 %

9.34 A 7.5 kW, 400 V, 4-pole induction motor gave the following test results:

No-load test
V_0 **= 400V,** P_0 **= 330 W,** I_0 **= 3.52 A**

Blocked rotor test:

V_{sc} = 110 V, P_{sc} = 615W, I_{sc} = 13 A,

The effective ac resistance between the stator terminals is 2.2 Ω and the full-load slip is 4%.
Determine:

(a) the parameters of the per phase circuit model.
(b) The stator current and its pf when the motor is delivering full-load.
(c) The efficiency in part (b).

No load test

$y_0 = (3.52 / (400/\sqrt{3})) = 0.0152$ mho

$g_m = 330 / (400)2 = 0.0021$ mho

$r_c = 476.2\ \Omega$

$b_m = 0.01505$ mho

Block rotor test:

$Z = (110 / \sqrt{3}) / 13 = 4.885\ \Omega$

$R = 615 / (3 X (13)^2 = 1.213\ \Omega$

$r_1 = 2.2/2 = 1.1\ \Omega$

$r'_2 = 1.113\ \Omega$

$X = x_1 + x'_2 = \sqrt{((4.885)^2 - (1.213)^2)}$
$= 4732\ \Omega$

a) diagram
b) s = 0.04

$r'_2 / s = 1.113/0.04 = 27.83\ \Omega$

$R = 1.1 + 27.83 = 28.93\ \Omega$
$I'_2 = 231 / (28.93 + j\ 4.732) = 231 / (29.31 \angle\ 9.3^\circ)$
$= 7.88 \angle (- 9.3^\circ)$
$= 7.776 - j\ 1.273$

$I_0 = (231/476.2)\ X\ -j\ X\ (231 / 66.43)$

$= 0.485\ X\ -j\ X\ 3.477$
Therefore $I_1 = 7.776 - j1.272$

$(0.485 - j\ 3.477) / \ (8.261 - j\ 4.749) = 9.53 \angle (-29.9^\circ)$

$I1 = 9.53$ A
$Pf = \cos 29.9°$
$= 0.867$ lagging

c) Pm(net output) $= 3 \times ((1/0.04) – 1) \times (7.88)^2 \times 1.113$
$= 4.976$ kw

$Pi = \sqrt{3} \times 400 \times 9.53 \times 0.867$
$= 5.724$ kw

$\eta = 4.976 / 5.724 = 86.9$ %

9.35 A 30 kW, 440 V squirrel-cage induction motor has a starting torque of 182 Nm and a full-load torque of 135 Nm. The starting current of the motor is 207 A when rated voltage is applied.

Determine:

- **(a) the staring torque when the line voltage is reduced to 254 V.**
- **(b) the voltage that must be applied for the motor to develop a starting torque equal to the full-load torque.**
- **(c) The starting current in parts (a) and (b).**
- **(d) The starting voltage to limit the starting current to 40 A, and the corresponding starting torque.**

$Ts / Tfl = (Is/Ifl)^2$ sfl

a) $182 / 135 = (207 / Ifl)^2$ sfl ---------------- (i)

Is(reduced voltage) $= 207 \times (254/440) = 119.5$ A

$Ts / 135 = (119.5 / Ifl)^2$ Sfl ----------------- (ii)

Dividing (ii) by (i)

$Ts /135 \times 135 / 182 = (119.5/207)2$

$Ts = 135 \times (119.5/207)2 = 45$ Nm

b) $Ts = Tfl$

$1 = (Is/Ifl)^2$ sfe -------------- (iii)

Dividing (iii) by (i)

$135/182 = (Is/ 207)^2$

Or Is = $\sqrt{(135/182)}$ X 207
= 178.3 A

Voltage to be applied = 440 X (178.3/207)
= 379 V

9.36 A 400V, 4-pole, 7.5 kW, 50 Hz, 3-phase induction motor develops its full-load torque at a slip of 4%. The per phase circuit parameters of the machines are

$R_1 = 1.08\ \Omega$ $\quad$ $R'_2 = ?$

$X_1 = 1.41\ \Omega$ $\quad$ $x'_2 = 1.41\ \Omega$

Mechanical, core and stray losses may be neglected.

(a) Find the rotor resistance (as referred to stator)

(b) Find the maximum torque, slip at maximum torque and the corresponding rotor speed.

S = 0.04
N = 0.96 X 1500 = 1440 rpm
W = (2Π X 1440) / 60
= 150.8 rad / s

Ws = (2Π X 1500) / 60
= 157.1 rad / s

Pout (μ) = 7.5 kw
T(μ) = 7500 / 150.8 = 49.73 Nm

$T = (3 / ws) \times (I'^2_2\, r'_2/s)$

$49.73 = 3/157.1 \times [((231)^2 \times r'_2) / (1.08 + 25r'_2)^2 + (2.82)^2\ ] \times 1 / 0.04$

$(1.08 + 25\, r'_2)^2 + 625\, r'_2 + 7.952 = 512.3\, r'_2$

$1.166 + 54r'_2 + 625\, r'_2 + 10.23 = 0$

$r'^2_2 - 0.733\, r'_2 + 0.0164 = 0$

$r'_2 = [0.733 \pm \sqrt{(0.537 - 0.0656)}] / 2$
$= 0.71\ \Omega, 0.0232\ \Omega$

Smaller value is rejected as it would regain to large current for development of the required torque. Hence
$r'_2 = 0.71\ \Omega$

b) For max torque

$r'_2 = sx'_2$

or $s = 0.71 / 1.41 = 0.504$

$n = (1 – 0.504) X 1500 = 744$ rpm

$r'_2/s = 0.71 / 0.504 = 4.41$

$r'_2 + (r'_2/s) = 0.17 + 1.41 = 2.11\Omega$

$X = 2 X 1.41 = 2.82\Omega$

$I'_2 = 231 / (2.11 + j 2.82)$
$= 65.6\ \Omega$

$Tmax = 3 / 157.1 X (65.6)^2 X 1.41$
$= 116$ Nm

9.37 A 3 – phase, 440 V, 4-pole 50 Hz induction motor has a star-connected stator and rotor. The rotor resistance and standstill reactance/phase are 0.22 Ω and 1.2 Ω respectively; the stator to rotor turn ratio being 1.3. The full-load slip is 4%. Calculate the full-load torque and power developed. Find also the maximum torque and the corresponding speed.

$r'_2 = 0.22 X (1.3)^2 = 0.372\ \Omega$

$x'_2 = 1.2 X (1.3)^2 = 2.03\Omega$

Stator impedence is neglected

$V = 440\sqrt{3} = 254$ V
$S = 0.04$
$r'_2 /s = 0.372 / 0.04 = 9.3\ \Omega$
$I'_2 = 254 / (9.3 + j 2.031)$
$= 26.68$ A

$\omega s = (2\Pi X 1500) / 60 = 157.1$ rad/s

$T_{(fl)} = (3/157.1) X (26.68)^2 X 9.3$
$= 126.5$ Nm

$S_{max,T} = 0.372/2.05 = 0.18$

$r'_2 / s_{max,T} = 0.372/0.18 = 2.05$

$I'_2 = (254/2.05\sqrt{2}) = 87.62A$

$Tmax = 3 / 157.1 X (87.62) X 2.05$
$= 300.5$ Nm

9.38 A 3-phase, 3.3kV, 6-pole wound rotor induction motor has the following test data:

No-load test	**3.3kV**	**18.5 A**	**15.1 kW**
Blocked-rotor test	**730 V**	**61 A**	**3.5 kW**

The resistance of the stator winding is 1.6 Ω and the rotational loss is 6.2 kW. Calculate the circuit model parameters (rotational loss not to be accounted in Ri core loss resistance).

Assume $X_1/X'_2 = R_1/R'_2$ Calculate

(a) the slip at maximum developed torque
(b) the maximum developed torque and the corresponding shaft torque
(c) the starting torque at half the rated voltage

Note: Do not approximate the circuit model.

No load test:

$F0 = 18.5 / (3300 / \sqrt{3}) = 9.71 \times 10^{-3}$
Core + rotational loss = 15.1 kw

Rotational loss = 6.2 kw

Therefore core loss = 15.1 – 6.2 = 8.9 kw

$G_i = 8900 / (3300)^2 = 3.17 \times 10^{-4}$ mho
$r_i = 1224\ \Omega$

$Bm = \sqrt{[(9.71 \times 10^{-3})^2 + (8.17 \times 10^{-4})^2}$
$= 9.676 \times 10^{-3}$ mho
$xm = 103.35\ \Omega$

Blocked rotor tests:

$Z = (730 / \sqrt{3}) / 61 = 6.91\Omega$

$R = (30.5 \times 1000) / (3 \times (61)^2) = 2.732\ \Omega$

$r_1 = 1.6\ \Omega$
$r'_2 = 1.132\ \Omega$

$X = \sqrt{[(6.91)^2 - (2.732)^2]}$
$= 6.347\Omega$

$x_1 = 6.347 \times (1.6 / 2.732) = 3.717\ \Omega$

$x'_2 = 2.63\ \Omega$

Series equivalent of the shunt branch

= 10 3/(0.817 – j 9.676)
= 10 3 / (9.71 $\angle$ (-85.20)
= 103 $\angle$(+85.2 $^\circ$)

= 8.62 + j 102.6
1.6 + j 106.32

10.22 + j 106.32

Nth = [(1905 X 103 $\angle$(85.2°) / (10.22 + j 106.32)]
= 1837 V

Zth = [(1.6 + j 3.72) X (8.62 + j 102.6)] / (10.22 + j 106.32)

= [4.05$\angle$ (66.7°) X 103$\angle$(85.2°) / 106.8 (84.5)
= 3.906 $\angle$ (67.4°)
= 1.5 + j3.61

a) Slip at maximum torque

$s = r'_2 / x'_2 = 1.132 / 2.63 = 0.43$

b) $r'_2/s = 1.13 / 0.43 = 2.63\ \Omega$

Z(total) = 4.13 + j 6.24
= 7.48 (56.5°)Ω

I'_2 = 1837 $\angle$(0°) / (7.48 $\angle$(56.5°)) = 245.6 A

N = (1-0.43) X 1000 = 570 rpm

w = 59.7 rad/s
ws = 104.7 rad/s

Tmax (developed) = (3/104.7) X $(245.6)^2$ X 2.63
= 4.546 X 10 3 Nm

Rotational loss = 6.2 kw

T(loss) = 6200 / 59.7 = 104
= 0.104 X 10 3 Nm

T(Shift) = (4.546 – 0.104) X 10 3
= 4.442 X 10 3 Nm

c) Starting torque at half rated voltage

Z (total) = 2.73 + j 6.24
= 6.81 (66.4°) Ω

I'_2 = 1837 / (2 X 6.81) = 135 A

T(start) = 3/104.7 X $(135)_2$ X 1.13
= 590 Nm

9.39 A 6-pole, 50 Hz induction motor has a rotor resistance of 0.25 Ω and a maximum torque of 180 Nm while it runs at 860 rpm. Calculate:

(a) the torque at 4.5% slip

(b) the resistance to be added to the rotor circuit to obtain the maximum torque at starting.

$S_{max,T}$ = (1000 – 800) / 1000 = 0.14

$r'_2 = 0.14x'_2$ for max torque

Tmax = 3/cos $(0.5V^2 / x'_2) = kx'_2$

180 = kx'_2

0.25 = 0.14 x'_2
X'_2 = 1.786 Ω
Now k = $(1.5 / \omega s)V^2$ = 180 / 1.786 = 100.8

(i) s = 0.045

T = $(3/\omega s)$ X $(V^2 / (r'_2/s)^2 + x'_2{}^2)$ X (r'_2/s)

= (100.8 X 2) / [$(0.25/0.045)^2 + (1.786)^2$] X (0.25/0.045)
= 32.9 Nm

(ii) For max torque at starting

r'_2 + Rext = x'_2 = 1.786
Rext = 1.786 – 0.25 = 1.536Ω (referred to stator)

9.40 At rated voltage the blocked rotor current of and induction motor is five times its full-load current and full-load slip is 4%. Estimate its starting torque as a percentage of full-load torque when it is started by means of

(a) a star-delta starter, and
(b) by an autotransformer with 50% tapping.

$S_f = 0.04$, $I_{SC} / I = 5$

(a) Y- Δ => T_{St} / T_f

$= (I_{St} / I_f)^2 S_f$
$= 1/3 \ (I_{SC} / I_f)^2 S_f$
$= 1/3 \ (5)^2 \text{ X } 0.04$
$= 0.33$
=> 33 %

Auto T/X => T_{St} / T_f

$= K^2 \ (I_{SC} / I_f)^2 S_f$
$= 0.5^2 (5)^2 \text{ X } 0.04$
$= 0.25$
= **25 %**

9.41 squirrel-cage induction motor has a full-load slip of 4% and a blocked-rotor current of six times the ful-load current. Find the percentage of tapping of the autotransformer starter to give full-load torque on starting and the line current as a percentage of full-load current.

$I_{sc} / I_{fl} = 6$,

$S_{fl} = 0.04$

$T_s/T_{fl} = x^2 \ (I_{sc} / I_{fl}) \ S_{fl}$

$1 = x^2 \text{ X } 36 \text{ X } 0.04$
X = 0.694 or 69.4%

$I(\text{line}) = x^2.I_{fl}$
$= (0.694)2 \ I_{fl}$
$= 0.482 \ I_{fl}$ or 4.82 of I_{fl}

9.42 A 440 V, 22 kW, 50 Hz, 8-pole induction motor has its rotor and stator winding star-connected. The effective stator to rotor turn ratio is 2.5/1. The parameters of its circuit model are

$\mathbf{R_1 = 0.4\ \Omega}$, $\mathbf{R_2 = 0.07\ \Omega}$
$\mathbf{X_1 = 1.03\ \Omega}$ $\mathbf{X_2 = 0.18\ \Omega}$
$\mathbf{R_i = 127.4\ \Omega}$ $\mathbf{X_m = 25.9\ \Omega}$

Turn ratio, a = 2.4
(includes rotational loss)

Neglecting any changes in mechanical losses due to changes in speed, calculate the added rotor resistance required for the motor to run up to the speed 675 rpm for a constant load torque of 300 Nm. At what speed would the motor run if the added rotor resistance is:

(a) left in the circuit
(b) subsequently shorted out. Also compare the motor efficiency under these two conditions.

$r'_2 = (2.5)^2 \text{ X } 0.07 = 0.4375\Omega$
$r_1 = 0.4$
$r_1 + r'_2 = 0.8375\Omega$

$x_1 + x'_2 = 1.03 + (2.5)^2 \text{ X } 0.18 = 2.155\ \Omega$

R'ext = ?

$V = 440 / \sqrt{3} = 254$ V
$n_s = 750$ rpm
$\omega_s = 78.54$ rad/s
$T = (3/\omega_s) \text{ X } [V^2 / (r_1 + r'_2 + R'_{ext})^2] \text{ X } [(r'_2 + R'_{ext})/1]$

$300 = 3/78.54 \text{ X } (254)^2 / [(0.8375 + R'_{ext})^2 + 4.644] \text{ X } (0.4375 + R'_{ext})$

$(0.837s + R'_{ext})^2 + 4.644 = 8.214\ (0.4375 + R'_{ext})$

0.701 + 1.67s R'ext + R'ext + 4.644 = 3.594 + 8.214 R'ext

R'ext – 6.539 R'ext + 1.751 = 0

R'ext = (6.539 ± 5.98) / 2

= 6.26, 0.28 Ω
$R'ext = 0.28 / (2.9)^2 = 0.045\ \Omega$

(i) Resistance left in circuit

$r'_2 + R'ext = 0.4375 + 0.28 = 0.7175\Omega$

T(load) = 300 Nm

$T(dev) = (3/\omega s) \text{ X } [V^2/ (r_1+r'_2 +R'ext) / s)^2 + (x_1 + x'_2)^2] \text{ X } (r'_2 + R'ext)/s$

$300 = 3/78.54 \text{ X } (254)^2 / (0.4 + (0.7175/s)^2 + 4.644 \text{ X } (0.7175 / s)$

$300 = (3 / 78.54) \text{ X } (254)^2 \text{ X } s^2 / (0.4 s + 0.7175)^2 + 4.644 s^2) \text{ X } 0.7175/s$

$(0.4s + 0.7179)^2 + 4.644s)^2 = 5.894s$

$0.16s^2 + 0.544s + 0.512 + 4.644s^2 = 5.894s$

$4.804 s^2 – 5.32s + 0.512 = 0$

$S^2 – 1.107s + 0.1066 = 0$

$S = (1.107 \pm 0.893) / 2 = 1.0315 . 0.107$

$S = 0.107$, speed = (1-0.107) X 750 = 670 rpm

(ii) External rotor resistance cut out

$300 = (3 / 78.54) \text{ X } (254)^2 / [(0.4 + 0.4375/s)^2 +4.644] \text{ X } 0.4375 / s$

$(0.4s + 0.4375)^2 + 4.644s^2 = 3.594s$

$0.16s^2 + 0.35s + 0.1914 + 4.644s^2 = 3.594s$

$4.304 s^2 – 3.244s + 0.1914 = 0$

$s^2 – 0.675s + 0.04 = 0$

$s = (0.0675 \pm 0.5437) / 2 = 0.0609, 0.0656$

s= 0.0656 ,

speed = (1 – 0.0656) X 750 = 701 rpm

<u>Comparison of efficiencies</u>

(i) $r'_2 + R'ext = 0.4375 + 0.28$
$= 0.7175\Omega$

$S = 0.107$

$I'_2 = 254 / [(0.4 + 6.71) + 21.55]$

$= 34.21$ A

$P_m = 3 \text{ X } (34.21)^2 \text{ X } 0.7175 \text{ X } ((1/0.107) -1)$

$= 21.02$ kw

$P_i = 3 \text{ X } (254)^2/127.4 + 3 \text{ X } (34.21)^2 \text{ X } (0.4 + 6.71) = 26.48$ kw

$\eta = (21.02/26.48) \text{ X } 100 = 79.38$ %

(ii) R'ext cut out

$r'_2 = 0.4375\Omega$

$s = 0.0656$

$I'_2 = 254 / [(0.4 + 6.67) + j\ 2.155]$
$= 34.37$ A (Notice current is the same as in part (ii)

$P_m = 3 \text{ X } (34.37)^2 \text{ X } 0.4375 \text{ X } ((1/0.0656) – 1) = 22.08$ kw

$P_i = 3 \text{ X } (254)^2 / 127.4 + 3 \text{ X } (34.37)^2 \text{ X } (0.4 + 6.67)$
$= 25.565$ kw

$\eta = 22.08 / 25.565 \text{ X } 100 = 86.64$ %

9.43 A 40 kW, 400 V, 3-phase, 6-pole, 50 Hz wound rotor induction motor develops a maximum torque of 2.75 times full-load torque at a slip of 0.18 when operating at rated voltage and frequency with slip rings short-circuited. Stator resistance and rotational losses may be ignored.
Determine:
(a) the full-load slip.
(b) the full-load rotor copper loss.
(c) the starting torque at half the rated voltage. The rotor circuit resistance is now doubled by adding an external resistance through the slip rings. Determine:
(d) the developed torque at full-load current.
(e) the slip in part (d).

Smax, $T = r'_2 / (x1 + x'_2) = 0.18$

Or $(x1 + x'_2) / r'_2 = 5.56$
$T = (3/\omega s) \text{ X } (V^2 / [(r'_2/s)^2 + (x_1 + x'_2)^2] \text{ X } r'_2/s$

$= (3/\omega s) \text{ X } [(sV^2r'_2) / (r'_2{}^2 + (x_1 + x'_2)s^2]$

$T_{fl} = (3/\omega s) \text{ X } (s_{fl} \text{ V2 } r'2) / r'_2{}^2 + (x_1)x'_2)^2 S^2_{fl}$ ---------- (i)

$T\text{max} = (3/\omega s) \text{ X } (0.5V^2 / (x_1 + x'_2)$ ----------------------- (ii)

$Tmax / T_{fl} = 0.5 [r'^2_2 + (x_1 + x'_2) s^2_{fl}] / [(x_1 + x'_2) r'_2 s_{fl}]$ --------- (iii)

$Tmax / T_{fl} = 0.5 [1 + ((x_1 + x'_2)/r'_2)^2 s^2_{fl}] / [((x_1 + x'_2) / r'_2) s_{fl}$

Substituting the value

$2.75 = 0.5(1+ (5.56)^2 s^2_{fl}) / 5.56 s\mu$

$30.58 s_{fl} = 1 + 30.91 s^2_{fl}$

$S_{fl} = 0.989 \pm \sqrt{}(0.978 – 0.129) / 2$

$= 0.989 \pm \sqrt{}(0.978 – 0.129) / 2$

$= 0.989 \pm 0.921 / 2$

$= 0.905, 0.034$

$S_{fl} = 0.034$

(b) $Pm = 3 X [(V^2 s_{fl} r'_2)] / [(r'^2_2 + (x_1 + x'_2)^2 s^2_{fl}]$
$= 3 X (V^2 s_{fl}) / [1+ ((x_1 + x'_2)/r'_2) s_{fl}] X 1/ r'_2$

$40 X 1000 = 3 X [(400 /\sqrt{}3)^2 X 0.034] / [1+ (5.56)^2 X 0.034] X (1/r'_2)$

Or $40 X 1000 = [(400)^2 X 0.034 / 2.051] X (1/r'_2)$

Or $r'_2 = 0.0663\Omega$

$r'_2 / s_{fl} = 0.0663 / 0.034 = 1.95\Omega$

$x_1 + x'_2 = 0.0663 / 0.18 = 0.3684 \Omega$

Rotor copper loss $= 3I'^2_2 r'_2 = 3 X [V^2 / [(r'_2 / s_{fl})^2 + (x_1 + x'_2)^2]] X r'_2$

$= 3 X [(400 / \sqrt{}3)^2 / [(1.95)^2 + (0.3684)^2]] X 0.0663$

$= 2.694$ kw

(c) Ts (half voltage) $= (3/\omega s) X I'^2_2 r'_2$
ns = (120 X 50) / 8 = 750 rpm
ws = 78.54 rad/s

$Ts(voltage) = (3 / 78.54) X [(200 / \sqrt{}3)^2 / (0.0663)^2 + 10.3684)] X 0.0663$

= 241 Nm

(c) $I'_{2\,(fl)} = V/\sqrt{3} / [\sqrt{\{(r'_2 / s_{fl})^2 + (x_1 + x'_2)^2\}}]$

$= (400 / \sqrt{3}) / \sqrt{\{(1.95)^2 + (0.3684)^2\}}$

$= 116.4$ A

Now $R'2(total) = 2r'_2 = 2 \text{ X } 0.0663 = 0.1326\Omega$

For full – load current, total impedance must be same.

Thus

$(1.95)^2 + (0.3684)^2 = ((0.1326/s)^2 + (0.3084)^2)$

Or $0.1326/s = 1.9s$

Or $s = 0.068$

$T(dev) = 3/78.54 \text{ X } (116.4)^2 \text{ X } (0.1326 / 0.068)$

$= 1.95$

$= 1009$ Nm

9.44 Determine the slip at maximum torque and ratio of maximum to full load torque for a 3 phase star connected 6.6kV, 20pole, 50 Hz induction motor has rotor resistance of 0.12 Ω and stand still reactance of 1.12 Ω. The motor speed at full load is 292.5 rpm.

a) $S_{max} = 0.12 / 1.12 = 0.107$

b) $T_{max} = (3/\omega_s) \text{ X } (0.5 V^2 / (x_2^{'})^2)$

$T_{fl} = (3/\omega_s) \text{ X } (V^2 / ((V_2^{'} / S_{fl})^2 + (x_2^{'})^2$

$N_s = 1200 \text{ X } 50 / 20 = 300$ rpm

$N = 292.5$ rpm

$S_{fl} = (300 – 292.5) / 300 = 0.025$

Therefore $T_{max} / T_{fl} = 0.5 [((r_2^1 / S_{fl})^2 + (x_2)^2)] / x_2(r_2 / S_{fl})$

$= 0.5 [(0.12 / 0.025)^2 + (1.12)^2] / 1.12 \text{ X } (0.12 / 0.025)$

$T_{max} / T_{fl} = 0.5 [23.04 + 1.2544] / 5.376$

$= 12.1472 / 5.376$

Therfore $T_{max} / T_{fl} = 2.2595$

CHAPTER 10: FRACTIONAL-KILOWATT MOTORS

10.1 *A 220 V, 50 Hz, 6-pole, single-phase induction motor has the following circuit model parameters:*

$$r_{1m} = 3.6\ \Omega, \quad (x_{1m} + x_2) = 15.6\ \Omega$$
$$r_2 = 6.8\ \Omega, \quad x = 96\ \Omega$$

The rotational losses of the motor are estimated to be 75 W. At a motor speed of 940 rpm, determine the line current, the power factor, the shaft power, and the efficiency.

Solution

$$s = \frac{1,000 - 940}{1000} = 0.06$$

The circuit model is drawn in Fig. P10.1

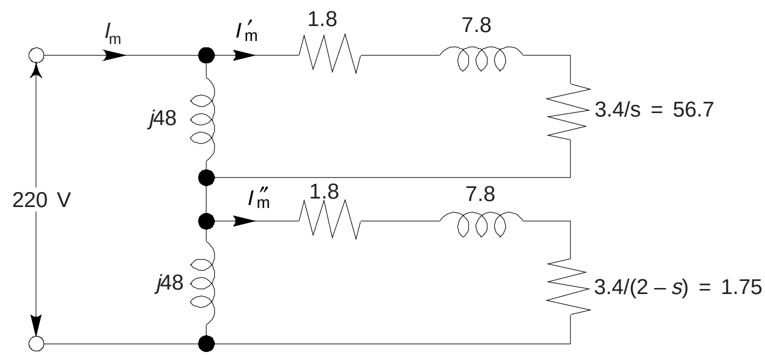


Fig. P10.1

$$\bar{Z}_f\ (\text{total}) = j\ 48 \parallel (1.8 + 56.7 + j\ 7.8) = j\ 48 \parallel (58.5 + j\ 7.8)$$
$$= 35\angle 54° = 20.6 + j\ 28.3$$

$$\bar{Z}_b\ (\text{total}) = j\ 48 \parallel (1.8 + 1.75 + j\ 7.8) = j\ 48 \parallel (3.55 + j\ 7.8)$$
$$= 7.36\angle 69.1° = 2.63 + j\ 6.88$$

$$\bar{Z}\ (\text{total}) = (20.6 + j\ 28.3) + (2.63 + j\ 6.88)$$
$$= 23.23 + j\ 35.18 = 42.16\angle 56.6°$$

$$\bar{I}_m = \frac{220}{42.16\angle 56.6°} = 5.22\angle -56.6°$$

$$I_L = I_m = 5.22\ \text{A}, \quad \text{pf} = \cos 56.6° = 0.55\ \text{lagging}$$

$$I'_m = 5.22\angle -56.6° \times \frac{j48}{58.5 + j\,55.8}$$
$$= 3.1\angle -10°$$

$$I''_m = 5.22\angle -56.6° \times \frac{j48}{3.55 + j\,55.8}$$
$$= 4.48\angle -53°$$

$$n_s = 1,000\ \text{rpm}; \quad \omega_s = 104.7\ \text{rad/sec}$$

$$T = \frac{1}{104.7}\,[(3.1)^2 \times 56.7 - (4.48)^2 \times 1.75]$$
$$= 4.87 \text{ Nm}$$
$$P_m = 104.7\,(1–0.06) \times 4.87 = 479.3 \text{ W}$$
$$P_{out} = 479.3 - 75 = 404.3 \text{ W}$$
$$P_{in} = 220 \times 5.22 \times 0.55 = 631.6 \text{ W}$$
$$= \frac{404.3}{631.6} = 64\%$$

10.2 *A 1/4 kW, 230 V, 50 Hz, 4-pole split-phase motor has the following circuit model parameters:*

$$r_{1m} = 10.1\ \Omega \qquad x_{1m} = 11.6\ \Omega$$
$$r_{1a} = 40.30\ \Omega \qquad x_{1a} = 9.65\ \Omega$$
$$x = 236\ \Omega \qquad a = 0.92\ \Omega$$
$$r_2 = 9.46\ \Omega \qquad x_2 = 6.86\ \Omega$$

Friction, windage and core loss = 45 W

(a) Calculate the starting torque and current of the motor.

(b) Calculate the performance of the motor at a slip of 0.035 (the auxiliary winding is open-circuited).

Solution

(a) $s = 1$

$$\bar{Z}_f = \bar{Z}_b = j\,236 \parallel (9.46 + j\,6.86)$$
$$= 11.34\ \angle 38.1° = 8.92 + j\,7.0$$
$$\frac{\bar{z}_{1a}}{a^2} = \frac{1}{(0.92)^2}\,(40.3 + j\,9.65)$$
$$= 47.6 + j\,11.4$$
$$\bar{Z}_{12} = \frac{1}{2}\,(47.6 + j\,11.4 - 10.1 - j\,11.6)$$
$$= 18.75 - j\,0.1 = 18.75\angle 1°$$
$$\bar{V}_{mf} = \frac{230}{2}\left(1 - \frac{j}{0.92}\right) = 169.8\angle –47.4°$$
$$\bar{V}_{mb} = \frac{230}{2}\left(1 + \frac{j}{0.92}\right) = 169.8\angle 47.4°$$
$$\bar{z}_{1m} + \bar{Z}_f + \bar{Z}_{12} = z_{1m} + Z_b + Z_{12} = 10.1 + j\,11.6$$
$$+ 8.92 + j\,7.0$$
$$+ 18.75 - j\,0.1$$
$$= 37.77 + j\,18.5 = 42.06\angle 26.1°$$

Substituting in Eqs (10.10) and (10.11)

$$\bar{I}_{mf} = \frac{169.8\angle -47.4° \times 42.06\angle 26.1° + 169.8\angle 47.4 \times 18.75\angle 1°}{(42.06)^2\ \angle 52.2° - (18.75)^2\ \angle 2°}$$
$$= \frac{8{,}771\angle 1.4°}{1{,}567\angle 62.2} = 5.6\angle –60.8 = 2.73 - j\,4.89$$

$$\bar{I}_{mb} = \frac{169.8\angle 47.4° \times 42.06\angle 26.1° + 169.8\angle -47.4° \times 18.75\angle 1°}{(42.06)^2 \angle 52.2° - (18.75)^2 \angle 2°}$$

$$= \frac{6,203\angle 47.1°}{1,567\angle 62.2°} = 3.96\angle{-15.1°} = 3.82 - j\ 103$$

$$n_s = 1,500 \text{ rpm}, \qquad \omega_s = 157.1 \text{ rad/s}$$

$$T_s = \frac{2}{157.1} \times 8.92 \times [(5.6)^2 - (3.96)^2]$$

$$= 1.78 \text{ Nm}$$

$$\bar{I}_m = \bar{I}_{mf} + \bar{I}_{mb} = 2.73 - j\ 4.89 + 3.82 - j\ 1.03 = 6.55 - j\ 5.92$$

$$I_a = \frac{j}{a}\left(\bar{I}_{mf} - \bar{I}_{mb}\right) = \frac{j}{0.92}\ (2.73 - j\ 4.89 - 3.82 + j\ 1.03)$$

$$= 4.20 - j\ 1.18$$

$$\bar{I}_L = \bar{I}_m + \bar{I}_a = 6.55 - j\ 5.92$$

$$+ 4.20 - j\ 1.18$$

$$= 10.75 - j\ 7.10 = 12.88\angle{-33.4°}$$

$$I_L(\text{start}) = 12.88 \text{ A}$$

(b) With reference to Fig. P10.2

$$\bar{Z}_f = j\ 236 \parallel \left(\frac{9.46}{0.035} + j\ 6.86\right)$$

$$= j\ 236 \parallel (270.3 + j\ 6.86) = 175.7\angle 49.6° = 113.9 + j\ 133.8$$

$$\bar{Z}_b = j\ 236 \parallel \left(\frac{9.46}{1.965} + j\ 6.86\right)$$

$$= j\ 236 \parallel (4.81 + j\ 6.86) = 8.14\angle 56.1° = 4.54 + j\ 6.76$$

$$\bar{Z}(\text{total}) = \bar{z}_{1m} + \frac{1}{2}\left(\bar{Z}_f + \bar{Z}_b\right)$$

$$= (10.1 + j\ 11.6) + \frac{1}{2}\ [(113.9 + j\ 133.8) + (4.54 + j\ 6.76)]$$

$$= 69.32 + j\ 81.88 = 107.3\angle 49.7°$$

$$\bar{I}_L = \bar{I}_m = \frac{V_m = V_L}{\bar{z}(\text{total})} = \frac{230}{107.3\angle 49.7°}$$

$$= 2.14\angle{-49.7°}$$

$$I_L = 2.14 \text{ A} \qquad \text{pf} = 0.647 \text{ lagging}$$

From Eq. (10.4),

$$T = \frac{I_m^2}{2\omega_s}\ (R_f - R_b)$$

$$= \frac{(2.14)^2}{2 \times 157.1}\ (113.9 - 4.54) = 1.59 \text{ Nm}$$

$$P_m = (1 - s)\ \omega_s T$$
$$= (1 - 0.035) \times 157.1 \times 1.59 = 241 \text{ W}$$
$$P_{in} = 230 \times 2.14 \times 0.647 = 318.5 \text{ W}$$
$$\eta = \frac{241}{318.5} = 75.67\%$$

10.3 *A 400 W, 220 V, 50 Hz, 6-pole, permanent capacitor motor has the following circuit model parameters:*

$r_{1m} = 9.2\ \Omega$ $\qquad x_{1m} = 8.7\ \Omega$
$r_{1a} = 15.5\ \Omega$ $\qquad x_{1a} = 13.5\ \Omega$
$x = 138.5\ \Omega$
$\bar{Z}_C = -j\ 257\ \Omega$ *(series capacitive reactance in auxiliary winding)*
$a = 1.25$
$r_2 = 14.3\ \Omega$
$x_2 = 6.84\ \Omega$

The windage friction and core loss is 45 W.
(a) Calculate starting torque and current
(b) Calculate motor performance at s = 0.1.

Solution

(a) $s = 1$ $\qquad$ With reference to Figs 10.3(a), (b) and (c)

$$\bar{Z}_f = \bar{Z}_b = j\ 138.5 \parallel (14.3 + j\ 6.84)$$
$$= 15.03\angle 31.2°$$
$$= 12.86 + j\ 7.79\ \Omega$$
$$\bar{z}_{1m} = 9.2 + j\ 8.7$$
$$\bar{z}_{1a} = 15.5 + j\ 13.5 - j\ 257$$
$$= 15.5 - j\ 243.5$$
$$a^2 = (1.25)^2 = 1.5625$$
$$\frac{\bar{z}_{1a}}{a^2} = 9.92 - j\ 155.9$$
$$\bar{Z}_{12} = \frac{1}{2}\left(\frac{\bar{z}_{1a}}{a^2} - \bar{z}_{1m}\right) + \frac{1}{2}\ (9.92 - j\ 155.9 - 9.2 - j\ 8.7)$$
$$= 0.36 - j\ 82.3 = 82.3\angle -89.7°$$
$$V_{mf} = \frac{1}{2}\left(\bar{V}_m - j\ \frac{\bar{V}_a}{a/V}\right)$$
$$\bar{V}_{mb} = \frac{1}{2}\left(\bar{V}_m + j\ \frac{a}{a}\right)$$
$$\bar{V}_m = \bar{V}_a = 220\angle 0°\text{ V}$$
$$\bar{V}_{mf} = \frac{1}{2} \times 220\ (1 - j\ 0.8) = 140.9\angle -38.6°$$
$$\bar{V}_{mb} = \frac{1}{2} \times 220\ (1 + j\ 0.8) = 140.9\angle 38.6°$$

$$\bar{z}_{1m} + \bar{Z}_f + \bar{Z}_{12} = 9.2 + j\ 87$$

$$+ 12.86 + j\ 7.79$$

$$+ 0.36 - j\ 82.3$$

$$= 22.42 - j\ 65.81 = 69.52\angle{-71.2^\circ}$$

$$\bar{z}_{1m} + \bar{Z}_b + \bar{Z}_{12} = 69.52\angle{-71.2^\circ}$$

$$\bar{I}_{mb} = \frac{\bar{V}_{mb}\left(\bar{z}_{1m} + \bar{Z}_f + \bar{Z}_{12}\right) + \bar{V}_{mf}\bar{Z}_{12}}{\left(\bar{z}_{1m} + \bar{Z}_f + \bar{Z}_{12}\right)\left(\bar{z}_{1m} + \bar{Z}_b + \bar{Z}_{12}\right) - \bar{Z}_{12}^2} = \frac{\bar{N}_2}{\bar{D}}$$

$$\bar{D} = (69.52^2)\angle{-142.4^\circ} = -3829 - j\ 2949$$

$$\bar{Z}_{12}^2 = (82.3)^2\angle{-179.4^\circ} = -6773 - j\ 71$$

$$\bar{D} - \bar{Z}_{12}^2 = 2944 - j\ 2878 = 4{,}117\angle{-44.4^\circ}$$

$$\bar{V}_{mb}\left(\bar{z}_{1m} + \bar{Z}_f + \bar{Z}_{12}\right) = 140.9\angle 38.6^\circ \times 69.52\angle{-71.2^\circ}$$

$$= 9795\angle{-32.6^\circ} = 8252 - j\ 5277$$

$$\bar{V}_{mf}\bar{Z}_{12} = 140.9\angle{-38.6^\circ} \times 82.3\angle{-89.7^\circ}$$

$$= 11596\angle{-128.3^\circ} = -7187 - j\ 9100$$

$$N_2 = 1065 - j\ 14{,}377 = 14{,}416\angle{-85.8^\circ}$$

$$\bar{I}_{mb} = \frac{\bar{N}_2}{\bar{D}} = \frac{14{,}416\angle -85.8^\circ}{4{,}177\angle -44.4^\circ}$$

$$= 3.45\angle{-41.4^\circ} = 2.59 - j\ 2.28$$

$$\bar{I}_{mf} = \frac{\bar{V}_{mf}\left(\bar{z}_{1m} + \bar{Z}_b + \bar{Z}_{12}\right) + \bar{V}_{mb}\bar{Z}_{12}}{\bar{D}} = \frac{\bar{N}_1}{\bar{D}}$$

$$\bar{V}_{mf}\left(\bar{z}_{1m} + \bar{Z}_b + \bar{Z}_{12}\right) = 140.9\angle{-38.6^\circ} \times 69.52\angle{-71.2^\circ}$$

$$= 9{,}795\angle{-109.8^\circ} = -3{,}318 - j\ 9{,}216$$

$$\bar{V}_{mb}\bar{Z}_{12} = 140.9\angle 38.6^\circ \times 82.3\angle{-89.7^\circ}$$

$$= 11{,}596\angle{-51.1^\circ} = 7{,}282 - j\ 9025$$

$$\bar{N}_1 = 3{,}964 - j\ 18{,}241 = 18{,}667\angle{-77.7^\circ}$$

$$\bar{I}_{mf} = \frac{18{,}667\angle{-77.7}}{4{,}117\angle{-44.4^\circ}} = 4.53\angle{-33.3^\circ}\ \text{A}$$

$$= 3.79 - j\ 2.49$$

$$\bar{I}_m = \bar{I}_{mf} + \bar{I}_{mb} = 3.79 - j\ 2.49$$

$$+ 2.59 - j\ 2.28$$

$$= 6.38 - j\ 4.77$$

$$\bar{I}_a = \frac{j}{a}\left(\bar{I}_{mf} + \bar{I}_{mb}\right) = \frac{j}{1.25}\ (3.79 - j\ 2.49 - 2.59 + j\ 2.28)$$

$$= 0.168 + j\ 0.96$$

$$\bar{I}\,(\text{line}) = \bar{I}_m + \bar{I}_a = 6.55 - j\,3.81 = 7.58\angle{-30.2}°\text{A}$$

$$n_s = 1{,}000 \text{ rpm}, \qquad \omega_s = \frac{2\pi \times 1{,}000}{60} = 104.7 \text{ rad/s}$$

$$T_s = \frac{2}{\omega_s}\left(I_{mf}^2 R_f - I_{mb}^2 R_b\right)$$

$$= \frac{2}{104.7}\,[(4.53)^2 \times 22.4 - (3.45)^2 \times 22.4]$$

$$= \frac{2 \times 22.4}{104.7} \times 8.62 = 3.69 \text{ Nm}$$

(b) $s = 0.1$

$$\frac{r_2}{s} = \frac{14.3}{0.1} = 143\ \Omega; \qquad \frac{r_2}{(2-s)} = \frac{14.3}{1.9} = 7.53$$

$$\bar{Z}_f = j\,138.5 \parallel (143 + j\,6.84); \quad \bar{Z}_b = j\,138.5 \parallel (7.53 + j\,6.84)$$

$$= \frac{138.5\angle 90° \times 143.2\angle 2.7°}{203.8\angle 45.4°}; \quad \frac{138.5\angle 90° \times 10.17\angle 42.3°}{(7.53 + j\,145.34) = 145.5\angle 87°}$$

$$= 97.29\angle 47.3°; \quad = 9.68\angle 45.3°$$

$$\bar{Z}_f = 65.98 + j\,71.5; \quad \bar{Z}_b = 6.81 + j\,6.88$$

$$\begin{aligned}\bar{z}_{1m} + \bar{Z}_f + \bar{Z}_{12} &= 9.2 + j\,8.7 \\ &\quad + 65.98 + j\,71.5 \\ &\quad + 0.36 - j\,82.3 \\ &= 75.54 - j\,2.1 \\ &= 75.57\angle{-16}°\end{aligned}$$

$$\begin{aligned}\bar{z}_{1m} + \bar{Z}_b + \bar{Z}_{12} &= 9.2 + j\,8.7 \\ &\quad + 6.81 + j\,6.88 \\ &\quad + 0.36 - j\,82.3 \\ &= 16.37 - j\,66.72 \\ &= 68.70\angle{-76.2}°\end{aligned}$$

$$\bar{V}_{mf} = 140.9\angle{-38.6}°; \quad \bar{V}_{mb} = 140.9\angle 38.6°$$

$$\bar{Z}_{12} = 82.3\angle{-89.7}°$$

$$\bar{N}_1 = 140.9\angle{-38.6}° \times 68.70\angle{-76.2}° = 9680\angle{-114.8}° = -4{,}060 - j\,8787$$

$$140.9\angle 38.6° \times 82.3\angle{-89.7}° = 11{,}596\angle{-51.1} = \frac{7{,}282 - j\,9{,}025}{3222 - j\,17812 = 18101\angle{-79}°}$$

$$\bar{D} = 75.57\angle{-16}° \times 68.70\angle{-76.2}° = 5192\angle{-78}° = 1{,}080 - j\,5{,}079$$

$$\bar{Z}_{12}^2 = (82.3)^2\angle{-17.9}°.4° = -6{,}773 - j\,71$$

$$\bar{D} - \bar{Z}_{12}^2 = 7{,}853 - j\,5{,}008 = 9314\angle{-32.5}°$$

$$\bar{I}_{mf} = \frac{18{,}101\angle{-79.8°}}{9{,}314\angle{-32.5°}} = 194\angle{-47.3°}$$

$$= 1.32 - j\ 1.42$$

$$\bar{N}_2 = 140.9\angle 38.6° \times 75.57\angle{-1.6°} = 10{,}648\angle 37.0° = 8{,}504 + j\ 6{,}408$$

$$140.9\angle{-38.6°} \times 82.3\angle{-89.7°} = 11{,}596\angle{-128.3°} = -7187 - j\ 9{,}100$$

$$= 1{,}317 - j\ 2{,}692$$

$$= 2{,}998\angle{-63.9°}$$

$$\bar{I}_{mb} = \frac{2{,}998\angle{-63.9°}}{9{,}314\angle{-32.5°}} = 0.322\angle{-31.4°}$$

$$= 0.275 - j\ 0.167$$

$$\bar{I}_m = \bar{I}_{mf} + \bar{I}_{mb} = 1.32 - j\ 1.42$$

$$= 0.275 - j\ 0.167$$

$$= 1.6 - j\ 1.59 = 2.25\angle{-44.8°}\ \text{A}$$

$$\bar{I}_a = \frac{j}{a}\left(\bar{I}_{mf} - \bar{I}_{mb}\right) = \frac{j}{1.25}\ (1.32 - j\ 1.42 - 0.28 + j\ 0.17)$$

$$= 1 + j\ 0.83$$

$$\bar{I}_L = \bar{I}_m + \bar{I}_a = 1.69 - j\ 1.59 + 1 + j\ 0.83$$

$$= 2.67 - 0.76 = 2.78\angle{-15.9°}\ \text{A}$$

$$I_L = 2.78\ \text{A} \quad \text{pf} = 0.96 \text{ lagging}$$

$$\text{Power input} = 220 \times 2.79 \times 0.96$$

$$= 589\ \text{W}$$

$$\text{Torque in syn watts} = 2\ \left(I_{mf}^2 R_f - I_{mb}^2 R_b\right)$$

$$= 2[(1.94)^2 \times 65.98 - (0.322)^2 \times 6.81]$$

$$= 495.2\ \text{W}$$

$$\text{Mech. power developed} = 495.2\ (1 - 0.1) = 445.7\ \text{W}$$

$$\text{Mech. power output} = 445.7 - 45 = 400.7\ \text{W}$$

$$\eta = \frac{4000}{589} = 68.0\%$$

$$\text{Torque output} = \frac{400.7}{104.7} = 3.83\ \text{Nm}$$

10.4 *Show that if the stator voltages of a 2-phase induction motor are V_m and V_a with a fixed phase difference of 90°, the starting torque is the same as for a balanced voltage of $\sqrt{V_m V_a}$ per phase.*

Solution

$$\bar{Z}_f(\text{total}) = (r_1 + jx_1)\ \bar{Z}_f$$

$$\bar{Z}_b(\text{total}) = (r_1 + jx_1)\ \bar{Z}_b$$

At $s = 1$

$$\bar{Z}_f = \bar{Z}_b = \bar{Z} + (R + j\ X)$$

$$\bar{Z}_f(\text{total}) = \bar{Z}_b(\text{total}) = \bar{Z}(\text{total})$$

$$V_{mf} = \frac{V_m - V_a}{2} \quad V_{mb} = \frac{V_m - V_a}{2}$$

$$T_f = \frac{2}{\omega_s}\left(\frac{V_m + V_a}{2Z(\text{total})}\right)^2 R$$

$$T_b = \frac{2}{\omega_s}\left(\frac{V_m - V_a}{2Z(\text{total})}\right)^2 R$$

Hence

$$T_s = T_f - T_b = \frac{2}{\omega_s}\frac{R}{4Z^2(\text{total})}\left[(V_m + V_a)^2 - (V_m - V_a)^2\right]$$

$$= \frac{2}{\omega_s}\frac{R}{Z^2(\text{total})}\left(\sqrt{V_m V_a}\right)^2$$

Hence, balanced voltage for the same starting torque is

$$V = \sqrt{V_m V_a}$$

10.5 *For a 2-phase servo motor (with high resistance rotor), find approximate expressions for forward and backward torques in terms of phase voltages (differing 90° in phase) and motor speed. Assume stator impedance and rotor reactance to be negligible.*

Solution

$$V_{mf} = \frac{V_m + V_a}{2}; \qquad V_{mb} = \frac{V_m - V_a}{2}$$

$$Z_f = \frac{r_2'}{s} \quad Z_b = \frac{r_2'}{R - s}$$

$$T_f = \frac{2}{\omega_s}\left(\frac{V_m + V_a}{2r_2/s}\right)^2\left(\frac{r_2'}{s}\right) = \frac{2}{\omega_s r_2'}\left(\frac{V_m + V_a}{2}\right)^2 s$$

$$T_b = \frac{2}{\omega_s}\left(\frac{V_m - V_a}{2r_2'/(2 - s)}\right)^2\left(\frac{r_2'}{2 - s}\right) = \frac{2}{\omega_s r_2'}\left(\frac{V_m - V_a}{2}\right)^2 (2 - s)$$

$$s = 1 - \frac{\omega_o}{\omega_s} \quad \omega_o = \text{motor speed} \quad \omega_s = \text{synchronous speed}$$

Hence

$$T_f = k\left(\frac{V_m - V_a}{2}\right)^2\left(1 - \frac{\omega_o}{\omega_s}\right)$$

$$T_b = k\left(\frac{V_m - V_a}{2}\right)^2\left(1 + \frac{\omega_o}{\omega_s}\right)$$

10.6 *Show that in a 2-phase tachometer with high resistance rotor, the voltage induced on the open-circuited phase (a) is proportional to rotor speed and leads the other phase (m) voltage by 90°. Neglect the stator impedance and rotor reactance.*

Solution

$$\bar{I}_{\text{mf}} = \bar{I}_{\text{mf}} \text{ (magnetizing current is left out)}$$

$$\therefore \qquad \frac{\bar{V}_{\text{mf}}}{\bar{V}_{\text{mb}}} = \frac{r_2'/s}{r_2'/(2-s)} = \frac{2-s}{s}$$

$$\bar{V}_{\text{mf}} + \bar{V}_{\text{mb}} = \bar{V}_{\text{m}} \angle 0°$$

$$\bar{V}_{\text{mb}}\left(1+\frac{2-s}{s}\right) = V_{\text{m}}$$

or

$$\bar{V}_{\text{mb}} = \frac{V_{\text{m}}\, s}{2}$$

$$\bar{V}_{\text{mf}} = \frac{2-s}{s}\,\frac{V_{\text{m}} s}{2} = V_{\text{m}}\left(1-\frac{s}{2}\right)$$

$$\bar{V}_{\text{a}} = j\left(\bar{V}_{\text{mf}} - \bar{V}_{\text{mb}}\right)$$

$$= jV_{\text{m}}(1-s)$$

$$= j\ V_{\text{m}}\,\frac{\omega_{\text{o}}}{\omega_{\text{s}}} = jk\omega_{\text{o}}$$

CHAPTER 12: MOTOR CONTROL BY STATIC POWER CONVERTERS

12.1 *A separately excited dc motor is fed from a 230 V, 50 Hz source via a single-phase full-converter. The motor armature resistance and inductance are respectively 2 Ω and 0.05 mH. The motor's torque constant is 1 Nm/A and the voltage constant is 1 V rad/s.*

If the current through the armature flows for 160° after the commencement of current flow at α = 60° (causing discontinuous current operation), calculate the average voltage across the load, the average current through the load and the torque developed. Assume ω = 150 rad/s.

Solution

The voltage waveform at motor terminals fed from a single-phase full-converter is drawn in Fig. P12.1.

$$E_a = 150 \times 1.0 = 150 \text{ V (motor induced emf)}$$

$$V_a = \frac{1}{\pi}\left[\int_{60°}^{160°+60°} 230\sqrt{2}\sin \omega t \, d(\omega t) + \frac{150\,(180-160)}{180} \times \pi\right]$$

$$= 130.65 \text{ V (average motor terminal voltage)}$$

$$I_a = \frac{130.6}{2} = 65.325 \text{ A}$$

$$T_a = 65.325 \times 1$$

$$= 65.325 \text{ Nm}$$

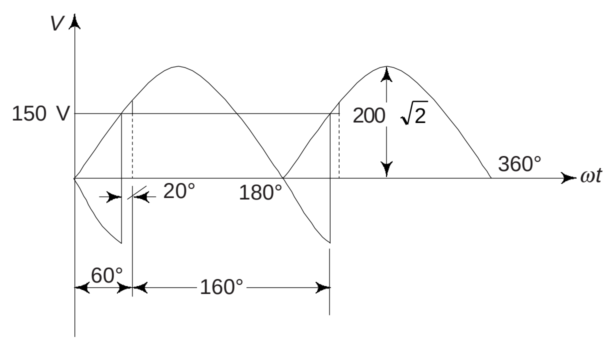


Fig. P12.1

12.3 *Figure 12.3 shows a three-phase full-converter feeding a separately excited dc motor. The thyristors are fired at intervals of 60° (Th_1, Th_6, Th_2, Th_4, Th_3, Th_5). Draw the voltage and current waveforms for the firing angle α = 60°.*

Derive an expression for the average motor terminal voltage as function of α.

The converter is fed from a 400 V, 50 Hz, supply. The rated motor armature current, I_a = 50 A and R_a = 0.1 Ω, L_a = 6.5 mH and $K_a\,\Phi$ = 0.3 V/rpm. Calculate the no-load speed for α = 30° assuming the no-load current to be 5 A.

Also calculate the value of α to obtain a speed of 1,600 rpm at rated current.

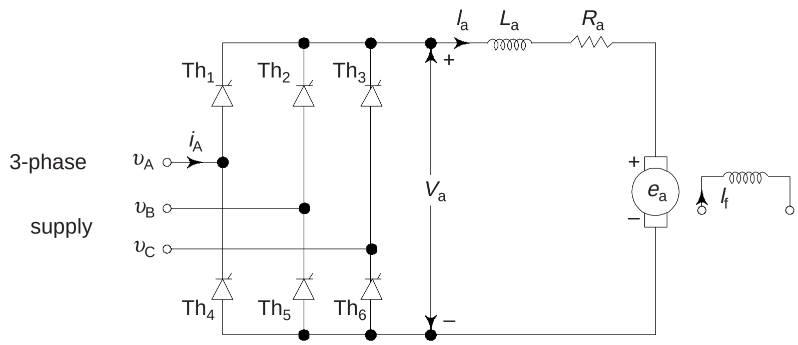


Fig. P12.3

Solution

With reference to the waveform of Fig. P12.3(a), firing instants of thyristors and output voltage waveform (V_a)

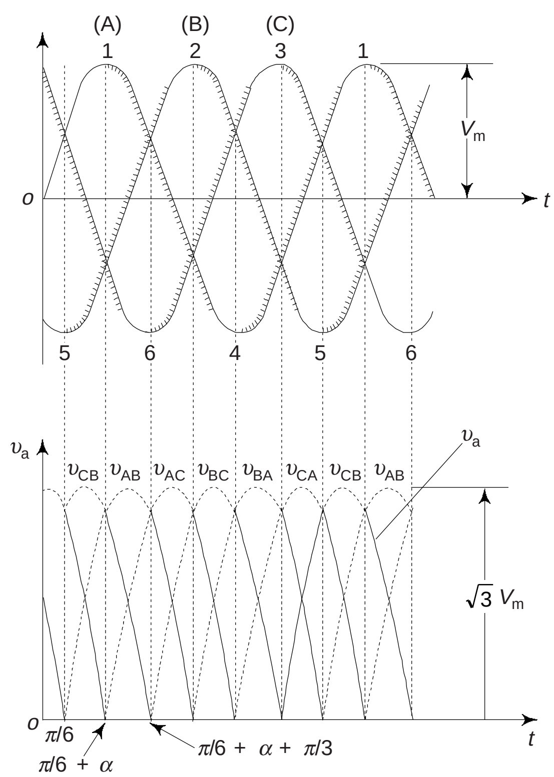


Fig. P12.3(a)

$$V_a(\alpha) = \frac{3}{\pi}\int_{\pi/6+\alpha}^{\pi/6+\alpha+\pi/3} (v_a - v_B)d\ (\omega t)$$

$$= \frac{3\sqrt{6}V}{\pi} \cos \alpha \qquad (V = \text{rms value of phase voltage})$$

Given $\alpha = 30°$

$$V_a = \frac{3 \times \sqrt{6} \times (400/\sqrt{3})}{\pi} \cos 30°$$

$$= 467.82 \text{ V}$$

$$E_a = 467.82 - 5 \times 0.1 = 467.32 \text{ V}$$

$$\text{No-load speed} = \frac{467.32}{0.3} = 1{,}558 \text{ rpm}$$

For speed of 1,500 rpm,

$$E_a = 1{,}600 \times 0.3 = 480 \text{ V}$$
$$V_a = 480 + 50 \times 0.1 = 485 \text{ V}$$

Now

$$485 = \frac{3\sqrt{6} \times 400/\sqrt{3}}{\pi} \cos \alpha$$

or $\alpha = 26.1°$

12.2 *A chopper circuit as shown in Fig. P12.2 is inserted between a battery, V_{dc} = 100 V and a load of resistance R_L = 10 Ω. The turn-off time for the main thyristor Th_1 is 100 μs and the maximum permissible current through it is 25 A. Calculate the value of the commutating components L and C.*

Hint: (see page 837)

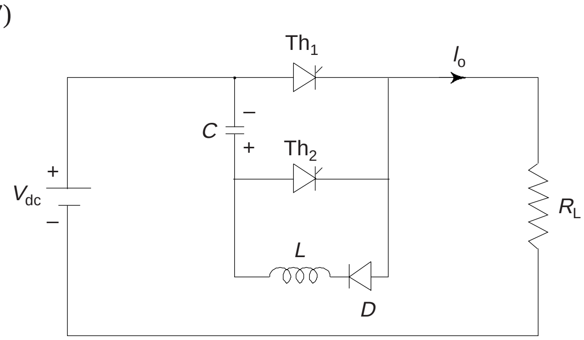


Fig. P12.2

Solution

With Th_1 in conducting state as Th_2 is fired, the capacitor discharges via R_L and battery source, while Th_1 behaves as an open circuit to C as it is conducting in the forward direction. The voltage of C which is also the voltage across Th_1 changes according to the following equation.

$$V_{Th_1} = V_C = V_{dc} \left(\frac{1 - 2e^{-t}}{R_L C}\right) \qquad \text{(i)}$$

For turn off in 100 μs, V_{Th1} must reduce to zero at the end of this period, i.e.

$$V_{Th_1} = 0 = V_{dc} \,(1 - 2e^{-100 \times 10^{-6}/10C}) \qquad \text{(ii)}$$

which gives $C = 14.4\ \mu\text{F}$

With Th_1 turned off the capacitor voltage keeps changing till it is reverse charged as $-V_{dc}$. As Th_1 is fired once again, diode D is positively biased by the capacitor and begins conducting so that the LC resonant circuit oscillates through Th_1. The maximum value of the oscillatory current is given as

$$i_C(\max) = V_{dc}\ C\omega = V_{dc}\ C/\sqrt{LC} = V_{dc}\sqrt{C/L} \tag{iii}$$

Hence the maximum current through Th_1 is

$$i_{\text{Th}_1}(\max) = I_0 + V_{dc}\sqrt{C/L} \tag{iv}$$

or
$$25 = \frac{V_{dc}}{R_L} + V_{dc}\sqrt{C/L}$$

Substituting values, we get

$$L = 640\ \mu\text{H}$$

12.4 *A 4 pole, 3-phase, 400 V, 50 Hz, star-connected induction motor is fed from an inverter such that this phase voltage is a six-step waveform. The motor speed is controlled by maintaining V/f constant at a value corresponding to the rated voltage and frequency.*

(a) Determine the expression for the fundamental and harmonics of the inverter output voltage waveform.

(b) Calculate the dc input voltage required to feed the inverter for operating the motor at 60 Hz, 50 Hz and 40 Hz.

(c) Calculate the firing angles if the dc input voltage to the inverter is obtained from a 3-phase semi-converter fed from a 500 V (line-to-line), 50 Hz source while the inverter output corresponds to 60 Hz.

Solution

The output voltage waveform is drawn in Fig. P12.4.

(a)
$$\alpha_n = \frac{4}{\pi}\left[\int_0^{\pi/3}\frac{1}{3}V_{dc}\sin n\omega t\, d(\omega t) + \int_{\pi/3}^{\pi/2}\frac{2}{3}V_{dc}\sin n\omega t\, d(\omega t)\right]$$

$$= \frac{4V_{dc}}{3\pi}\left[\left\{-\frac{\cos n\omega t}{n}\right\}_0^{\pi/3} + 2\left\{-\frac{\cos n\omega t}{n}\right\}_{\pi/3}^{\pi/2}\right]$$

$$= \frac{4V_{dc}}{3n\pi}\left(1+\cos\frac{n\pi}{3}\right)$$

The fundamental is given by

$$V_{01} = \frac{4V_{dc}}{3\pi}\left(1+\cos\frac{\pi}{3}\right)\sin\omega t$$

$$= \frac{2V_{dc}}{\pi}\sin\ \omega t$$

$$V_{01} = \frac{\sqrt{2}}{\pi}\ V_{dc}\ \text{(rms phase)}$$

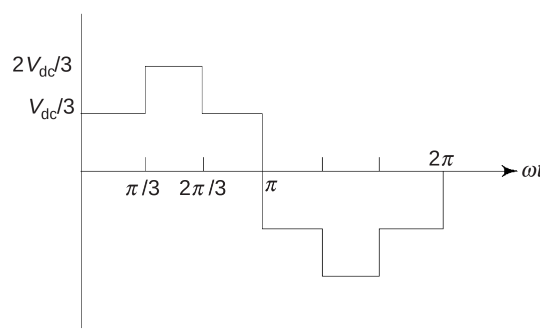


Fig. P12.4

(b) For 60 Hz output,

$$V_0 = 400 \times \frac{60}{50} = 480\text{V (line)}$$

$$\frac{480}{\sqrt{3}} = \frac{\sqrt{2}}{\pi} V_{dc}$$

or $\quad V_{dc} = 615.6$ V

For 50 Hz output,

$$\frac{400}{\sqrt{3}} = \frac{\sqrt{2}}{\pi} V_{dc}$$

or $\quad V_{dc} = 513$ V

For 40 Hz output,

$$V = 400 \times \frac{40}{50} = 320 \text{ V}$$

$$\frac{320}{\sqrt{3}} = \frac{\sqrt{2}}{\pi} V_{dc}$$

or $\quad V_{dc} = 410.4$ V

(c) For a 3-phase semi-converter

$$V_{dc} = \frac{3\sqrt{6}V}{2\pi} (1 + \cos \alpha)$$

or
$$513 = \frac{3\sqrt{6} \times (500/\sqrt{3})}{2\pi} (1 + \cos \alpha)$$

$$1 + \cos \alpha = 1.52$$

$$\alpha = 58.7°$$

12.5 *A single-phase bridge inverter of Fig. P12.5(a) with the quasi-square wave of Fig. 11.5(b) as output (with an on-period of 5 ms) feeds a load of R = 8 Ω in series with L = 0.044 H from a 200 V dc source. The output frequency of the inverter is 50 Hz. Determine:*

(a) The load current waveform for the first two half-cycles. Also find expressions for the steady-state current.

(b) The expression for the fundamental component of the load current from the general expression describing the harmonic content of the output voltage waveform.

Remark: This output waveform of Fig. P12.5(b) can be obtained from the bridge inverter circuit of Fig. 11.5(a) by the firing sequence

$$Th_1\ Th_3,\ Th_1,\ Th_4,\ Th_2\ Th_4,\ Th_2\ Th_3,$$

also,

$$V_0 = \frac{4V}{n\pi} \sin \frac{n\delta}{2} \sin n\,\omega t$$

Solution

$$f_o = 50 \text{ Hz or } 314 \text{ rad/s}$$

$$\text{Period} = 20 \text{ m}$$

$$t_{ON} = 5 \text{ m}$$

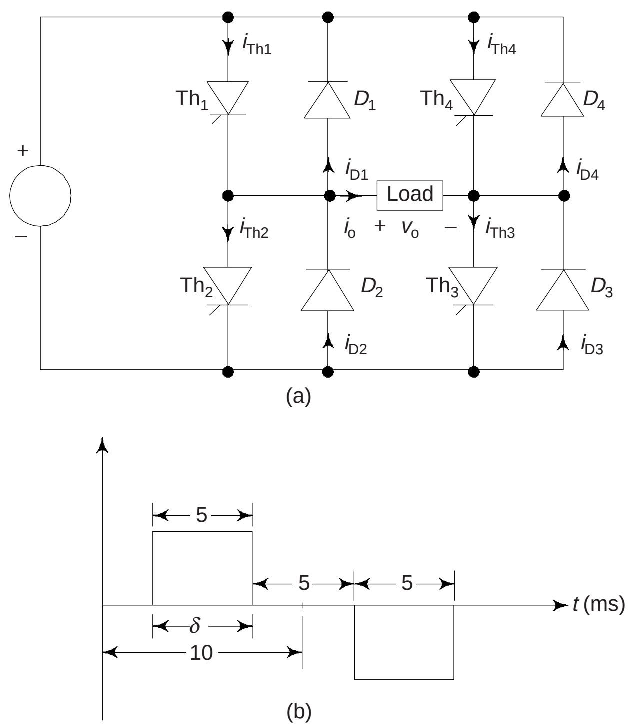


Fig. P12.5

$$\tau = \frac{L}{R} = \frac{0.044}{8} = 5.5 \text{ m}$$

Period 1

$$i(1) = \frac{200}{8}\left(1 - e^{-t/\tau}\right) = 25\left(1 - e^{-t/\tau}\right)$$
$$= 14.93 \text{ A at the end of period 1}$$

Period 2

$$i(2) = 14.93e^{-t/\tau}$$
$$= 6.01 \text{ A at the end of period 2}$$

Period 3

$$V_o = -200 \text{ V}$$
$$i(3) = -25 + (25 + 6.01)e^{-t/\tau}$$
$$= -12.47 \text{ A at the end of period 3}$$

Steady State Current

The steady-state current waveform is drawn in Fig. P12.5(a). It follows from this figure that

$$[(25 - (25 + I_{01})e^{-t/\tau)}|_{t = 5\text{ms}}]e^{-t/\tau}|_{t=5\text{ms}} = I_{01}$$
$$[(25 - (25 + I_{01}) \times 0.403] \times 0.403 = l_{01}$$
$$25 - 10.08 - 0.403I_{01} = 2.48I_{01}$$
$$I_{01} = 5.18 \text{ A}$$

(b) $\delta = 314 \times 5 \times 10^{-3} = 1.57$ rad

$$V_{01} = \frac{4 \times 200}{1 \times \pi} \sin \frac{1 \times 157}{2} \sin \omega t$$

$$= 180 \sin 314\ t$$

$$Z = \sqrt{R^2 + \omega^2 L^2} = 16$$

$$\phi = \tan^{-1} \frac{\omega L}{R} = 60°$$

$$\therefore \quad i_o = 11.25 \sin (314\ t - 60°)$$

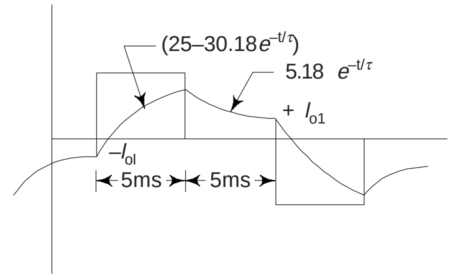


Fig. P12.5(a)